LINEAR SCALING ON CHOOSING A BUNCH COMPRESSION RATIO FOR AN FEL DRIVER*

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Abstract

In this paper, a proper initial bunch length as a function of the main rf frequency and rf phase is estimated analytically by several approaches, assuming that no harmonic rf section is needed to linearize the energy modulation introduced during main rf acceleration. Next the upper limit of the bunch compression ratio in a single stage is evaluated analytically. The analytical relations derived on choosing a proper initial bunch length as a function of main rf frequency are confirmed by numerical simulation. These simple limits provide rough analytical estimations and may be beneficial for choosing bunch compression ratios in different stages of an FEL driver, especially in a first-stage bunch compression system where an harmonic rf linearization is usually applied.

OVERVIEW

A magnetic bunch compression system is normally adopted during the acceleration process of the electron beam, usually in several (two) stages. The basic idea is to first introduce an energy modulation (chirp) along the electron bunch in its longitudinal direction by an rf acceleration off-crest, then let the electron beam pass by a dispersive region where electrons in the head and tail of the bunch all move relatively towards the bunch center. Chicane- and wiggler-based bunch compressor designs were proposed and studied thoroughly in the 1990s, mainly for linear collider projects [1] [2] [3].

Due to the nonlinear nature of the rf sinusoidal wave and an initially long bunch length, higher-order energy chirps can not be neglected. Harmonic rf linearization of the longitudinal phase space of the electron bunch is normally employed to generate a more uniformly compressed bunch in a first-stage bunch compression. This paper discusses what is the limit on the bunch compression ratio in order to achieve a quasi-linear bunch compression without harmonic rf.

PROPER INITIAL BUNCH LENGTH WITHOUT HARMONIC RF

In this section, only the main rf acceleration section is considered, and a proper initial bunch length is derived as a function of rf frequency and rf phase, given that a quasi-linear compression is achieved with a reasonable bunch compression ratio adopted for each stage. There are two possible ways to quantify a quasi-linear compression. First, the contribution to final bunch length from nonlinearities (from rf acceleration and bunch compressors) could be evaluated and limited to a certain percentage. A second way is to compare the final bunch length calculated as the root mean square of the distribution with the one derived from a numerical fit of the final distribution (Gaussian fit if the initial distribution is Gaussian), and limit the difference between these two to a certain amount.

Energy Chirp Approach

First let us recall the energy modulation formula. For any one particle in a bunch, its relative energy offset after passing by an rf acceleration off-crest can be expressed as shown below:

$$\begin{aligned}\delta(z) &= \delta_i\frac{E_{i0}}{E_{f0}} + \frac{eV_0\cos(\phi+kz_i)}{E_{f0}} \\ &= a\cdot\delta_i + \frac{eV_0\cos(\phi+kz_i)}{E_{f0}}. \end{aligned} \quad (1)$$

where δ_i denotes the initial uncorrelated energy offset, E_{i0} is the central energy before rf acceleration, E_{f0} is the central energy after rf acceleration, e is the electron charge, V_0 is the rf voltage, ϕ is the rf phase, $k=\frac{2\pi}{\lambda}$ is the rf wave number, λ is the rf wavelength, z_i is the particle's longitudinal coordinate relative to the bunch center, and $a=E_{i0}/E_{f0}$ is the energy damping ratio.

The first- and second-order energy chirp from the main rf acceleration could be described by the rf wave number, rf voltage, and rf phase, solved from the same Taylor expansions discussed above, as shown below:

$$h_1 = -\frac{keV_0\sin\phi}{E_{f0}} \quad (2)$$

$$h_2 = -\frac{k^2eV_0\cos\phi}{2E_{f0}}. \quad (3)$$

Neglecting the energy chirp above the third order, the overall energy modulation from this main rf section can then be rewritten as

$$\delta(z) = a\cdot\delta_i + (h_1+h_2\cdot z)z, \quad (4)$$

where $h_1+h_2\cdot z$ denotes the effective first-order energy chirp.

One then observes that if z is small enough to make $h_1+h_2\cdot z\approx h_1$, the overall energy modulation is almost linear; in that case harmonic rf linearization is no longer necessary. This condition could be interpreted as $|h_1|\gg|h_2\cdot\sigma_z|$ and further derived to be

$$\frac{c\cdot\tan\phi}{\pi\cdot f_{rf}\cdot\sigma_z}\gg 1. \quad (5)$$

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At this step, a proper initial bunch length can be expressed as a function of the main rf frequency and rf phase, as shown below. Here one assumes that a reasonable bunch compression ratio is adopted, with the detailed requirements discussed in the following section.

$$\sigma_{z,max} = D_0 \frac{2\tan\phi}{k_{rf}}, \tag{6}$$

where D_0 denotes an empirical constant whichthat is a small number much less than 1, which could be fitted from numerical simulation studies employing rf systems with different frequency (such as from L-band rf to X-band rf). As illustrated by the numerical simulation results below, in general D_0 should be less than 0.01 in order to maintain a relatively linear longitudinal phase space and a quasi-linear bunch compression without harmonic rf assistance. In general, the required value of D_0 depends on rf frequency, average rf phase adopted for introducing energy correlation, total compression ratio in all the stages, and the number of bunch compression stages. D_0 is inversely proportional to the rf phase and total compression ratio, while it is proportional to the number of bunch compression stages.

To generate a final hard X-ray FEL using normal photoinjector and undulator configurations, usually the electron bunch length needs to be compressed by 30-200 times of its initial value. If more stages of bunch compression are adopted, one only needs a smaller bunch compression ratio in each stage. This in turn requires only a small rf phase and small longitudinal dispersion (momentum compaction) of the bunch compressor systems. In this case, a larger constant D_0 and a longer initial bunch length can be tolerated.

Final Bunch Length Approach

A proper initial bunch length can also be evaluated by examing the expression of a final bunch length after bunch compression. Under a condition of linear optimal compression $1 + h_1 R_{56} = 0$, a similar required relationship is derived, $|h_1| \gg |h_2\sigma_z|$. Assuming the electron bunch preserves a Gaussian distribution after passing by the dispersive region, the RMS bunch length can then be calculated as the integral shown below:

$$\sigma_z{}^2 = \int\int z_f{}^2(z,\delta)\cdot f(z,\delta)dzd\delta. \tag{7}$$

Then the expression $z_f(z,\delta)$ is derived with dispersion terms of the bunch compressor up to third order. After passing by the dispersive region, the longitudinal coordinate (relative to the bunch center) of any particle can be expressed as shown below:

$$z_f(\delta) = z_i + R_{56}\delta + T_{566}\delta^2 + U_{5666}\delta^3 + \dots. \tag{8}$$

Take the correlated relative energy offset $\delta(z)$ in terms of energy chirp up to second order, keep the terms up to second order bunch compressor dispersion, and neglect the initial uncorrelated energy offset δ_i in higher-order terms above δ^2. The square of $z_f(\delta)$ is then calculated to be a polynomial of z.

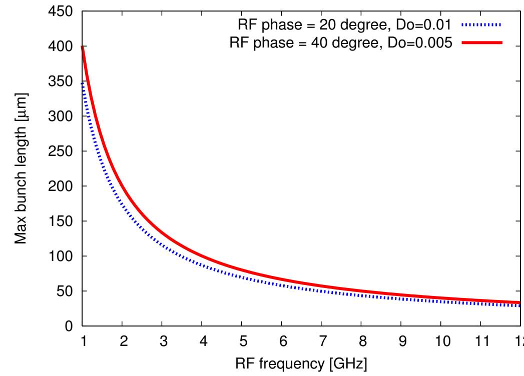

Figure 1: Proper initial bunch length as a funcion of rf frequency, for two given rf phases of 20 and 40 degrees. No harmonic rf linearization is included. A constant of $D_0 = 0.01$ is assumed for the 20-degree case, while $D_0 = 0.005$ is assumed for the 40-degree case.

One can then find the expression of the final bunch length by inserting $z_f^2(z,\delta_i)$ into the final bunch length formula andcalculate the integral. Here one needs to note that the integral of all the odd functions is zero, such as $\int\int z\cdot f(z,\delta)dzd\delta = 0$ and $\int\int z^3\cdot f(z,\delta)dzd\delta = 0$. The expression of final bunch length only has entries from the integral of all the even functions. Under a condition of linear optimal compression, $1+h_1R_{56} = 0$, one finds that the final bunch length equals

$$\begin{aligned}\sigma_{z,f}^2 &= a^2R_{56}^2\delta_i{}^2 \\ &+ [2R_{56}a\delta_i(h_2R_{56} + h_1^2T_{566} + h_2^2T_{566}\sigma_z^2) \\ &+ (h_1^2\cdot T_{566} + h_2\cdot R_{56})^2\cdot\sigma_z^2]\cdot\sigma_z^2.\end{aligned} \tag{9}$$

Again, assuming a normal four-dipole chicane is employed as the bunch compressor, there is a fixed ratio between its first-order and second-order dispersion terms R_{56} and T_{566}, as $T_{566} = -1.5R_{56}$. From the expression of final bunch length, one observes that the first term $a^2R_{56}^2\delta_i{}^2$ is from the initial uncorrelated energy spread, and the third term $(h_1^2\cdot T_{566}+h_2\cdot R_{56})^2\cdot\sigma_z^2$ implies that the second-order chirp h_2 always adds on a first-order effect given as $T_{566} = -1.5R_{56}$. However, a larger rf phase (close to on-crest point) can minimize the contribution from the third term. It is the second term that could be derived to give a similar final conclusion as discussed in the last section, with a required criteria of $|h_2R_{56} + h_1^2\cdot T_{566}| \gg |h_2^2T_{566}\cdot\sigma_z^2|$. To repeat, the required relationship associated with the maximum tolerable bunch length is $|h_1| \gg |h_2\sigma_z|$.

One could plot the proper initial bunch length as a function of rf frequency with different rf phase, as shown in Figure 1. One observes that for a lower rf frequency, the proper initial bunch length is longer, which is as expected. Another point is that if a higher rf phase can be employed, then the tolerable bunch length can be longer. In the next section, the analytical results derived above are checked and confirmed by numerical simulations with multi-stage bunch acceleration and compression based on rf systems with different frequencies.

Proper Compression Ratio in One Stage

In this subsection, a proper bunch compression ratio in one stage is discussed and an upper limit is drawn. Limited to first order, a compression ratio is approximated as shown below:

$$C = \frac{1}{1 + h_1 R_{56}}. \tag{10}$$

From the final bunch length expression one also observes that to keep an effectively linear bunch compression, there should be a limit applied on the bunch compression ratio. That means a bunch compression ratio can not be too large in one stage, otherwise the bunch compression is no longer linear.

Consider a first-order optimal bunch compression case. One has $1+h_1 R_{56} = 0$ and then could conclude that in this case, the final bunch distribution is only dependent on the nonlinear terms, which obviously breaks the linear compression requirement. So in general, there should be another criterion for a linear bunch compression that can be derived and shown in the following formula, up to second order. One needs to note that in the coefficient of term z^4, the relatively small uncorrelated initial energy offset δ_i and first-order associated term $4h_1 h_2 T_{566}(1 + h_1 R_{56})$ are neglected:

$$(1 + h_1 R_{56})^2 \gg (h_1^2 \cdot T_{566} + h_2 \cdot R_{56})^2 \cdot \sigma_z^2. \tag{11}$$

The above requirement in turn gives an upper limit on the bunch compression ratio in one stage. As there is a large acceleration in beam energy between subsequent stages, the nonlinear terms in the bunch energy chirp is also damped during acceleration. That damping effect helps to maintain a linear energy correlation. Also, as the bunch length becomes shorter from one stage to the next, the new contribution on the nonlinear energy chirp from the following linacs is smaller. Under this kind of configuration with more than two stages of bunch compression, a quasi-linear bunch profile should be easily achieved at the linac end. The requirement on bunch compression ratio in one stage (especially the first stage) is then approximated as shown below, which is a function of dispersion terms, energy chirp terms, and RMS bunch length:

$$C \ll \frac{1}{\sigma_z |h_1^2 \cdot T_{566} + h_2 \cdot R_{56}|}. \tag{12}$$

Combining all the above considerations, a bunch compression ratio between 5 and 10 may be adopted in one stage. The last point is that in general one should choose a weak chicane bunch compressor with small dispersion terms R_{56} and T_{566}, in order to minimize the contribution from higher-order terms.

Numerical simulation results [4] [5] are presented here, for two FEL driver designs based on different rf frequencies; these are the L-band rf (1.3 GHz) and the X-band rf (12 GHz) [6]. No harmonic rf correction is included here. The initial bunch length of $300\mu m$ and $40\mu m$ are adopted plus the rf frequency and phase. One could observe Figure 1 and find that $300\mu m$ and $40\mu m$ are the limited bunch lengths corresponding to the rf frequency of 1.3 GHz and 12 GHz, respectively. From the simulation results as shown below, one could conclude that the bunch compression process is quasi-linear, and a quasi-Gaussian bunch density profile is preserved. No spike in density profile is generated. These numerical simulation results confirm the validity of the above analytical derivations. It is noted that if one extends these two-stage bunch compression systems into three stages or more, the final bunch current profile could be more uniform, due to the reasons discussed above. In detail, the bunch compression ratio can be smaller in each single stage, and the dispersion terms R_{56} and T_{566} are also smaller in the bunch compressors. The relative energy-dependent-damping effect of the nonlinear energy chirp is also stronger given that more bunch compression stages are adopted.

CONCLUSION AND DISCUSSION

In this paper, analytical formulae are derived to estimate an initial proper bunch length as a function of rf frequency and rf phase in a linac-based FEL driver. These derivations may be benificial for an FEL driver design that has a mainly low bunch charge operation mode and that could achieve an initially short bunch length from the upstream photoinjector. It is also noted here that these limits only provide rough analytical estimations on the required bunch length and compression ratio, where assumptions have been made and no collective effects are included. A detailed bunch compression system design can start with these simple formulae and then be optimized by 3-D simulations with all effects included.

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