

# TRANSVERSE IMPEDANCE AND TRANSVERSE INSTABILITIES IN THE FERMILAB BOOSTER\*

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## Abstract

The Fermilab Booster is characterized by large space charge and large wake fields, leading to beam loss and instabilities at high intensity. Using the Synergia simulation package with a realistic lattice model and realistic wake functions we investigate the coherent tune shift and the instabilities at injection energy. We calculate the wake functions in the laminated magnets and find them to be four orders of magnitude larger than the wakes in the metallic straight sections. We find a large decrease of the vertical tune shift and a small increase of the horizontal tune shift with intensity, in good agreement with experiment. In agreement with measurements, we find that a large chromaticity is required to avoid a horizontal instability. The instability is caused by short-range bunch-bunch interaction and is due to the large coupling between the horizontal wake function and large lattice beta function at the locations of the focusing magnets.

## INTRODUCTION

The Fermilab Booster [1] is a 40-year-old machine which provides protons for the Fermilab Main Injector. It runs at an intensity of  $4.5 \times 10^{12}$  protons per batch, which is about two times larger than the originally designed intensity. The machine shows beam loss and instabilities at high intensity. For machine improvements required by the demand of future Fermilab programs for high intensity beams, it is essential to have a good understanding of these instabilities.

The particular thing about the Booster is the presence of the dipole-quadrupole combined-function focusing (F) and defocusing (D) magnets, which cover about 60% of the machine length. The rest of the machine is made by straight metallic beam pipe sections. Since there is no beam pipe in the combined-functions magnets, the beam is exposed directly to the magnets' laminations. A consequence of the presence of bare laminations is the formation of very large wake fields. Since the machine runs at low energy (injection  $E$ /final  $E = 0.4\text{GeV}/8\text{GeV}$ ), the space charge is also strong. The presence of both large wakes and space charge effects together with complex single-particle optics makes any attempt of analytical treatment very difficult.

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We address these effects by using Synergia [2], a state of the art simulation package developed at Fermilab. We perform single- and multibunch simulations at the injection energy. Since the RF sets the number of buckets in the Booster to 84, a full machine simulations requires simulations with 84 bunches.

In order to properly take into account the wake fields, we calculate the wake functions in the laminated magnets. We find that the wake functions in the laminated magnets have a non-metallic behavior and are four orders of magnitude larger than the wakes in the metallic pipe section.

The measurements of the coherent tune shift show a large negative shift for the vertical tune and a small positive shift for the horizontal tune [3, 4]. Close to injection, the beam is horizontally unstable unless the machine runs with a large horizontal chromaticity [5]. The result of our simulations are in good agreement with experiment. We find that the instability is due to short range bunch-bunch interactions. The instability is caused by the large value of the coupling between the lattice horizontal beta function and the horizontal wake at the location of the F magnets.

## SYNERGIA CODE

Synergia [2] is a simulation package for beam dynamics in accelerators developed at Fermilab. It can address single-particle optics by using direct symplectic integration through the lattice elements or/and by using arbitrary-order polynomial maps. Collective effects are incorporated by employing the split-operator method [6]. A variety of space-charge solvers (such as 3D Poisson solvers with different boundary conditions, 2.5D solvers and semi-analytical 2D solvers) can be used with Synergia. Synergia can accommodate arbitrary wake functions by reading them from external files.

In our Booster simulations, in order to account for the large nonlinear lattice effects, we used direct symplectic integration for particle propagation. Space charge was addressed by using three different 3D Poisson solvers with appropriate boundary conditions for the F magnets, D magnets and straight sections, respectively. We used different wake functions for the F and the D magnets. The wake functions for the laminated magnets were calculated separately, as is described in the next section.

## WAKE AND IMPEDANCE

The electromagnetic field created by a particle moving through the accelerator induces currents in the vacuum chamber walls. The field created by these currents will affect a trailing particle. This effect can be addressed by a

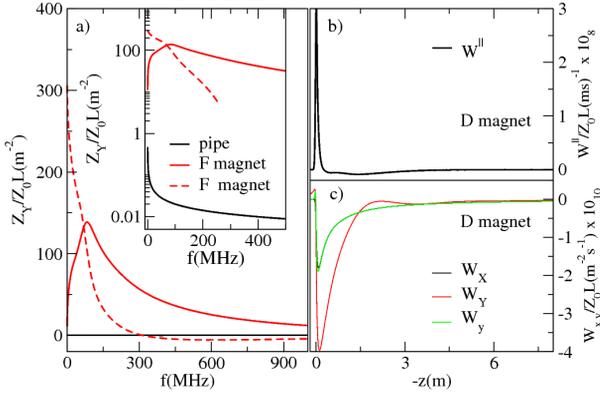


Figure 1: a) The vertical impedance caused by the source displacement in the F magnet ( $ReZ_Y(\omega)$  -solid line,  $Im(Z_Y(\omega))$  -dashed line) and in the pipe section. (See inset for the plot using a logarithmic scale). The impedance in the laminated magnets is four orders of magnitude larger than in the pipe. b) Longitudinal wake in the D magnet. c) Transverse horizontal and vertical wakes in the D magnet. The vertical wake caused by the source displacement (red) is about two times larger than the corresponding horizontal one (black). Due to nonultrarelativistic effects, the wakes are nonzero up to a small distance ( $\approx 0.1m$ ) in front of the source particle.

simple formalism [7],

$$\beta c \Delta p_z = -qQW^{||}(z) \quad (1)$$

$$\beta c \Delta p_x = -qQ(W_X(z)X + W_x(z)x) \quad (2)$$

$$\beta c \Delta p_y = -qQ(W_Y(z)Y + W_y(z)y). \quad (3)$$

Here  $Q, X, Y$  ( $q, x, y$ ) represent the charge and horizontal and vertical displacements of the source (witness) particle. The momentum of a witness particle going through a lattice element is kicked with a term proportional to a wake function which depends only on the distance,  $z$ , between the source and the witness particle. Only the terms up to the first order in the particles displacements are considered. Note that for highly symmetric chambers, such as circular or rectangular, and in the ultrarelativistic limit, the terms proportional to the witness particle displacement are zero. For a flat chamber like the one we use to model the Booster magnets,  $W_X = -W_x$  due to the horizontal translational symmetry.

For a realistic simulation it is important to have an accurate estimate of the wake functions. Calculation of the wakes requires solving the electromagnetic problem for the vacuum chamber. This is usually done in the frequency domain. The impedance, which is related to the wake function via a Fourier transform is proportional to the force acting on the witness particle. The solution is dependent on the chamber geometry and on the boundary conditions for the electromagnetic field at the vacuum chamber walls.

Due to the exposure of the laminations, the impedance and wakes are orders of magnitude larger in the combined-

function magnets than in the metallic pipes. The ultrarelativistic impedance calculation in the laminated structures is presented in [8], while the nonultrarelativistic effects for flat chambers are addressed in [9]. In Fig. 1 a) we show the vertical impedance  $Z_Y$  in the F magnet and compare it with the straight section vertical impedance. While the straight section impedance shows the conventional (i.e. metallic)  $\omega^{-0.5}$  behavior in the frequency domain of interest, the real part of the F magnet impedance shows a broad peak around  $80MHz$  and is four orders of magnitude larger. In Fig. 1 b) and c) we show the longitudinal and transverse wakes in the D magnet. The vertical wake  $W_Y$  is about two times larger than the corresponding horizontal wake,  $W_X$ , while the vertical wake caused by the witness particle displacement  $W_y$  is close to the corresponding horizontal wake  $W_x$ . (They are equal in the ultrarelativistic limit). Note that due to the nonultrarelativistic effects, the wakes are nonzero up to a small distance ( $\approx 0.1m$ ) in front of the source particle.

## SIMULATION RESULTS

To ensure agreement between the lattice model and the real lattice the parameters of the dipole and quadrupole correctors were determined using Orbit Response Measurement fitting [10]. We performed single- and multi-bunch simulations up to 2000 turns at the injection energy ( $0.4GeV$ ).

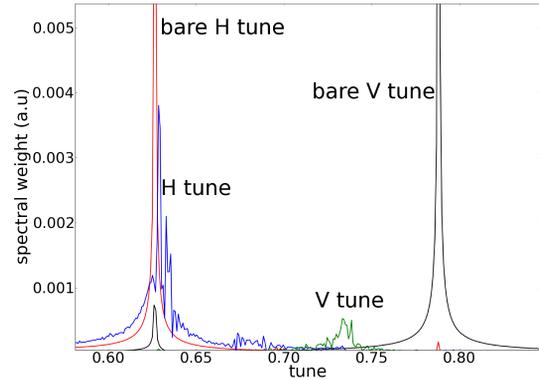


Figure 2: Fourier transform of the beam centroid horizontal and vertical displacements at intensity  $4 \times 10^{10}$  p per bunch for the full machine (84 bunches). When the collective effects are neglected (red and black), the spectral weight exhibits sharp peaks for frequencies corresponding to the bare tunes. The spectral weight shows small positive horizontal (blue) and large negative vertical (green) tune shifts when the collective effects are present. Note the wide spectral features when the collective effects are included.

The coherent tune was determined by Fourier transforming the beam centroid displacement as a function of the trajectory length. The spectral analysis is shown in Fig. 2. When no collective effects are included, the spectrum shows sharp peaks at frequencies corresponding to

the bare tunes. With the collective effects taken into account, the spectral weight shows small positive horizontal and large negative vertical tune shifts. Aside from that, the spectral features are broad, indicating the evidence of an interaction between multiple modes.

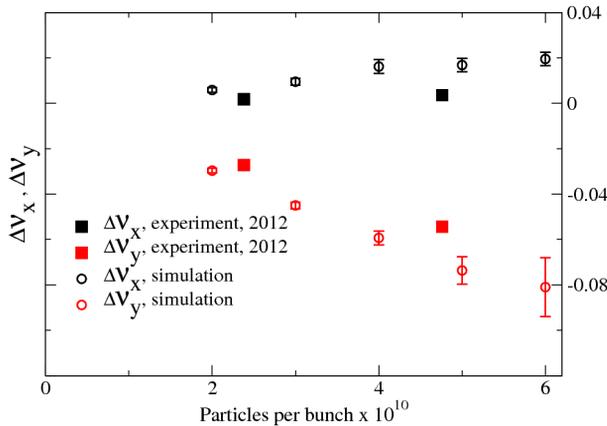


Figure 3: Comparison of the calculated coherent tune shifts for a full machine at injection with measurements. The simulation shows a slightly larger tune shift.

A comparison of the simulated coherent tune shift at injection with measurements [4] is shown in Fig. 3. As in the experiment, the simulations show a large decrease of the vertical tune and a small increase of the horizontal tune with increasing intensity. Our simulation shows a slightly larger tune shift ( $\approx 10\% - 15\%$ ) than the experiment. However, note the large error bar of the simulated tune shift which increases with increasing the intensity. The error bar is a consequence of the difficulty to uniquely determine the tune for a wide spectral feature.

Single-bunch simulations (not shown) suggest that, regarding beam loss, it is most favorable to have small values of chromaticity,  $\frac{\omega_{\xi x}}{\beta c}, \frac{\omega_{\xi y}}{\beta c} \leq 2\pi \times 0.023m^{-1}$ . An increase of chromaticity results in an increase of the bunch transverse spatial dimensions. More particles hit the vacuum chamber walls and the beam loss is larger.

However, full machine (i.e. 84 bunch) simulations find that the beam is horizontally unstable for small horizontal chromaticity. The beam centroid horizontal displacement versus the turn number is shown in Fig.4 for different values of the horizontal chromaticity. This behavior is in very good agreement with the experiment [5]. In order to stabilize the beam, a large horizontal chromaticity,  $\frac{\omega_{\xi x}}{\beta c} = 2\pi \times 0.091m^{-1}$  [11] (which corresponds to  $\xi_x = -19$  in Ref. [5]), should be chosen.

Simulations with a smaller number of bunches, occupying subsequent buckets also exhibit horizontal instability. In Fig. 5 the horizontal centroid displacement for different bunches in a 14-bunch train is shown. The instability is more pronounced for the bunches traveling at the end of the train. This result indicates that the instability is caused by short-range bunch-bunch interaction rather than by coupling to a resonant mode.

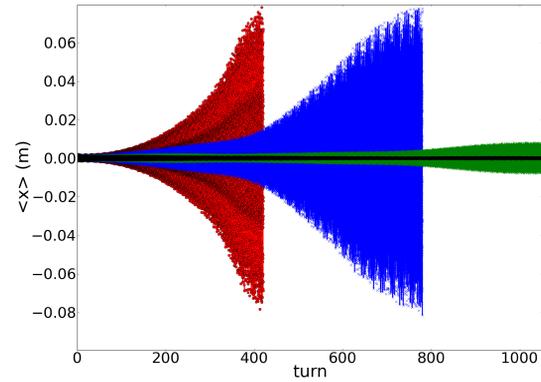


Figure 4: Full machine simulation. Beam centroid horizontal displacement at BPMs location versus turn number for different horizontal chromaticities,  $\frac{\omega_{\xi x}}{\beta c} = 2\pi \times 0.023m^{-1}$  (red),  $\frac{\omega_{\xi x}}{\beta c} = 2\pi \times 0.046m^{-1}$  (blue),  $\frac{\omega_{\xi x}}{\beta c} = 2\pi \times 0.069m^{-1}$  (green) and  $\frac{\omega_{\xi x}}{\beta c} = 2\pi \times 0.091m^{-1}$  (black). The vertical chromaticity is kept constant,  $\frac{\omega_{\xi y}}{\beta c} = 2\pi \times 0.023m^{-1}$ . The intensity is  $5 \times 10^{10}$  p per bunch (i.e.  $4.2 \times 10^{12}$  p per batch). The beam shows horizontal instability unless a large horizontal chromaticity is considered, similar to the experiment [5]. In our units, the chromaticity is  $\frac{\omega_{\xi x}}{\beta c} = 2\pi \times 0.091m^{-1}$ , which stabilizes the beam, corresponding to  $\xi_x = -19$  in Ref. [5].

In order to understand the contribution of the different wake terms to the instability we performed simulations with modified wakes. First, we turned direct space charge [12] off in our simulations. The instability is still present, but the growth rate is smaller. Next, we ran simulations with the wake function terms (see Eqs. 1, 2, 3) modified. The instability growth rate is influenced very little by the multiplication of the longitudinal wake,  $W^{\parallel}$ , or of the transverse wakes caused by the displacement of the witness particle,  $W_x$  and  $W_y$ , with a factor of two or by neglecting them (not shown). We also find that the vertical wake  $W_Y$  caused by the source displacement, has little contribution to the instability. This can be seen from Fig. 6, where the number of particles and centroid horizontal displacement are shown for both the original wakes and for enhanced  $W_Y$ . Nevertheless, we find that the instability is very sensitive to the horizontal wake  $W_X$ , as illustrated in Fig. 6. An increase of  $W_X$  by a factor of 1.5 strongly enhances the instability.

Considering that the vertical wake in the laminated magnets is about two times larger than the horizontal wake (remember Fig. 1 -c), it might seem counter intuitive that the system is prone to horizontal instability and not to vertical instability. However, note that the momentum kick is proportional to the wake coupled to the displacement. In a rough approximation, the instability growth rate should be proportional to the average over the machine length of the lattice beta function multiplied by the wake. Taking into

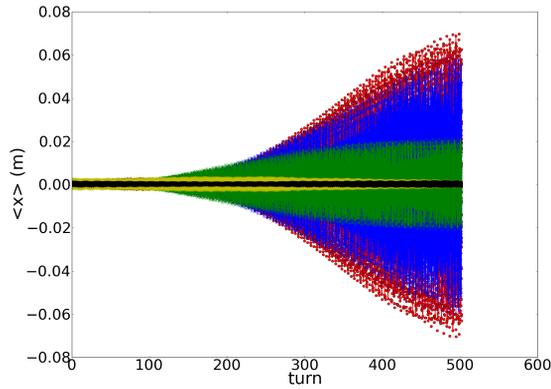


Figure 5: 14 bunch simulation with subsequent buckets occupied.  $\frac{\omega_{\xi x}}{\beta_c}, \frac{\omega_{\xi y}}{\beta_c} = 2\pi \times 0.023m^{-1}$ . The intensity is  $5 \times 10^{10}$  p per bunch. The bunch centroid horizontal displacements versus turn number. 14<sup>th</sup> (red, the last bunch), 13<sup>th</sup> (blue), 9<sup>th</sup> (green), 4<sup>th</sup> (yellow), 1<sup>st</sup> (black, the leading bunch). The motion of the trailing bunches is unstable. The instability is caused by short-range bunch-bunch interaction rather than by a coupling to a resonant element.

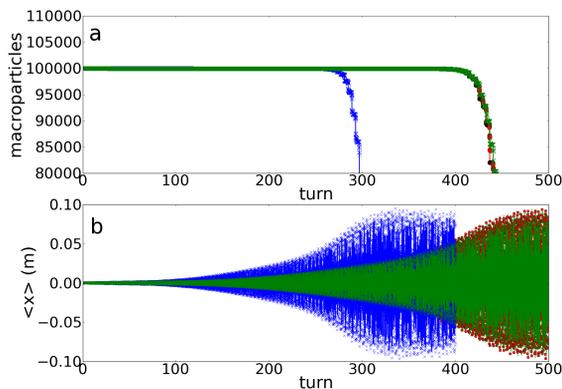


Figure 6: The number of macroparticles a) and beam centroid horizontal displacement b) versus turn number for different values of the wake functions.  $\frac{\omega_{\xi x}}{\beta_c}, \frac{\omega_{\xi y}}{\beta_c} = 2\pi \times 0.023m^{-1}$ . The intensity is  $5 \times 10^{10}$  p per bunch. The direct space charge [12] is turned off. The instability is present for unmodified wakes (the ones corresponding to the real machine, red) and is very little affected by the increase of the vertical wake. ( $W_Y$  increased two times shown with green). However it is strongly enhanced by the increase of the horizontal wake caused by the source particle. ( $W_X$  increased 1.5 times shown with blue).

account that the average horizontal beta over the F magnets length is about 3.5 times larger than the corresponding average vertical beta ( $\langle \beta_x \rangle_F = 27.75$ ,  $\langle \beta_y \rangle_F = 8.15$ ), while the averages over the D magnets are of similar magnitude ( $\langle \beta_x \rangle_D = 12.78$ ,  $\langle \beta_y \rangle_D = 16.78$ ), one can conclude that the instability is caused by the large horizon-

tal beta wake coupling at the location of the F magnets.

## CONCLUSIONS

Experimental measurements show the presence of large wake fields and horizontal beam instability in the Fermilab Booster. We calculate the wakes in the laminated magnets and find them to be four order of magnitude larger than in the metallic straight section. Using Synergia we ran single- and multibunch simulations with a realistic lattice model, space charge and wake fields. The simulated coherent tune shift and the instability behavior are in good agreement with experiment. We find that the horizontal instability is caused by short-range bunch-bunch interaction rather than by a coupling to a resonant element. The instability is due to the large coupling between the horizontal wake and the horizontal lattice beta function at the location of the F magnets.

## ACKNOWLEDGMENT

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- [10] Meghan McAteer, private communication.
- [11] We define the chromaticity as  $\omega_{\xi} = \omega_0 \frac{\xi}{\eta}$ , where  $\omega_0$  is the revolution angular frequency,  $\eta = \frac{p\delta\nu}{\delta p}$  and  $\eta$  is the slippage factor.
- [12] By direct space-charge we mean the effect of the electromagnetic field created in a perfect conductor beam pipe. It is the result of the space-charge 3D Poisson solver with closed boundary conditions. The remaining effects of the electromagnetic field are included in the wake functions.