

# IDENTIFICATION OF INTRA-BUNCH DYNAMICS USING CERN SPS MACHINE MEASUREMENTS\*

Ozhan Turgut<sup>†</sup>, John Cesaratto, John Dusatko, John D. Fox  
Kristin Pollock, Claudio Rivetta, SLAC, Menlo Park 94025, USA

## Abstract

Modern control techniques can be used to design feedback systems for stabilizing the intra-bunch dynamics in the presence of electron cloud (ECI) and transverse mode coupling (TMCI) instabilities. These techniques require reduced models of the bunch dynamics. We present a methodology to identify reduced order linear models representing single bunch dynamics using CERN SPS machine measurements. Vertical motion, in response to a wideband excitation signal, is sampled multiple times across the 5 ns bunch. The data and an observable canonical structure [1] are used to identify the dynamics, which is represented as a discrete time multi-input multi-output (MIMO) system. We focus on mode 0 (barycentric) and mode 1 (head-tail) data to identify a reduced order model. Results show that models clearly capture dominant dynamics and replicate machine measurements with corresponding central tune, damping value for each mode and correct separation between the modes.

## INTRODUCTION

Electron cloud and machine impedance can cause intra-bunch instabilities [2]. Modern control techniques can be used to mitigate these problems but require reduced order models of intra-bunch dynamics to design optimal and robust controllers for wideband feedback systems [3]. We use system identification techniques to estimate parameters of linear models representing single bunch dynamics. We briefly explain the reduced order model and identification method. Experimental data was collected from a single bunch with  $1 \times 10^{11}$  protons at 26 GeV with low chromaticity configuration at CERN SPS. This paper includes results from mode 0 and mode 1 dynamics analysis but the methods are in principle applicable to  $N$  modes.

## MODEL AND IDENTIFICATION

The physical system analyzed in this study is a nanosecond scale SPS bunch. The interaction with the bunch is done via control variables (momentum kick / driving signal) and measured variables (vertical displacement). The control variables and measured variables are discretized to represent the physical system in a MIMO form. The number of samples is arbitrary and depends on the sampling rate of the data acquisition system. These studies use 3.2

GS/s sampling rate allowing us to sample 16 different locations across 5 ns RF bucket [3]. The goal is to estimate the intra-bunch dynamics using these input and output signals.

## Reduced Order Model

Any linear dynamical system can be represented in state space matrix form. A discrete time system sampled at every revolution period  $k$  with  $p$  inputs and  $q$  outputs is:

$$\begin{aligned} X_{k+1} &= AX_k + BU_k \\ Y_k &= CX_k \end{aligned} \quad (1)$$

where control variable (external excitation)  $U \in R^p$ , vertical displacement measurement  $Y \in R^q$ , system matrix  $A \in R^{n \times n}$ , input matrix  $B \in R^{n \times p}$ , and output matrix  $C \in R^{q \times n}$ . For a MIMO system, the model order determines the complexity.

## Identification

Identification of a linear dynamical system is done by casting this problem into a linear least squares form. Assuming full observability, we can represent our state space in discrete time observable canonical form. This will enable us to estimate a minimum number of parameters [1]. However, even for linear systems there are two well known limitations for identification. These are the effect of noise and lack of persistent excitation.

## Persistent Input

Input signal design and persistent excitation are critical aspects of system identification. Given a quasi-stationary input of  $u$  with a dimension  $nu$  and with a spectrum  $\phi_u(\omega)$ ,  $\phi_u(\omega) > 0$  should hold for at least  $n$  distinct frequencies for  $u$  to be a persistent excitation [4]. Random noise would be ideal to excite all the modes in the system but requires high excitation power and bandwidth. The hardware used in these measurements puts constraints on both power and bandwidth. The design of an input signal for identification under given constraints becomes an important question for the future studies.

## Noise Sensitivity

Noise affects the performance of the identification, and in certain cases can make identification impossible. We can quantify the effect of the noise by adding noise on a known system until the identification can no longer clearly estimate the known dynamics. We drive a synthetic  $2 \times 2$  coupled MIMO system using a frequency chirp signal with random noise added to the output signals. The effect of noise is tested by running the identification algorithm

\*Work supported by U.S. Department of Energy under contract DE-AC02-76SF00515 and the US LHC Accelerator Research Program

<sup>†</sup> oturgut@stanford.edu

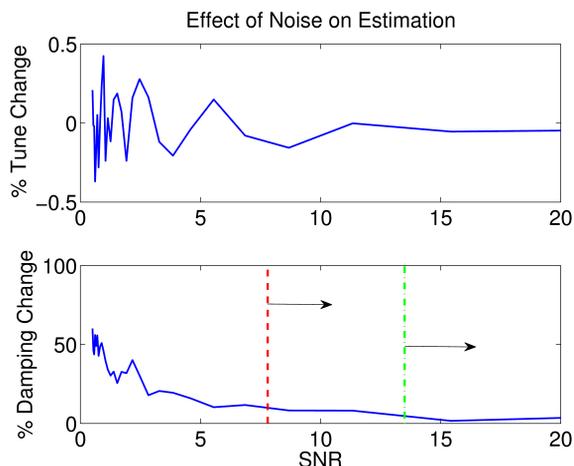


Figure 1: Deviation of estimated natural tune and damping of the 1<sup>st</sup> mode from the true value for different SNR values. Red line shows min SNR to get errors less than 10%, green line is for errors less than 5%.

on input - output data as we increase the noise level. Parameters of the model and the corresponding modes of the identified model are estimated for different noise cases. We observe the effect of noise on the mode estimation by looking at the natural frequency and damping values of the first mode for different values of signal to noise ratio (SNR). Figure 1 shows the impact of noise on estimation of system parameters. For identification algorithm to perform well, we need to have  $\text{SNR} > \sim 8$ .

## APPLICATION : SPS MEASUREMENTS

In this case the persistent excitation condition is satisfied because we are analyzing the dynamics of mode 0 and mode 1 excited by a chirp excitation, where the frequency changes over 15000 turns.

Our data processing uses a time varying bandpass filter to improve SNR to  $\sim 20$ . A window of 500 turns is used to calculate corresponding center frequency of the filter, which follows the chirp excitation.

We focus on mode 0 (barycentric) and mode 1 (head-tail) dynamics. Dynamics, input-output relation of momentum kick and vertical displacement, can be defined by 2 coupled 2<sup>nd</sup> order differential equations and represented as  $2 \times 2$  MIMO system with  $p = 2$ ,  $q = 2$  and  $n = 4$ . As mentioned before the number of inputs and outputs depends on the sampling across the bunch. Measurement data should be arranged in such a way that we have  $2 \times 2$  system. As an initial approach we separate the bunch into two parts. The samples lying in the first part (head) are represented by a macro particle. Its motion is calculated by taking the average motion of the all samples in the head. The same approach is used for the second part (tail) of the bunch. Similarly we calculate an effective kick for the head and the tail by using same technique. At each turn we calculate the effective kick that the head and tail experience by averag-

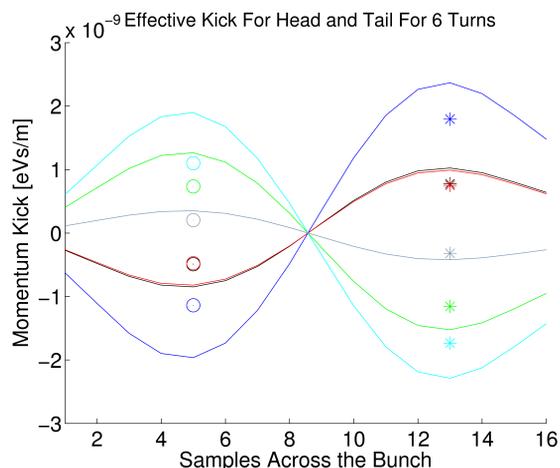


Figure 2: Amplitude modulated momentum signal used to kick particles in vertical plane. Each color represents the kick applied at a single turn. Average kicks, o : head, \* : tail.

ing the waveform between samples 1-8 and 9-16. Figure 2 shows 6 sequential traces (each represents a different turn) of the momentum waveform across the bunch and corresponding effective kicks for head and tail.

## Results

In driven measurements we use both mode 0 and mode 1 excitations [3]. Excitations and corresponding bunch measurements are used to estimate the parameters of the model. In Fig. 3 the top left plot shows the vertical displacement of the centroid of the head with measurements in blue and the response of the model in red. The bottom left plot shows the same for the tail. Note that our reduced order model is linear time invariant. It cannot capture external perturbations or parameter variations in the bunch. Still the envelope of the amplitude of the centroid motions is captured in the time domain. The figures on the right show measurements and response of model in frequency domain. As an example of external perturbation, we see modulations in FFT data. One of mechanisms for these effects can be tune modulation. From both the time and frequency domains we observe that in this specific measurement, mode 0 is dominantly excited in the tail of the bunch. Although it is a frequency chirp sweeping both mode 0 and mode 1 frequencies, the excitation signal was generated and time aligned to dominantly excite mode 0. Figure 4 shows another data with strong excitation of mode 1. On the left, we see the RMS spectrogram of the driven measurement with clear mode 0 and mode 1 excitation around turns  $\sim 4000$  and  $\sim 9000$ . On the right side, we show the RMS spectrogram of bunch's vertical motion predicted by reduced model. The model is able to capture dominant characteristics such as motions at mode 0 and mode 1 tune together with the effect of chirp and mode 0 residual motion.

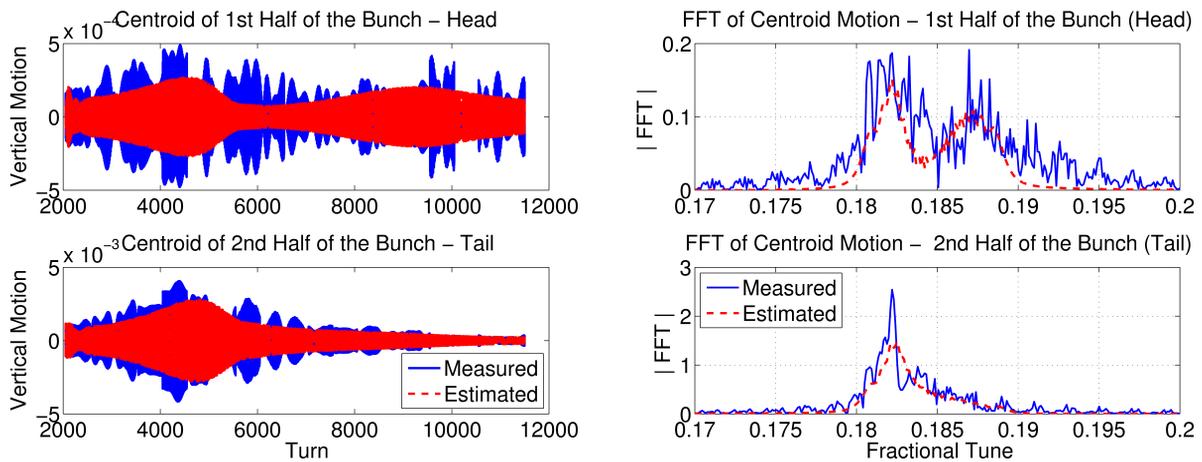


Figure 3: Comparison of the reduced order models response with machine measurements in time and frequency domain.

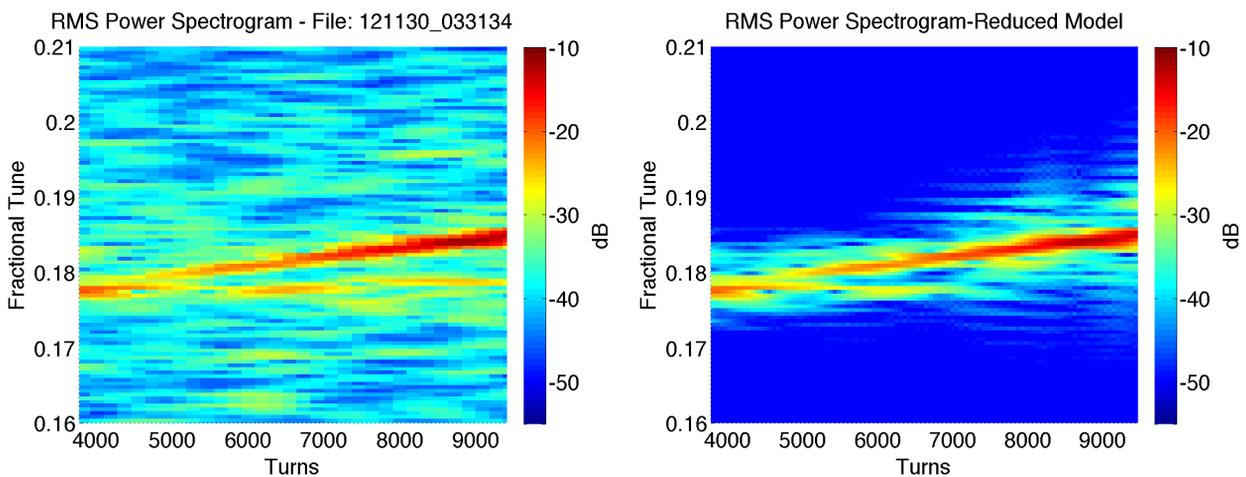


Figure 4: On the left we see the spectrogram of physical measurement showing both chirp excitation where we excite mode 0 and mode 1 around turns  $\sim 4000$  and  $\sim 9000$ , respectively. On the right, we see the same excitation and analysis applied to the reduced order model.

## CONCLUSION

Control of intra-bunch instabilities via a wideband feedback control system with optimal control algorithms requires a model of the intra-bunch dynamics. We show initial promising system identification results. We identify parameters of a reduced order model that captures mode 0 and mode 1 dynamics from CERN SPS machine measurements. The natural tunes, damping values and the separation of modes associated with the motion seen in measurements are estimated correctly using a linear model. Future work is aimed at estimating more internal modes as the wideband kicker is available late 2014. The techniques used are also applicable to explore optimal and robust control techniques using nonlinear simulation codes [5].

## ACKNOWLEDGMENT

We thank H. Bartosik, A. Drago, G. Kotzian, U. Wehrle, CERN AB RF group, the CERN operations team and US-

ISBN 978-3-95450-138-0

Japan Cooperative Program for their vital help.

## REFERENCES

- [1] NF. Al-Muthairi et al., "Identification of discrete-time MIMO system using observable canonical-form", IEE Proceedings - Control Theory and Applications, 2002.
- [2] G. Rumolo et al., "Experimental Study of the Electron Cloud Instability in the CERN-SPS", EPAC 08, Genoa, Italy, pp TUPP065 June 2008.
- [3] J. D. Fox et al., "A 4 GS/s Feedback Processing System for Control Of Intra-Bunch Instabilities", IBIC13, Oxford, UK 2013.
- [4] M. A. Dahleh, "Lecture Notes - 6.435 System Identification", EECS, MIT, Spring-2005.
- [5] M. Pivi et al., "Simulation Code Implementation to Include Models of a Novel Single-bunch Instability Feedback System and Intra-beam Scattering", IPAC 2012 conference, New Orleans, LA May 2012