

TRAJECTORIES OF LOW ENERGY ELECTRONS IN PARTICLE ACCELERATOR MAGNETIC STRUCTURES*

E.E. Cowan, Dept of Physics, Syracuse University, NY, USA
 Kiran G. Sonnad[†], CLASSE, Cornell University, NY, USA
 Seth Veitser, TechX Corp. Boulder, CO, USA

Abstract

It has been observed recently that low energy electrons, or electron clouds formed within quadrupoles can remain trapped for time periods beyond a beam revolution period at CEsr [1]. This paper provides some results obtained from analysis and numerical tracking of low energy electrons in quadrupoles under conditions similar to those when the above mentioned data was collected.

MOTION OF PARTICLES IN A NONUNIFORM MAGNETIC FIELD

Electron clouds generated in positively charged beams are known to negatively interfere with the beam. Most of the simulations done for various accelerators for estimating the cloud buildup have been for the duration of the passage of a single train of bunches. However if an electron remains trapped for long enough in a magnetic structure such as a quadrupole after the passage of a train, this trapped electron will encounter the next passage of a bunch train. The electron may then impinge upon the walls and potentially produce secondary electrons. Such a mechanism could lead to a progressive enhancement in cloud densities with repeated train passages and can have adverse consequences on the properties of the beam.

Trapping of electrons has been observed in the PSR [2]. In this experiment the proton beam was allowed to circulate and produce electrons and the beam was then deflected out of the storage ring, while the electron cloud was allowed to evolve. It was observed that electrons in quadrupoles persisted for a time period exceeding 50-100 μ s. More recently, experiments performed at CEsr observed electron trapping in quadrupoles in the presence of a beam [1]. The revolution period of the beam in CEsr is about 2.5 μ s.

In this paper, we report results obtained from single particle tracking and compare them with theoretical expectations. The parameters of our computations were similar to those of the operating conditions when the data reported in Ref. [1] was taken. A similar study was performed earlier for the KEKB [3]. This study clearly showed that trapping of electron in quadrupoles can be important and requires a careful study.

Adiabatic Invariants

In dynamical systems exhibiting periodic motion, the quantity $\oint p \cdot dq$, which is the phase space area encom-

passed by the trajectory, also known as the Poincare invariant is a conserved quantity [4]. If a system undergoes a very gradual change, this quantity remains conserved so long as the relative value of this change over a full period of motion is very small, (see for example Ref. [4]). A classic example is a simple pendulum where the length of the string is gradually altered at a rate such that over the course of the oscillation period, $\delta l/l \ll 1$. Then the quantity E/ω remains constant and is the adiabatic invariant. The same principal of adiabatic invariance can be applied to a particle executing cyclotron motion in a magnetic field (see for example Ref. [5]). For a particle in this situation, the magnetic moment

$$\mu = IA = \frac{mv_{\perp}^2}{2B} \quad (1)$$

is conserved as long as $dB/B \ll 1$ over the cyclotron period T_c . This is equivalent to the following condition,

$$\frac{|\nabla B|}{B} r_c \ll 1, \quad (2)$$

where r_c is the cyclotron radius. On the other hand, energy which is given by

$$\mathcal{E} = \frac{1}{2}m(v_{\perp}^2 + v_{\parallel}^2) \quad (3)$$

being an absolute invariant always remains conserved. In the above equation, v_{\perp} and v_{\parallel} are velocities perpendicular and parallel with to the magnetic field, respectively.

Magnetic Mirroring

If energy and the magnetic moment are conserved, one can obtain a formula for magnetic mirroring. As B increases, v_{\perp} increases and v_{\parallel} decreases. If v_{\parallel} decreases to zero, the trajectory turns around. This phenomenon is referred to magnetic mirroring. Since v_{\perp} is rotating around the field line, one can associate this condition with a cone in velocity space. A particle would lie within the "loss" cone if

$$v_{\parallel}/v_{\perp} > 1 - (B_{bd}/B_{in})^{1/2}. \quad (4)$$

Here B_{in} is the magnetic field value at the initial point, and B_{bd} is the magnetic field on the same field line at the boundary of the vacuum chamber. A particle lying outside this cone in velocity space at the given point will remain trapped.

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[†] kgs52@cornell.edu

Motion Transverse to Magnetic Field

Magnetic mirroring is a consequence of the effect of the particle motion along the magnetic field lines. Additionally, in the presence of a nonuniform field, the particle undergoes a "grad B" and a "curvature drift", which are perpendicular to the field lines. In the absence of any current sources, where $\nabla \times \mathbf{B} = \mathbf{0}$, the curvature of the field can be related to its gradient, and the expressions of the two drifts may be combined in the following form,

$$v_{curv} + v_{\nabla B} = \frac{m \mathbf{B} \times \nabla B}{q |\mathbf{B}|^3} (v_{\parallel}^2 + \frac{1}{2} v_{\perp}^2) \quad (5)$$

The values of the magnetic field here correspond to points along the motion of the guiding center. While it is clear that a particle can never be trapped indefinitely because of this drift, what matters in our study is if the particle is trapped for the period of one beam revolution. Fringe field effects have been disregarded throughout this study, which would become important for electrons that are either formed in this region or drift into this region due to the effect described above. Nevertheless, we show that for electrons to undergo a prominent longitudinal drift, they have to acquire energies close to 1 KeV, which are typically high for electrons formed from multipacting.

RESULTS FROM TRACKING STUDIES IN QUADRUPOLE FIELDS

In order to test the validity of the loss cone and Eq. 4, five different initial positions were chosen. These points lay on a horizontal line parallel to the cross-section of the beam pipe and passing through the center of the cross-section. In order to determine the loss cone angle for these points, one needs to determine the field value at the given point as well as the "escape" point. The latter was determined with the help of a program we developed that traced the path along the field line. The point beyond which the particle was considered lost was located on this field line, very close to the boundary of the chamber. The chamber shape consists of two circular arcs (radius 0.075m) on the top and bottom, connected with flat side planes. It is about 0.090m from side to side and 0.050m between the apices of the arcs. The magnetic field is given by $B_x = ky$ and $B_y = kx$ with $k = 7.4T/m$, which is close to the value set during the operation of Cesium at 5.3 GeV.

Table 1: Cases Studied to Confirm Trapping

x(m)	B_{in} (T)	B_{bd}	θ_m (deg)	$\angle 1, \angle 2$
0.0005	0.0365	0.2261	66.2989	65, 67
0.015	0.1095	0.2386	47.3414	45, 48
0.025	0.1825	0.2626	33.507	32, 34
0.035	0.2555	0.2972	21.9773	20, 23
0.04	0.2920	0.3184	16.7044	15, 17

Table 1 shows all the initial positions from where electrons were tracked. The critical angle θ_m is equal to

$90deg - (\text{loss-cone-angle})$. Since the points lay on the horizontal axis, the magnetic field direction for all of them was vertical. Thus, if the particle launch angle with respect to the horizontal axis was less than θ_m one would expect it to be trapped. The tracking was performed using the plasma simulation program Vorpil [6]. The particles were launched at angles (with respect to the horizontal axis) just above and below the critical angle, indicated as $\angle 1$ and $\angle 2$ respectively. For each case, the energies used were 1eV, 10eV, 100eV and 1 KeV. In most cases, particles that started at $\angle 2$ remained confined while those with $\angle 1$ were lost. The only exceptions were when the particles were at $x = 0.05cm$ with energies 1KeV and 100eV, when particles at both the starting angles were lost. It was also confirmed that when particles were incrementally close to the center, they were lost within $2.5\mu s$ even at an energy of 1eV regardless of the initial direction of motion.

For a quadrupole magnetic field, one can easily verify that the condition of adiabatic invariance, given by Eq. (2) becomes weaker as one approaches the center. Depending on the particle energy, it becomes invalid at a certain point close enough to the origin. This explains the poor confinement for particles that cross points close enough to the center.

Overall, one could divide the trajectories into three classes, (1) Those that escape because their parallel and perpendicular velocities are distributed such that they lie within the loss cone. (2) Particles that remain trapped because they lie outside the loss cone, and the condition for adiabatic invariance remains satisfied. (3) Particles that cross points sufficiently close to the center that the motion becomes non-adiabatic in the vicinity of the center. As these particles move away from the center due to inertia, they begin to move into an area of higher magnetic field, where the motion may become adiabatic. If they are within the loss cone, they escape. If they are outside the loss cone, they get reflected back toward the center. Depending upon the phase of the cyclotron motion they acquire near the center, they may stray away from the center in an entirely different direction. This process repeats itself until it enters the loss cone and impinges against the wall.

Figure 1 is an example of a particle that escapes because it is within the loss cone. Figure 2 is an example of particle that remained trapped for the period of $2.5\mu s$ because it was located outside the loss cone. Figure 3 was a particle very close to the center that exhibited non-adiabatic motion. The energy in all these cases was 1 KeV.

Figure 4 shows the three dimensional motion of a trapped particle at 1 KeV, clearly indicating the drift along the longitudinal direction. Figure 5 is a plot of the same z motion as a function of time. This plot compares the motion of the full trajectory, as computed by the tracking, the guiding center motion obtained from the tracking by numerically averaging over the cyclotron periods, and the motion as predicted by Eq. 5. We see that the analytic results agrees well with the numerically computed guiding center motion. The longitudinal distance covered by the

particle in $2.5\mu\text{s}$ was found to be about 0.34m , while the length of the quadrupole is about 0.6m . Additionally, it is known from build up simulations that the fractional population of particles with energies of 1KeV are not more than a few percent. Based on Eq. 5, one can state that the distance covered by the particles is roughly proportional to the energy (disregarding the details of the parallel vs perpendicular distribution of the velocities). Typical energies of the electrons is of the order of 100eV . One can thus argue that the longitudinal drift of trapped electrons does not contribute significantly to their escape before the arrival of a subsequent bunch train.

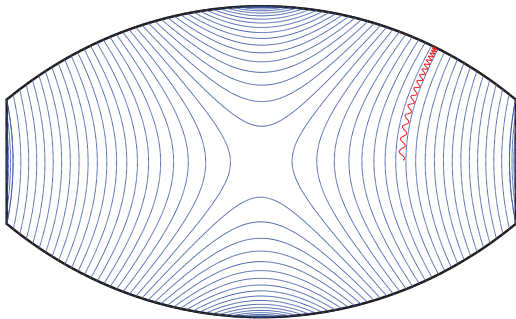


Figure 1: Escaped particle lying within the loss cone.

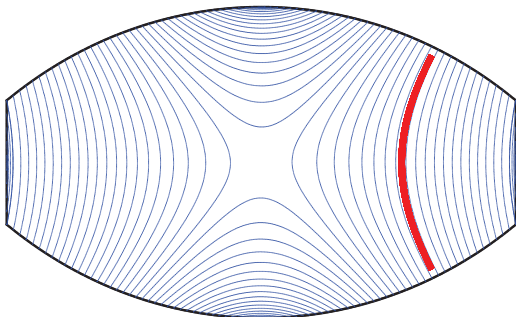


Figure 2: Trapped particle lying outside the loss cone.

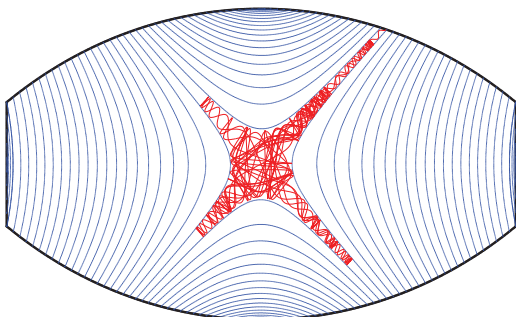


Figure 3: Particle executing non-adiabatic motion.

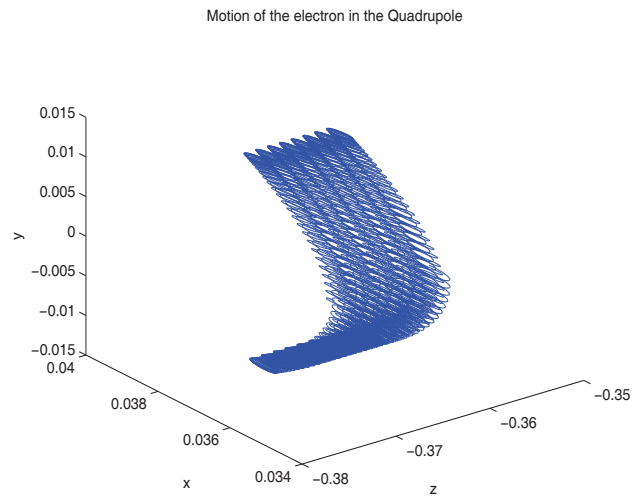


Figure 4: Three dimensional motion of a trapped particle showing longitudinal drift.

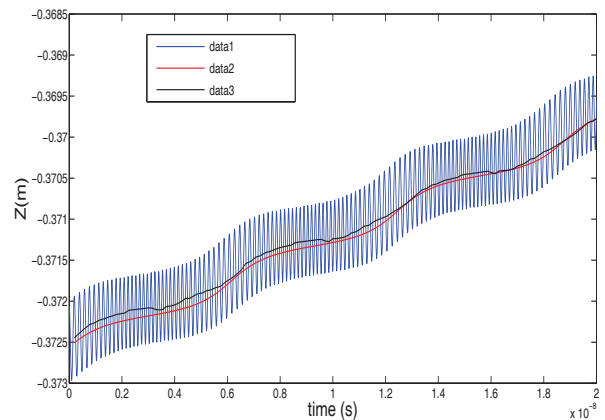


Figure 5: Validation of the combined grad-B and curvature drift, data1 is the full particle trajectory obtained from tracking, data2 the trajectory obtained from Eq. 5, data3 the tracked data averaged over the cyclotron motion.

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