KINETIC THEORY OF HALO FORMATION IN CHARGED PARTICLE BEAMS*

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Abstract

The proceeding proposes a derivation of the Vlasov-Poisson equation where the distribution function is splitted in a main part standing for the core and a fluctuating part standing for the halo, with which the kinetic theory of halo formations and core-halo dynamics are analysed.

INTRODUCTION

High-power hadron beams are more and more requested in multiple research fields like fusion material studies, neutrino physics, nuclear transmutation, accelerator-driven systems. Among the most recent projects, the IFMIF accelerators [1] break the record of high intensity, leading to a multi-MW beam power at relatively low energy. The concern for such accelerated beams is the predominance of the self-field energy upon the beam energy. In these conditions, the space charge effect is at its maximum, which triggers different non-linear mechanisms implying emittance growth, halo formation and sudden particle lost. The challenge is then to be able to control the total beam size, to maintain it well inside the beam pipe, and to avoid particle losses on its wall at the level of 10^{-6} in order to preserve hands-on maintenance. Careful studies of the dynamics of high intensity charged particles are thus mandatory to better understand and characterise the beam non-linear behaviour.

In the space-charge dominated regime, the space-charge effects result in emittance growth and halo formation, which contribute to beam losses. A change of focusing lattice or inadequate knowledge of proper injection conditions can cause a mismatch between the beam and the transport system. This mismatch may result in an oscillation of the beam envelope and generally excite a superposition of the envelope eigenmodes. These envelope modes possess additional free energy compared with the stationary distribution. Particles with appropriate oscillation frequencies can resonate with these envelope modes through the so called parametric $2:1$ resonance and attain large amplitude to form a halo [2]. These halo particles extract the energy from the envelope modes and convert free energy from mismatch into thermal energy, which causes beam emittance growth. The goal of this proceeding is proposes a derivation of the Vlasov-Poisson equation where the distribution function is splitted in a main part standing for the core and a fluctuating part standing for the halo, with which core-halo interactions will be analysed.

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CORE-HALO EQUATIONS

The beam is assumed to propagate with a constant axial velocity $v_z\hat{\mathbf{e}}_z$, so that the axial coordinate $s = z = v_z t$ play the role of time. The external focusing field is given by $\mathbf{B} = B_0\hat{\mathbf{e}}_z$ and is used to compensate the repulsive Coulomb force between the beam particles. It is convenient to work in the Larmor frame [3] which rotates with respect to the laboratory frame with angular velocity $\Omega_L = qB_0/2\gamma_b mc$, where c is the speed of light in vacuo, and q, m, and $\gamma_b = 1/\sqrt{1-\beta_b^2}$ are the charge, mass and relativistic factor of the beam particles respectively.

A collisionless charged particle beam is usually described in a self-consistent mean-field approximation by the Vlasov-Poisson system :

$$\frac{df_b}{dt} = \frac{\partial f_b}{\partial t} + \mathbf{v}\cdot\frac{\partial f_b}{\partial \mathbf{r}} + \mathbf{F}\cdot\frac{\partial f_b}{\partial \mathbf{v}} = 0, \quad (1a)$$

$$\mathbf{F} = -\kappa_z\mathbf{r} - \nabla\phi, \quad (1b)$$

$$\nabla^2\phi = -4\pi q n_b(\mathbf{r}, s). \quad (1c)$$

where N_b is the number of particles per unit axial length, $\mathbf{r}$ is the position vector in the tranverse plane, $\mathbf{v} = \frac{d\mathbf{r}}{ds}$ is the transverse velocity, $n_b(\mathbf{r}, s) = q_b\int d\mathbf{v}^2 f_b(\mathbf{r}, \mathbf{v}, s)$ is the transverse beam density, $\kappa_{z_0} = qB_0/2\gamma_b\beta_b mc^2$ is the focusing field, and $K = 2q^2B_0^2/\gamma_b^3 v_z^2 m^2 c^2$ is the perveance, which is a measure of the beam space charge. In Eq. (1), ϕ is a scalar potential that incorporates both self-electric and self-magnetic fields, $\mathbf{E}^s$ and $\mathbf{B}^s$. We shall take zero of the scalar potential to be at r_w, the position of the conducting channel wall. The distribution function is normalized so that $\int d\mathbf{v}^2 d\mathbf{r}^2 f_b = 1$. In the Larmor frame, the system corresponds to a two dimensional non-neutral plasma of pseudo particles of mass $m = 1$ and charge $q = \sqrt{K/N_b}$ interacting by a repulsive logarithmic potential $\phi(\mathbf{r}) = -q^2\ln(r/r_w)$ confined in a parabolic potential well of $U(\mathbf{r}) = \kappa_z r^2/2$.

The Vlasov equation (1) express the conservation of the distribution function in phase space. We assume that the initial condition in phase space consists of a patch of uniform distribution function ($f_0 = \eta_0$) surrounded by vacuum ($f_0 = 0$). After some evolution, the patch mixes with vacuum and the coarse-grained (i.e. locally averaged) distribution function $\bar{f}$ takes values between 0 and η_0. The fact that the coarse-grained distribution function cannot exceed the maximum value η_0 of the initial condition is responsible for an effective "exclusion principle "leading to degeneracy effects. This degeneracy, resulting from the incompressibility of the Vlasov equation in phase space , was first recognized by Lynden-Bell [4] is his study of equilibrium.

Suppose that the *coarse-grained* [5] distribution function is the distribution f_c for *core* of the beam and the fluctuations of the distribution function is the distribution f_h for the beam *halo* [6, 7].

We introduce the decomposition $f_b = \bar{f}_b + \tilde{f}_b$ $[f_b = f_{b_c} + f_{b_h}]$ where $\bar{f}_b$ $[f_{b_c}]$ is the *coarse-grained* distribution function and $\tilde{f}_b$ $[f_{b_h}]$ is the fluctuations. We assume that $\bar{f}_b$ $[f_{b_c}]$ results from the statistical average so that $\overline{\bar{f}_b} = \bar{f}_b$. Taking the local average of the Vlasov equation we obtain a convection-diffusion equation:

$$\frac{\partial f_{b_c}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_{b_c}}{\partial \mathbf{r}} + \mathbf{F_c}\cdot\frac{\partial f_{b_c}}{\partial \mathbf{v}} = -\frac{\partial \mathbf{J}}{\partial \mathbf{v}}, \quad \text{(2a)}$$

$$\mathbf{J} = \overline{\mathbf{F_h} f_{b_h}}, \quad \text{(2b)}$$

for the *core* distribution function, with a diffusion current $\mathbf{J} = \overline{\mathbf{F_h} f_{b_h}}$ related to the correlations of the fine-grained fluctuations [8] (to describe the perturbations inside the *core* and to describe the retroaction of the *halo* on the *core* [7]). Note that the diffusion occurs only in velocity space. There is no diffusion in position space since the velocity $\mathbf{v}$ is a pure coordinate and therefore does not fluctuate. The problem in hand consists in determining the diffusion current $\mathbf{J}$. Since the diffusion current $\mathbf{J} = \overline{\mathbf{F_h} f_{b_h}}$ is related to the fine-grained fluctuations of the distribution function, any systematic calculation starting from the Vlasov equation must necessarily in corporate the evolution equation of f_{b_h}. This equation is simply obtained by substracting equation (1a) from (2a) :

$$\frac{\partial f_{b_h}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_{b_h}}{\partial \mathbf{r}} + \mathbf{F_c}\cdot\frac{\partial f_{b_h}}{\partial \mathbf{v}} = -\mathbf{F_h}\cdot\frac{\partial f_{b_c}}{\partial \mathbf{v}} - \mathbf{F_h}\cdot\frac{\partial f_{b_h}}{\partial \mathbf{v}} + \frac{\partial \overline{\mathbf{F_h} f_{b_h}}}{\partial \mathbf{v}} \quad \text{(3a)}$$

The essence of the quasi-linear theory is to assume that the fluctuations are weak [9] and neglect the non-linear terms in (3) altogether. In that case (2a) and (3) reduce to the coupled system:

$$\frac{\partial f_{b_c}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_{b_c}}{\partial \mathbf{r}} + \mathbf{F_c}\cdot\frac{\partial f_{b_c}}{\partial \mathbf{v}} = -\frac{\partial \overline{\mathbf{F_h} f_{b_h}}}{\partial \mathbf{v}}, \quad \text{(4a)}$$

$$\frac{\partial f_{b_h}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_{b_h}}{\partial \mathbf{r}} + \mathbf{F_c}\cdot\frac{\partial f_{b_h}}{\partial \mathbf{v}} = -\mathbf{F_h}\cdot\frac{\partial f_{b_c}}{\partial \mathbf{v}} \quad \text{(4b)}$$

Physically, these equations describe the coupling between the *core* (*coarse-grained* distribution f_{b_c}) and the *halo* (the small-scale fluctuations f_{b_h}).

To resolve the equations (4a) and (4b) we assume during beam evolution, the system tends to maximize its rate of entropy production while satisfying all the constraints imposed by the dynamics.

Relaxation Process of the Beam Core and Halo Formation

The evolution of the system equations (4a) e (4b) is extremely complicated. Although the dynamics is collisionless, the fluctuations of the electromagnetic potential are able to redistribute energy between beam particles and provide an effective relaxation mechanism on a very short timescale (violent relaxation [4]).

The system develops an intricate filamentation in phase space (*phase mixing*). In physical space, this *mixing* process is associated with the heavily damped oscillations of beam away from mechanical equilibrium. In a strict sense, the distribution function $f_b = f_{b_c} + f_{b_h}$ does not reach an equilibrium distribution but develops smaller and smaller filaments. However if we introduce a coarse-graining procedure and locally average over the filaments, the coarse-grained distribution function $\bar{f}_b$ is expected to reach a stationary state on a short timescale. It is usually advocated that this metaequilibrium state is a particular stationary solution of the Vlasov equation [10]. A statistical theory appropriate for this process of violent relaxation has been developed by Lynden-Bell [4]. Unfortunately, the statistical prediction of Lynden-Bell is limited by the problem of incomplete relaxation [11]. Beams tend towards the equilibrium state during violent relaxation but cannot attain it: the fluctuations of the potential die away before the relaxation process is complete [12].

Equation (4b) is first solved formally with the help of Green functions [13]. This gives rise to an iterative process. This iterative process is truncated to quadratic order and the result is substituted back into (4a). The diffusion current $\mathbf{J}$ does not have yet a closed form as it involves the equal-time correlation function. To close the system, we assume that the fluctuations of the distribution function in two different macrocells are decorrelated. The resulting equation is now self-consistent [5].

Our working hypothesis is that during the stage of violent relaxation, the particles of beam extract their energy from the rapid fluctuations of the eletromagnetic field. By this process, some particles may acquire very high energies and escape from the beam forming the *halo*.

Process of the Core-Halo Interactions

With the halo formation, the relaxation of system from equilibrium becomes incomplete [14]. It starts a process of interaction *core-halo* where the terms on the right side of the equations (4a) and (4b) dominate the dynamic. The field energy generated by the core interact with the halo and the fluctuations tend to become weaker. The core reaches a stationary state [15].

Relaxation processes of Beam Halo via thermal bath

As the core is quasi-equilibrium state and the fluctuations are weakening, the core will serve as a thermal bath to the halo. This allows us to we find a quasi-equilibrium solution for the system core-halo in a relaxed state [16, 17, 18].

CONCLUSIONS

The maximum transportable current density of an ion beam with high space-charge intensity propagating in a periodic focusing lattice is a problem of practical importance.

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Accelerator applications such as Heavy Ion Fusion (*HIF*), High Energy Density Physics (*HEDP*), and transmutation of nuclear waste demand a large flux of particles on target. A limit to the maximum current density can result from a variety of factors: instability of low-order moments of the beam describing the centroid and envelope, instability of higher order collective modes internal to the beam, growth in statistical phase-space area (rms emittance growth) and excessive halo generation.

The most important examples of a nonstationary beam are a beam with an rms mismatch, a nonstationary distribution function (the distribution function is not a solution of the stationary Vlasov equation), and a misaligned beam. If the beam is initially nonstationary, it has a higher average energy per particle than the stationary beam. The energy difference represents the free energy that can be thermalized by nonlinear space charge forces, collisions, or instabilities. As a result of such a process, the emittance increases as the beam relaxes toward its final quasistationary state.

The deviation of beam distribution from the stationary state may be understood as the main cause of halo formation. Whenever a beam is not in its own stationary state, it naturally tries to approach closer to there, minimizing the nonlinear field energy. During this process, plasma oscillations are excited in the beam core, driving a portion of it into a halo. Inversely speaking, if a beam injected into a uniform channel is fine grainedly matched to it, there is no reason to expect the development of a halo unless the distribution itself is intrinsically unstable against perturbation. While several different factors have been considered as the possible sources of halo formation, it seems, after all, that the important point is how much a beam is deviated from the stationary state

In Linac the effective phase space volume occupied by a beam can grow rapidly if the beam intensity is sufficiently close to the space charge limit and if a source of instability is available. In principle there are three sources of instability: in a continuously focused beam it can be a nonmonotonic distribution function, like the K-V distribution; in two or three dimensional beams it can be an anisotropic distribution with different emittance and/or energy in two phase planes; in addition, periodic focusing can act as a source if an eigenoscillation is in resonance with the period of focusing.

In this proceeding a derivation of the Vlasov-Poisson equation where the distribution function is splitted in a main part standing for the core and a fluctuating part standing for the halo, with which the kinetic theory of halo formations and core-halo dynamics are analysed.

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