A PLANAR SNAKE MUON IONIZATION COOLING LATTICE *

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Abstract

Muon Colliders require cooling in six dimensions (x, x',y, y', t, E). Several ionization 6D cooling lattices, using vacuum acceleration, are reviewed: RFOFO lattices, both curved [1] and rectilinear [2] cool one sign to low emittances; A Helical FOFO snake [3] cools both signs, but not to low emittances. A Planar Snake [4] lattice is proposed that cools both muon signs, and appears suitable for low emittances. The parameters and performance for an example suitable for the early stages of cooling is given.

INTRODUCTION

The Muon Accelerator Program (MAP) [5] is studying Muon Colliders with center of mass energies from 125 GeV to several TeV. All require 6 dimensional (x, x',y, y', E, t) cooling to transverse emittances of ≈ 0.31 mm, and longitudinal emittances ≈ 1.5 mm.

In transverse ionization cooling, in a low Z absorber (hydrogen or Lithium Hydride), both transverse and longitudinal momentum are lost. Radio-frequency (rf) cavities (only vacuum rf is considered in this paper) then restore the longitudinal momentum, leaving the reductions of the transverse components. Multiple scattering leads to a minimum (equilibrium) emittance that is proportional to the betatron $\beta_\perp$ divided by the relativistic β of the muon momentum.

For cooling in the longitudinal dimension, emittance exchange is required. Two methods are illustrated in Fig. 1a and b. In a), a wedge absorber is placed in a region with dispersion, so higher momentum muons have more energy loss than those lower. In b) a plane slab absorber is placed where there is angular dispersion. Emittance exchange can also be achieved in a Helical Cooling Channel [6] where absorbing material is everywhere while higher momenta have longer paths.

Periodic lattices allow lower transverse betatron lengths $\beta_\perp$, but introduce integer and half integer resonances defining the operating momentum range. Figure 2a, b, and c show three cases:

a) Simple lattices of equi-spaced solenoids operate above the single π resonance.

b) Bi-periodic lattices [7], with pairs of coils spaced by longer drifts operate between the 2π and π resonances and have an attractively level $\beta_\perp$ between these resonances.

c) Lattices with more complex coils sequences, such as the Helical FOFO Snake and Planar snake, which both operate between the 3π and 2π resonances.

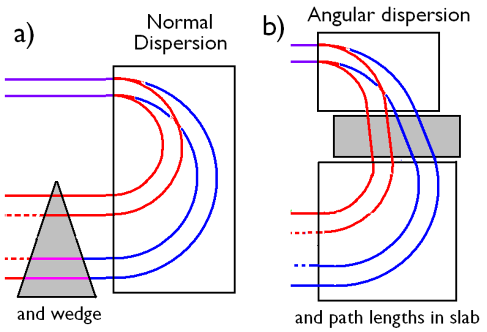

Figure 1: Emittance Exchange: a) using dispersion & wedge absorbers; b) using angular dispersion & slab absorbers.

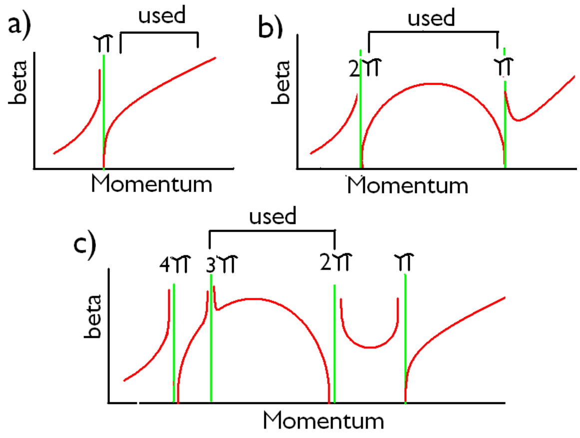

Figure 2: Lattice types: a) Simply periodic alternating solenoids; b) Bi-periodic alternating solenoids; c) Higher tune lattices (Helical Snake & Planar Snake).

GEOMETRIES

Figure 3 shows three lattice geometries. They could be curved, but we consider only straight geometries here.

RFOFO & RFOFO Rectilinear Lattices

Using alternating solenoids, Balbekov [2] has shown that the required dispersions can be generated by small tilts of the coils in either a simply periodic (FOFO) or bi-periodic (RFOFO) lattice. The tilts are small and alternate to generate periodic dipole fields (Fig. 3a). The performance of these lattices is essentially the same as with curved lattices [8], but with simpler engineering. Recent simulations [9, 10], have shown cooling from emittances close to those simulated from the initial phase rotation (15 mm transverse, 45 mm longitudinal) to that required in the baseline design (0.31 mm transverse and 1.5 mm longitudinal) with acceptable particle loss (of order 50%).

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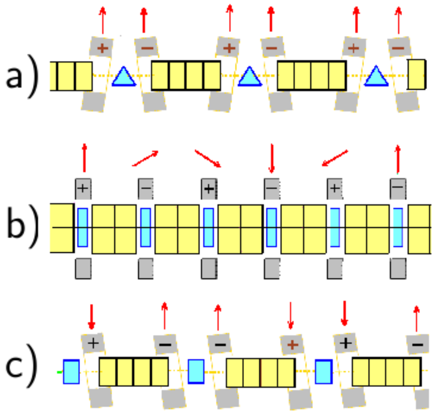

Figure 3: Geometries: a) Rectilinear RFOFO; b) Helical FOFO Snake; c) Planar Snake.

Despite the success of such simulations, there are reasons to seek alternatives. The current densities are high, in part because the coils on either side of the absorber are opposed; and for the same reason, the forces between them are outward, requiring supporting structures. The engineering of wedge shaped liquid hydrogen absorber will not be easy, and the fact that they only cool one sign is less than ideal.

Helical FOFO Snake

The Helical FOFO Snake [3] (Fig. 3b) is based on a periodic FOFO lattice with alternating focus solenoids, but the coil tilts are no longer in a single plane. Instead, the tilt directions rotate in azimuth giving closed orbits that perform an approximation to a helix. Because the lattice does not repeat for multiple sub-cells, there are more resonances, and the operating momenta are restricted to tunes between 3π and 2π - as in Fig.2c.

The lattice has two very attractive characteristics: 1) it uses absorbers with plane parallel windows that would be much easier to build than the wedges; and 2) it works for both signs simultaneously, possible because the lattice employs emittance exchange represented in Fig. 1b. Unfortunately the absorbers are located where the $\beta_\perp$s are at their maxima making it unsuitable for cooling to the smaller emittances.

Planar RFOFO Snake

This lattice[4] (see Fig. 3c) is, like the RFOFOs, based on bi-periodic focusing coils that give low betas at absorbers placed between the nearer pairs. But the polarities of the coils, instead of alternating, have the sequence $+|+-|-+|+$ etc., where the vertical bars indicate absorber locations. The number of field reversals is now only half that in an RFOFO, and the coils on either side of the absorber have the same polarity. Their fields no longer buck one another, and the magnet forces pull them together. A disadvantage is that the phase advances are doubled and the momentum acceptance, between 3π and 2π (see Fig. 2c) is reduced.

An attempt to find tilts that would give dispersions at the absorbers failed, but a solution was found, using very small tilts, giving large angular dispersions. The solution has tilts all in the same direction, resulting in dipole fields with the same pattern as the solenoids. This introduces a transverse resonance, at the tune 2π, at the high end of the momentum acceptance. This dispersion is highly non-linear: low over most of the momentum range, but very high near the 2π resonance, yet, at least for the example simulated, this was not a problem.

EARLY STAGE PLANAR RFOFO SNAKE

This example would be suitable for an early stage of 6D cooling for a Muon Collider. Parameters are given in Table 1. Figure 4 shows a cross section of one complete cell. The solenoid coils are tilted in the same direction alternatively by 7.3 and 17.9 mrad. The rf is at 201 MHz operating at 17 MV/m axial gradient, and a phase of approximately 30 degrees.

Figure 5 shows the betas as a function of length for a few cells. It is seen that, as in a normal RFOFO lattice, the betas are much lower at the absorbers than between them.

Figure 6 shows the dispersions as a function of length over a few cells. It is seen that, at the absorbers, the dispersion is not large, but the x dispersion at the absorber is changing rapidly as a function of length, corresponding to an angular dispersion of approximately 0.4.

Figure 7 shows the emittances and transmission. Cooling is good in both transverse and longitudinal emittance, with the rate of muon loss, after an initial miss-match, dominated by the inevitable decays. The quality factor $Q = (d\epsilon_6/\epsilon_6)/(dn/n)$ has a low initial value where matching losses dominate; a low value at the end as the emittances approach their equilibria; and a maximum value of over 9, which is approximately the same as that with a Rectilinear RFOFO lattice.

The results plotted in Fig. 7 and discussed above were obtained using a mode of ICOOL [11] simulation in which fields are defined along the axis, and the program derives the fields at other locations by a 5th order extrapolation. The lattice has since been simulated [12] with ICOOL using a field map, and also with G4BL [13] using the same map. The results are very similar or slightly better in these simulations.

CONCLUSION

Simulations of Rectilinear RFOFO 6 dimensional ionization cooling has given the required parameters, but there remain engineering challenges and two complete systems are needed for the two particle signs.

A Planar RFOFO Snake, using planar absorbers, can cool both signs simultaneously. An example lattice design of an early stage of cooling has been simulated and gives good performance. It uses coils on either side of the absorber, that have the same polarity, thus reducing the cur-

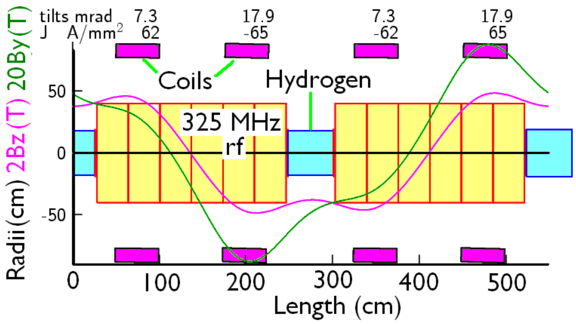

Figure 4: Section of an early stage of a Planar RFOFO Snake.

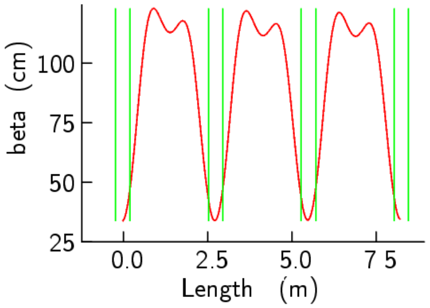

Figure 5: Beam betas as a function of length. The green lines define the location of the absorbers.

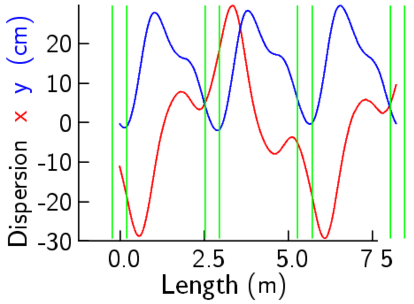

Figure 6: Dispersions as a function of length for the early stage lattice. The green lines define the location of the absorbers.

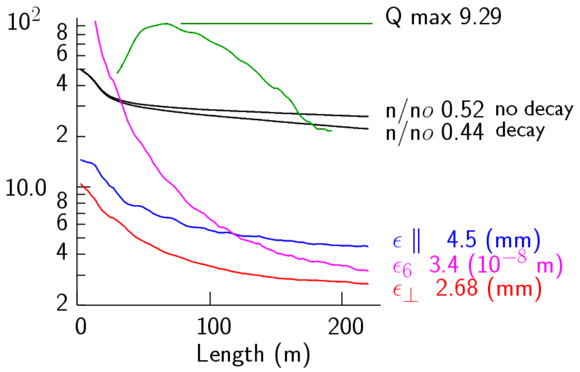

Figure 7: Emittances and transmission of early stage planar snake.

Table 1: Parameters of Early 6D Cooling Lattice

gap m	start m	dl m	rad m	dr m	tilt mrad	I/A A/mm^2
0.50	0.50	0.5	0.77	0.11	7.3	62.22
0.75	1.75	0.5	0.77	0.11	17.9	-65.45
0.50	3.25	0.5	0.77	0.11	7.3	-62.22
0.75	4.50	0.5	0.77	0.11	17.9	65.45

	material	length cm	radius cm
Half absorber	Liquid H_2	21.3	18
Absorber window	Aluminum	0.05	18
Gap	Vacuum	17.15	50
6 rf cavities	Vacuum	33	61
Gap	Vacuum	17.15	50
Absorber window	Aluminum	0.05	18
Half absorber	Liquid H_2	21.3	18

rent densities and forces. Non-linear angular dispersions are obtained with small tilts (0.5-1 deg.), enhanced by the 2π resonance at the high momentum end.

The concept is similar to that of a Helical FOFO Snake [3], but is here generated by alternating dipole fields in a single direction, conceptually giving planar snaking orbits, unlike the azimuthally rotating dipoles in the Helical Snake. The planar RFOFO snake allows betas at the absorbers to be much smaller than elsewhere. This result has been confirmed using field maps in both ICOOL and G4BeamLine. A design reaching the required final emittances has been tried, but its transmission is poor, and is not yet understood.

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