LASER-UNDULATOR FEL WITH NEARLY COPROPAGATING LASER PULSE*

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Abstract

A free-electron laser (FEL) may be obtained by interaction of an electron bunch and a nearly copropagating laser pulse, where the laser pulse is sheared to extend the interaction length. When the angle between the electron beam axis and the propagation direction of the laser pulse greatly exceeds the reciprocal of the relativistic factor, the output radiation is nearly on the axis, and the FEL may be approximated by an axially aligned magnetostatic undulator. For a feasible laser-undulator FEL, we calculate gain using a one-dimensional model, as well as a three-dimensional model that includes degradation from diffraction and electron velocity spread.

INTRODUCTION

An electromagnetic plane wave that collides head-on with an ultrarelativistic electron bunch may create an FEL [1] that amplifies xrays whose frequency is ~$4\gamma^2$ times that of the incident "pump" wave [2], where γ is the electron bunch's relativistic factor. To obtain softer xrays, one can propagate a pump wave in nearly the same direction as the electron beam. By using a sheared laser pulse [3, 4], the electromagnetic field affecting the bunch is similar to a plane wave over an extended interaction length [5–8].

For the case where the propagation directions of the electron beam and the pump wave differ by an angle >> $1/\gamma$, a one-dimensional (1D) FEL model predicts maximum gain for xrays whose propagation direction differs from that of the electron beam by an angle << $1/\gamma$, and nearly identical gain for xrays that propagate on the electron beam axis. For this practical case, a magnetostatic undulator that is aligned with the electron beam approximates the laser undulator.

For a circularly polarized laser-undulator FEL with output wavelength of 2.5 nm described in Ref. [8], we use this magnetostatic undulator to calculate gain using the 1D model as well as a three-dimensional (3D) model [9].

ONE-DIMENSIONAL MODEL

The 1D model describes a uniform density of cold electrons interacting with a plane electromagnetic pump wave. In this model, maximum gain occurs for the electromagnetic wave that is a 180º reflection of the pump wave, when viewed in the beam frame of reference where the average electron velocity is zero [10].

In the laboratory frame, consider a uniform density of cold electrons drifting with average velocity β^*c, where c is the speed of light, interacting with a pump wave. Let the z-axis describe the direction of the average electron motion while interacting with the pump laser, which propagates at an angle of θ_{pump} w.r.t the z-axis. A head-on collision is described by $\theta_{pump} = 180º$.

For small laser intensity, the forced electron oscillations in the beam frame are nonrelativistic, and the average axial velocity of the electrons in the lab frame is given by $\beta^* \approx \beta = (1-1/\gamma^2)^{1/2}$. In this case, the Lorentz-invariant undulator parameter K is << 1, where [2, 5, 8]

$$K^2 = 2r_e I\lambda_{pump}^2 / (\pi mc^3)\,, \tag{1a}$$

$$K^2 = r_e I\lambda_{pump}^2 / (\pi mc^3)\,. \tag{1b}$$

Equation (1a) describes a planar undulator created by a linearly polarized pump wave, while eq. (1b) describes a helical undulator created by circular polarization. Here, r_e is the classical radius of the electron, I is the pump laser intensity, λ_{pump} is the pump wavelength and m is the electron mass. For larger values of K, the forced electron oscillations are relativistic in the beam frame. In the lab frame, $(K/\gamma)c$ is the maximum transverse electron velocity driven by the pump wave.

In the laboratory frame, the angular 4-frequency (c times the 4-vector) of the pump wave is

$$\overline{\omega}_{pump} = (\omega_{pump}, \omega_{pump}\sin\theta_{pump}, 0, \omega_{pump}\cos\theta_{pump})\,. \tag{2}$$

A Lorentz transform gives the angular 4-frequency in the beam frame, which is moving at the velocity β^*c [11]

$$\overline{\omega}'_{pump} = (\gamma^*\omega_{pump}(1-\beta^*\cos\theta_{pump}), \omega_{pump}\sin\theta_{pump}, 0, \gamma^*\omega_{pump}(\cos\theta_{pump}-\beta^*))\,, \tag{3}$$

where the axial relativistic factor is defined by $\gamma^* = (1-\beta^{*2})^{-1/2}$. In the beam frame, the pump wave has angular frequency

$$\omega'_{pump} = \gamma^*\omega_{pump}(1-\beta^*\cos\theta_{pump})\,, \tag{4}$$

and direction of propagation obeying

$$\cos\theta'_{pump} = \frac{\cos\theta_{pump}-\beta^*}{1-\beta^*\cos\theta_{pump}}\,. \tag{5}$$

For many cycles of interaction, the maximum FEL gain occurs for the wave given in the beam frame by a 180º reflection of the pump wave [10]. This wave is given in the beam frame by the same frequency as the pump

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$$\omega'_{xray} = \omega'_{pump} = \gamma^* \omega_{pump} (1-\beta^* \cos\theta_{pump}) , \quad (6)$$

and opposite direction of propagation as the pump

$$\cos\theta'_{xray} = -\cos\theta'_{pump} = -\frac{\cos\theta_{pump} - \beta^*}{1-\beta^*\cos\theta_{pump}} . \quad (7)$$

The 4-frequency of this wave, in the beam frame, is given by multiplying the spatial components of $\overline{\omega}'_{pump}$ by –1:

$$\overline{\omega}'_{xray} = (\gamma^* \omega_{pump} (1-\beta^* \cos\theta_{pump}), -\omega_{pump} \sin\theta_{pump}, 0, -\gamma^* \omega_{pump} (\cos\theta_{pump} - \beta^*)). \quad (8)$$

A Lorentz transformation gives the 4-frequency for maximum FEL gain in the laboratory frame

$$\overline{\omega}_{xray} = (\gamma^{*2} \omega_{pump}[1+\beta^{*2} - 2\beta^* \cos\theta_{pump}], -\omega_{pump} \sin\theta_{pump}, 0, \gamma^{*2} \omega_{pump}[2\beta^* - (1+\beta^{*2})\cos\theta_{pump}]). \quad (9)$$

In the laboratory frame, the wave with maximum FEL gain has angular frequency

$$\omega_{xray} = \gamma^{*2} \omega_{pump} (1+\beta^{*2} - 2\beta^* \cos\theta_{pump}) , \quad (10)$$

and propagation angle w.r.t. the *z*-axis of θ_{xray}, where

$$\cos\theta_{xray} = \frac{2\beta^* - (1+\beta^{*2})\cos\theta_{pump}}{1+\beta^{*2} - 2\beta^* \cos\theta_{pump}} . \quad (11)$$

MODELING BY AN AXIALLY ALIGNED MAGNETOSTATIC UNDULATOR

Consider the practical case where the pump propagation direction differs sufficiently from that of the bunch so that $\theta_{pump} >> 1/\gamma$. In this case $\theta'_{pump} >> \pi/2$ and $\theta'_{xray} << \pi/2$, while the output angle obeys $\theta_{xray} << 1/\gamma$. Provided that the undulator strength *K* does not greatly exceed one, $K\theta_{xray} << 1/\gamma$ is obeyed, so that the FEL gain for a wave propagating in the *z*-direction nearly equals the maximum gain occurring in the direction θ_{xray} [12]. We approximate the FEL with a magnetostatic undulator aligned to the *z*-axis, whose xray wavelength and *K* equal those of the laser undulator. For magnetostatic undulator period λ_u and electron oscillation frequency $\omega_u = 2\pi\beta^* c/\lambda_u$, the xray wavelength and frequency obey [1]

$$\lambda_{xray} = \lambda_u /(2\gamma^{*2}) \quad , \quad \omega_{xray} = 2\gamma^{*2} \omega_u / \beta^* , \quad (12)$$

Thus, the axially aligned magnetostatic undulator has period

$$\lambda_u = \frac{2}{1+\beta^{*2} - 2\beta^* \cos\theta_{pump}} \lambda_{pump} \approx \frac{\lambda_{pump}}{1-\beta^* \cos\theta_{pump}} , \quad (13)$$

and electron oscillation angular frequency

$$\omega_u = (\beta^*/2)(1+\beta^{*2} - 2\beta^* \cos\theta_{pump})\omega_{pump} \approx (1-\beta^* \cos\theta_{pump})\omega_{pump} . \quad (14)$$

For head-on laser interaction, $\omega_u \approx 2\omega_{pump}$ [2].

For a general value of *K*, the average axial electron velocity in the undulator is $\beta^* c$, where [1]

$$\beta^* = \sqrt{1-(1+K^2/2)/\gamma^2} , \quad (15a)$$

$$\beta^* = \sqrt{1-(1+K^2)/\gamma^2} . \quad (15b)$$

Equation (15a) describes a planar undulator, while eq. (15b) describes a helical undulator.

In the 1D model, the FEL gain is parameterized by the Pierce parameter ρ which obeys [1]

$$\rho^3 = (n_e e^2 K^2 [JJ]^2)/(32\omega_u^2 m\varepsilon_0 \gamma^3) , \quad (16a)$$

$$\rho^3 = (n_e e^2 K^2)/(16\omega_u^2 m\varepsilon_0 \gamma^3) , \quad (16b)$$

where eq. (16a) describes a planar undulator while eq. (16b) describes a helical undulator. Here, *e* is the electron charge, n_e is the electron density, and ε_0 is the permittivity of free space. The term [*JJ*] is a Bessel function factor

$$[JJ] \equiv J_0(K^2/[4+2K^2]) - J_1(K^2/[4+2K^2]) , \quad (17)$$

which is nearly equal to 1 for $K << 1$.

For low gain G << 1, the energy gain for a laser undulator with *N* cycles is [1]

$$G = 2(4\pi)^3 \rho^3 N^3 [(2-2\cos\phi - \phi\sin\phi)/\phi^3] , \quad (18)$$

where $\phi \equiv 2\pi N - 2\pi N_{rad}$ is the phase slippage through the undulator, in which N_{rad} is the number of cycles of amplified radiation which overtake an average electron while passing through the undulator. For this low-gain case, an evaluation of the laser-undulator gain in the electron beam frame [10] confirms that it equals that of the axially aligned magnetostatic undulator. The largest gain occurs for a slippage of 2.61 radians [10]

$$G_{max} = 0.27(4\pi)^3 \rho^3 N^3 . \quad (19)$$

In the high gain limit, the xray radiation power grows exponentially in a gain length [1, 2]

$$L_G = \lambda_u /(4\pi\sqrt{3}\rho) . \quad (20)$$

In principle, a crab cavity can be used to tilt the bunch so that the gain medium provided by the electrons is aligned in the direction of maximum gain.

Without a crab cavity, the gain medium extends farthest in the axial direction, and the gain in the axial direction nearly equals the maximum in the direction θ_{xray} [12]. The self-amplified stimulated emission (SASE) is expected to propagate at a small angle equal to or smaller than that given by eq. (11), with gain nearly equal to that of the axially aligned magnetostatic undulator. For a seeded FEL, amplification of an axially propagating xray is nearly the same as in the axially aligned magnetostatic undulator.

THREE-DIMENSIONAL MODELING

In Ref. [8], a 300-period circularly polarized laser undulator with *K* = 0.4336, produced by a 750-nm pump

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laser propagating at an angle of θ_{pump} = 0.0572 radian w.r.t. a 168.65-MeV electron beam, produces output at λ_{xray} = 2.5 nm. Equation (11) predicts maximum gain for θ_{xray} = 0.19 mrad << $1/\gamma$, in agreement with eqs. (5.42) and (5.43) of Ref. [6]. We approximate the laser undulator with an axially aligned helical magnetostatic undulator with period of 0.458 mm and length of 0.137 m.

In Ref. [9], M. Xie uses numerical fits to simulations to provide a 3D model for the degradation of high-gain FEL performance from diffraction and the electron axial velocity spread due to slice emittance and slice energy spread. Using 1D and 3D models, we evaluated gain for the ideal case where a cylindrically symmetric electron beam is focused in the center of a magnetostatic undulator where the horizontal and vertical beta functions equal one-half of the undulator length. The average beam size in the undulator is given by horizontal and vertical beta functions equal to 66% of the undulator length. Table 1 lists the parameters that were modeled.

Table 2 shows calculated FEL properties—the Pierce parameter, exponential gain length, gain saturation length, and saturation power—for the 1D and 3D models. Comparing the models shows that the gain length is increased 50% by diffraction and the bunch's axial velocity spread. According to the 3D model, the FEL gain is $e^{9.2}$ =10,000 within the 0.137-m interaction length. SASE saturation, which requires interaction over 19 gain lengths, is not achieved. Effects from nonuniformity of the laser undulator are not included in these models.

The FEL properties depend smoothly upon the beam parameters and *K*-value, indicating that a small deficiency in one parameter can be compensated by improving a different parameter. The 1D Pierce parameter of 0.00215 is comparable to the relative slice energy spread of 0.001, so that the 3D gain length increases substantially if the energy spread is increased, as shown in Fig. 1(a). Figure 1(b) shows that increasing the normalized emittance to 0.2 mm-mrad doubles the 3D gain length.

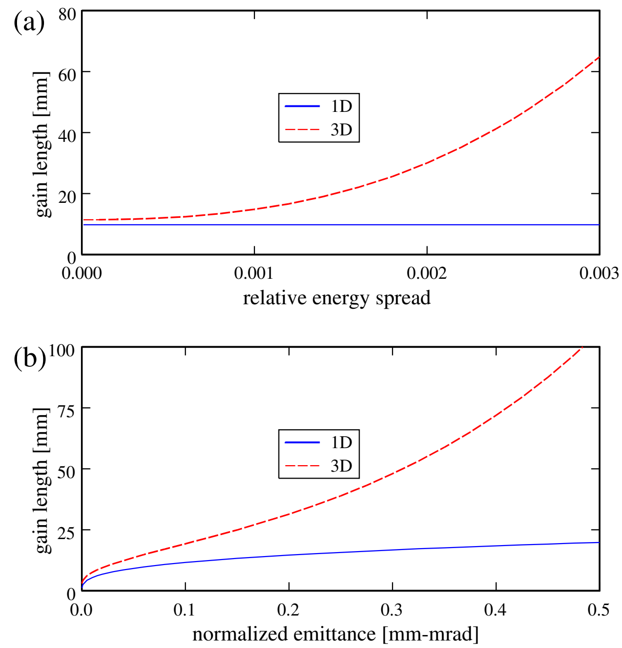

Figure 1: (a) Gain length vs. slice energy spread. (b) Gain length vs. normalized slice emittance.

SUMMARY

For a 300-period laser-undulator FEL created by interaction of an electron bunch with a nearly copropagating laser pulse, the FEL gain and saturation have been calculated by approximating the laser undulator with an axially aligned magnetostatic undulator. Xrays with a wavelength of 2.5 nm are predicted to by amplified by $e^{9.2}$ = 10,000 within the 0.137-m interaction length.

Table 1: Electron Beam and Undulator Properties

Beam energy	168.65 MeV
Slice energy spread	168.65 keV
Slice emittance (normalized)	0.06 mm-mrad
Peak current	800 A
β-function ($\beta_x = \beta_y$)	0.09 m
Undulator *K* value	0.4336
Output xray wavelength	2.5 nm

Table 2: Calculated FEL Properties

Calculated parameter	**1D model**	**3D model**
Pierce parameter (ρ)	0.00215	0.00143
Gain length [mm]	9.78	14.8
Gain saturation length [m]	0.186	0.275
Saturation power [GW]	0.29	0.204

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