

# ANALYTICAL EVALUATION OF CORRELATED TIMING JITTER CANCELLATION IN A STAGED BUNCH COMPRESSION SYSTEM\*

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## Abstract

In this paper, the wakefield cancellation scheme on the timing jitter is revisited. The correlated timing jitter effects between the photoinjector laser and the linac rf phase are evaluated analytically. It is possible to minimize its impact on the final bunch length (peak current) variation by employing a longer linac with a lower acceleration gradient between bunch compressors one and two.

## OVERVIEW

A staged magnetic bunch compression system is widely adopted in the acceleration process of the electron beam. It first introduces an energy modulation along the electron bunch in its longitudinal direction, then lets the electron beam pass by a dispersive region where electrons in the head and tail of the bunch all move relatively towards bunch center. Chicane- and wiggler-based bunch compressor designs were proposed and studied thoroughly in the 1990s, mainly for linear collider projects [1], [2], [3].

The stability of the final electron beam current depends on the timing and rf phase jitter, bunch compressor parameters, and electron beam charge variations. Gun laser to linac rf timing jitter (referred to as timing jitter)—the main timing error to be discussed in this paper—is considered to be correlated for all the linac sections as the sum of the uncorrelated (or random) rf phase jitter (different from rf source-to-source) tends to be small.

The bunch length variation due to jitter effects at the ends of the staged bunch compressors were studied in (and before) the LCLS initial design stage, and a longitudinal wakefield cancellation scheme was proposed with at least two stages of bunch compressions [4], [5], [6], [7], [8]. This scheme was also applied at the LCLS FEL [5], [7], [8]. A similar scheme is also proposed to reduce the energy chirp by employing the wake of coherent synchrotron radiation [9].

In this paper, this topic is revisited, while the associated formulae are rederived and extended analytically. The analytical result is verified by numerical simulations. It is possible to minimize the correlated timing jitter's impact on the final bunch length (peak current) variation by employing a longer linac with a lower acceleration gradient, between bunch compressors one and two.

## ANALYTICAL DERIVATION

Neglecting the small initial un-correlated energy spread and considering only linear terms, the RMS bunch length after bunch compressor one can be expressed as

$$\sigma_{z1} = (1 + h_1 R_{56(1)}) \cdot \sigma_{z0}, \quad (1)$$

where  $\sigma_{z1}$  denotes the RMS bunch length after bunch compressor one,  $h_1$  is the chirp of bunch compressor one,  $R_{56(1)}$  is the first-order longitudinal dispersion in bunch compressor one, and  $\sigma_{z0}$  is the initial bunch length; and

$$h_1 = -\frac{k_1 e V_0 \sin \phi}{E_{f0}} \approx -k_1 \tan \phi \approx -k_1 \phi_1, \quad (2)$$

where  $k_1 = \frac{2\pi_1}{\lambda}$  denotes the rf wave number in the first rf section,  $E_{f0}$  is the central energy after rf acceleration,  $e$  is the electron charge,  $V_0$  is the rf voltage,  $\phi_1$  is the rf phase,  $k = \frac{2\pi}{\lambda}$  is the rf wave number, and  $\lambda$  is the rf wave length.

Given a timing jitter  $\Delta\phi_1$ , the change in the final bunch length is

$$\Delta\sigma_{z1} = -k_1 \Delta\phi_1 R_{56(1)} \cdot \sigma_{z0}. \quad (3)$$

Neglecting the residual damped energy chirp from the first rf linac  $h_1 \cdot C_1 \cdot E_1/E_2$  ( $C_1 = 1/(1 + h_1 R_{56(1)})$ ) denotes the compression ratio in BC1. Also the longitudinal wakefield effects in Linac2 only consider an energy chirp established in Linac2, which is  $h_2$ . The final RMS bunch length after the second bunch compressor can be calculated as

$$\sigma_{z2} = (1 + h_2 R_{56(2)}) \cdot \sigma_{z1}, \quad (4)$$

$$h_2 \approx -k_2 \phi_2, \quad (5)$$

where  $\sigma_{z2}$  denotes the RMS bunch length after bunch compressor two,  $h_2$  is the chirp of bunch compressor two,  $R_{56(2)}$  is the first-order longitudinal dispersion in bunch compressor two, and  $k_2 = \frac{2\pi_2}{\lambda}$  is the rf wave number in the second rf section.

After inserting the expression  $\sigma_{z1}$  into Eq. (4), one finds

$$\sigma_{z2} = (1 + h_2 R_{56(2)}) \cdot (1 + h_1 R_{56(1)}) \cdot \sigma_{z0}. \quad (6)$$

Given a linac rf timing jitter of  $\Delta\phi_1$  and  $\Delta\phi_2$  in the first and second rf linacs, the final bunch length is

$$\sigma_{z2} = [1 - k_2(\phi_2 + \Delta\phi_2)R_{56(2)}] \cdot [1 - k_1(\phi_1 + \Delta\phi_1)R_{56(1)}] \cdot \sigma_{z0}. \quad (7)$$

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The change of bunch length after the second bunch compressor is

$$\begin{aligned} \Delta\sigma_{z2} = & [-k_2\Delta\phi_2 R_{56(2)}(1 - k_1\phi_1 R_{56(1)}) \\ & -k_1\Delta\phi_1 R_{56(1)}(1 - k_2\phi_2 R_{56(2)}) \\ & +k_1k_2\Delta\phi_1\Delta\phi_2 R_{56(1)}R_{56(2)}] \cdot \sigma_{z0}. \end{aligned} \quad (8)$$

Inserting the bunch compression ratio in the first and second stage of bunch compressors,  $C_1 = 1/(1 + h_1 R_{56(1)})$  and  $C_2 = 1/(1 + h_2 R_{56(2)})$ , into the above formulae, one finds

$$\begin{aligned} \Delta\sigma_{z2} = & [-k_2\Delta\phi_2 R_{56(2)}/C_1 \\ & -k_1\Delta\phi_1 R_{56(1)}/C_2 \\ & +k_1k_2\Delta\phi_1\Delta\phi_2 R_{56(1)}R_{56(2)}] \cdot \sigma_{z0}. \end{aligned} \quad (9)$$

Assuming the first and second rf linac have the same rf frequency  $k_1 = k_2 = k$  (which also means that the same time jitter introduces the same change in the rf phase,  $\Delta\phi_1 = \Delta\phi_2 = \Delta\phi$ ), one finds that the above formula can be simplified to

$$\begin{aligned} \Delta\sigma_{z2} = & k\Delta\phi[-R_{56(2)}/C_1 - R_{56(1)}/C_2 \\ & +k\Delta\phi R_{56(1)}R_{56(2)}] \cdot \sigma_{z0}. \end{aligned} \quad (10)$$

From the above formula, one observes that in order to minimize the impact from timing jitter on the final bunch length, one needs to employ a linac with lower frequency rf (smaller  $k$ ) and smaller longitudinal dispersion  $R_{56}$  in the bunch compressors. In order to keep similar compression ratios  $C_1$  and  $C_2$ , a larger rf phase should be employed, given a required smaller longitudinal dispersion  $R_{56}$  in the bunch compressors. Further, assuming the same timing jitter for both the first and second rf linacs, the impact from both compression stages on the final bunch length always adds up.

Another effect that could be used to partially compensate the timing jitter is the longitudinal wakefield in the second rf linac [4], [5], [6], [7], [8]. The basic idea is that if a timing jitter in the first rf linac introduces a stronger compression in bunch compressor one, which in turn results in a shorter bunch length in the second rf linac, the energy chirp established by the longitudinal wakefield in the second rf linac is larger. For an under-compression scheme with normal four-dipole chicane compressor, the rising slope of the rf wave is employed (electron bunch is ahead of rf crest), which means that the bunch head has a lower energy than its tail. As the longitudinal wakefield always makes the bunch tail lose energy with respect to its head, the energy chirp from the wakefield has a sign opposite the rf chirp in the second rf linac. The timing jitter in the second rf linac is assumed to be the same as the one in the first rf linac, so this relative change in rf phase also introduces a stronger compression in bunch compressor two. However,

the wakefield chirp fights the energy chirp established by the second rf linac and makes the bunch compression ratio smaller in this second stage, which partially compensates the timing jitter effects.

With the longitudinal wakefield effects in both the first and second rf linac included, the final bunch length is rewritten as

$$\begin{aligned} \sigma_{z2} = & [1 + (h_2 + h_{2w})R_{56(2)}] \\ & \cdot [1 + (h_1 + h_{1w})R_{56(1)}] \cdot \sigma_{z0}, \end{aligned} \quad (11)$$

where  $h_{1w}$  denotes the linear energy chirp induced by longitudinal wakefield in the first rf linac, and  $h_{2w}$  is the linear wakefield energy chirp established in the second rf linac.

Given an electron beam energy of 250 MeV at bunch compressor one, a short first rf linac is employed, which means that the wakefield-induced chirp  $h_{1w}$  is relatively small and could be negligible. As discussed above, the wakefield chirp  $h_{2w}$  has a sign opposite the rf chirp  $h_2$ , which should be taken into account when the FEL bunch compression system is designed. Basically, it is the change in wakefield-generated energy chirp that could compensate a timing jitter between laser (electron bunch generation) and linac rf. Based on this point, a second rf linac with longer length should have a stronger relative change in wakefield-induced chirp when the electron bunch length is affected in a first-stage bunch compressor by timing jitter effect. A final electron bunch length at the end of bunch compressor two is then expressed as shown below, with timing jitter and wakefield effects considered up to first order. The rf phase  $\phi_1$  and  $\phi_2$  denote the ones that already have wakefield-induced energy chirp subtracted:

$$\begin{aligned} \sigma_{z2} = & [1 - k_2(\phi_2 + \Delta\phi_2) \\ & -D(\sigma_{z1}, k_2) \cdot L_{linac2})R_{56(2)}] \\ & \cdot [1 - k_1(\phi_1 + \Delta\phi_1)R_{56(1)}] \cdot \sigma_{z0}, \end{aligned} \quad (12)$$

where  $D(\sigma_{z1}, k_2)$  denotes the unit-length change of wakefield-induced energy chirp in linac2, which is a function of rf structure (frequency) and electron bunch length in Linac2; and  $L_{linac2}$  is the length of Linac2.

In order to keep the same beam energy at BC2, the acceleration gradient in the second rf linac needs to be decreased. Another point is that, as the the second rf linac length is increased, the nominal wake-induced energy chirp is larger, which means that one has to adopt a larger rf phase in the second rf linac to cancel the wake effects and provide the necessary energy chirp. As discussed above, a larger rf phase also makes the timing jitter effect relatively smaller.

## NUMERICAL SIMULATION

ELEGANT [10] simulations of a low-charge X-band hard x-ray FEL [11] are performed with timing jitter set on all linac rf phases up to 50 fs, which is successfully achieved in LCLS operation [12]. This timing jitter of 50 fs changes

the Linac1 rf phase and in turn changes the electron bunch length at the end of bunch compressor one. The final relative RMS bunch length variation is around 20 % with the 35-meter-long Linac2, given a timing jitter of 50 fs between laser and linac rf.

One then can decrease the acceleration gradient in the second rf linac and increase the second rf linac length (employing more rf cavities), keep the same beam energy at bunch compressor two, and employ a stronger longitudinal wakefield in the second rf linac to cancel the timing jitter effect. However, a longer total accelerator length and a higher total cost are accompanied with increasing the second rf linac length and employing more rf cavities. A trade-off needs to be made between the factors mentioned above and the tolerated timing jitter. Analytical and numerical studies show that decreasing the average acceleration gradient from 80 MV/m to 65 MV/m in the second rf linac could drop the final peak current variation to 12% [11]. The timing-jitter-induced peak current variation could be fully compensated if an average acceleration gradient of 50 MV/m is adopted in the second rf linac [11].

A second example is an X-band rf-driven hard x-ray FEL design with LCLS injector [13]. Similarly, the length of the second rf linac is tuned to compensate (or even over-compensate) the timing jitter impacts. As shown in Figure 1, the LCLS injector plus X-band linac case (blue curve) is much more sensitive to timing jitter than the normal LCLS case (red curve). This is due to the higher frequency of X-band rf. For these nominal cases, an acceleration gradient of 80 MV/m is adopted for X-band rf, while it is 20 MV/m for S-band rf in the LCLS. However, if one doubles the length of the second rf linac and adopts an acceleration gradient of 40 MV/m in the second X-band rf linac, the black curve shows that one could possibly cancel the timing jitter effect. In this specific example, the timing jitter effect on the final bunch length is over-compensated by the wakefields in the second X-band rf linac. One should also note that the rf phase in the second X-band rf linac is increased from 22 degrees to 33 degrees.

## CONCLUSION

The longitudinal wakefield cancellation scheme is revisited in this paper. Analytical derivations are presented for bunch length and its variation in a staged bunch compression system. Under first-order approximation, the length of the second rf linac can be calculated analytically, which compensates the timing-jitter-induced bunch length variation. Simulation results of two X-band-rf-driven FEL drivers are presented where these cancellation schemes are applied.

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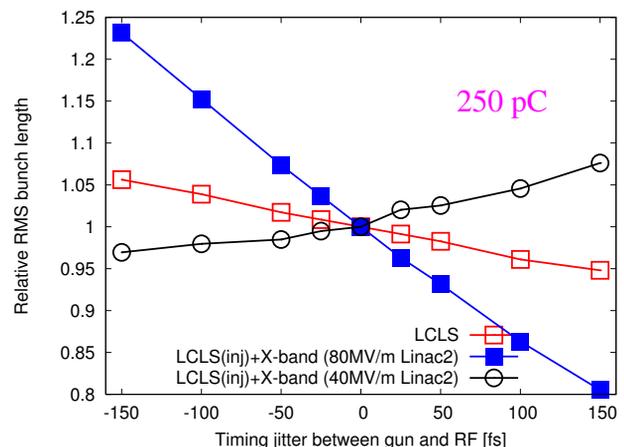


Figure 1: Relative bunch length after the final stage of bunch compression, as a function of timing jitter, for a bunch charge of 250 pC.

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