NONPARAXIAL TRANSVERSE EFFECTS ON THE PROPAGATION OF NONLINEAR ELECTROMAGNETIC PULSES *

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Abstract

In the present analysis we study the weakly nonlinear interaction between trains of electromagnetic pulses and space-charge fields in laser-plasma systems. We direct the analysis to regimes evolving with the co-moving coordinate of the beam frame, but do not make any assumptions on paraxial or underdense approximate conditions. The model thus constructed allows us to investigate regimes where transverse and longitudinal length scales of the pulses are comparable. Resonant and nonresonant regimes of space-charge wave excitation are analyzed. In both cases, with aid of analytical estimates and numerical simulations, we examine how far trains of electromagnetic pulses can travel before being affected by the destructive transverse effects.

INTRODUCTION

The propagation of localized radiation pulses in plasmas has been the subject of continuous interest in a variety of areas, including nonlinear wave excitation [1, 2] and particle acceleration [3, 4, 5]. Pulses are formed as large amplitude electromagnetic waves undergo the process of modulational instability in the plasma, breaking up into a series of narrow quasi-isolated structures. This is a typical behavior in soliton turbulence [6] and in self modulated laser accelerators, once an option in laser acceleration, whose interest has been reignited recently [7].

Pioneering works [8, 9] have studied the self-consistent problem of electromagnetic pulses propagating in plasmas and the space-charge waves generated by this propagation in one dimensional models. These models, however, usually refers to nonparaxial solutions (where dependence of the field on transverse coordinates is completely neglected) or the opposite, the paraxial limit (where transverse effects dominate the dynamics). In several occasions, a laser beam reaches operational conditions where longitudinal and transverse sizes become similar in magnitude [10, 11]. Under these conditions, an accurate account of the dynamics should treat transverse and longitudinal effects on the same footing.

The objective of the present work is to perform an analysis of the nonlinear interaction of electromagnetic and space-charge without invoking 1D, paraxial, or approximate nonparaxial approaches [12, 13]. Our main interest will be to determine how the the transverse structure affects a train of solitons, typically formed after an initially injected pulse breaks up through self-modulation [14].

THE MODEL

We can model this system with the following coupled equations for the laser field a and for the space-charge potential $\varphi \equiv v_g^2 n - a^2/2$,

$$\kappa \frac{\partial^2 a}{\partial \xi^2} = \delta a + \frac{1}{v_g^2}\varphi\, a + \frac{\kappa}{2v_g^2} a^3 - \nabla_\perp^2 a, \tag{1}$$

$$\frac{\partial^2 \varphi}{\partial \xi^2} + \frac{1}{v_g^2}\varphi = -\frac{1}{2v_g^2} a^2 + \frac{1}{2}\nabla_\perp^2 a^2, \tag{2}$$

derived in details in our previous work [15]. If the transverse structure is neglected, this set becomes similar to models analyzed in the past [9] and one can think of it as describing the coupled nonlinear dynamics of fields a and φ as a function of the co-moving coordinate ξ, which plays the role of time. With the transverse structure included, it describes the weakly nonlinear, spatio-temporal interaction of laser and space-charge field. Coordinate ξ can still be seen as time, and space is associated with the transverse structure itself. These Eqs. are solved numerically with the following initial conditions

$$a(0, x_\perp) = a_0\, e^{-\lambda x_\perp^2} \;\;,\;\; \partial a/\partial \xi|_{\xi=0} = 0 \tag{3}$$

$$\varphi(0, x_\perp) = -a(0, x_\perp)^2/2 \;\;,\;\; \partial \varphi/\partial \xi|_{\xi=0} = 0 \tag{4}$$

EVOLUTION OF THE TRAIN OF PULSES.

Examining Eqs. (1) and (2) together with the initial conditions we find that $(\delta/\kappa)^{-1/2}$ is the time scale for pulse formation along the propagation axis, $(\lambda/\kappa)^{-1/2}$ is the time scale for transverse effects. When $\lambda \ll \delta$, pulses are formed before being affected by transversal effects, and when the inequality reverses, one recognizes the typical instability associated with the transverse term, shown in a recent work to be of the form $a \sim e^{\lambda \xi^2/\kappa}$ [15]. Depending on the relative magnitude of the parameters δ and λ one thus may or may not observe pulses in the system.

In order to see how the electromagnetic pulses evolve as they move into the plasma, we will split the analysis into nonresonant and resonant regimes. Nonresonant regimes are the ones where the adiabatic approximation would remain valid throughout the entire dynamics, if the transverse derivatives were turned off. Resonant regimes, on the other hand, are the ones where adiabaticity is broken and proper plasma waves are excited, even in the absence of the transverse structure.

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Nonresonant Regimes

Nonresonant cases are those where the time scale for the dynamics is much longer than the plasma time scale scale. In our dimensionless system, this means that $1/v_g^2$ is sufficiently large that the the corresponding terms in Eq. (2) are much larger than the term with the ξ-derivative. In this case $\varphi = -a^2/2$ and one says that the φ field becomes enslaved to the field a, which allows to use Eq. (1) as a closed, governing equation (with transverse derivatives turned off) for field a in the adiabatic regime:

$$\kappa \frac{\partial^2 a}{\partial \xi^2} = \delta a - \frac{a^3}{2}. \quad (5)$$

As Eq. (5) describes a nonlinear oscillator with stable equilibria at $a = \pm\sqrt{2\,\delta}$, each transverse coordinate $x_\perp$ the laser oscillates with a different, local nonlinear frequency $\Omega = \Omega(x_\perp)$. When the transverse derivatives are turned on, it is only a matter of the time ξ until the gradient of the phase $\Omega(x_\perp)\,\xi$ along $x_\perp$ monotonically grows to any desired value. As one turns the transverse term on, even if initially small, the associated transverse derivatives will eventually grow and affect the dynamics. In the nonresonant case we are investigating, a quick estimate can be made on the value of ξ where the transverse term is expected to become comparable with the terms already present in Eq. (5). We first observe that for noise-like initial conditions, the right-hand-side of the energy conserving Eq. (5) keeps oscillating within the region $\mathcal{R}$ bounded by the limits $-2\delta^{3/2} \le \mathcal{R} \le 2\delta^{3/2}(2/27)^{1/2}$. To obtain the estimate, we simply integrate Eq. (5) for all $x_\perp$'s using the initial profile provided by Eq. (3), calculate $\nabla_\perp^2 a$ with basis on the solution for a yielded by the very same Eq. (5), and take note of the earliest time where $\nabla_\perp^2 a$ cross the boundaries of $\mathcal{R}$.

Figure 1 displays the corresponding behavior. Once again we choose $\delta = 10^{-2}$, along with $v_g^2 = 0.2$ and $\lambda = 10^{-3}$. The upper and lower straight lines represent the boundaries of $\mathcal{R}$ and we can see how the magnitude of $\nabla_\perp^2 a$ grows in time. At $\xi \sim 35$ the transverse Laplacian reaches the boundaries of $\mathcal{R}$ at $\lambda^{1/2}x_\perp \sim 1$, and at this time one can expect to see the effects of the transverse derivatives on the pulse dynamics.

Let us then integrate the full space-time system formed by Eqs. (1) and (2) to make comparisons with the estimates in some instances. Results are displayed in Fig. 2.

Panel (a) of Fig. 2 represents the case with the transverse term turned off. We repeat the parameters used in Fig. 1: $\delta = 10^{-2}$, $v_g^2 = 0.2$ and $\lambda = 10^{-3}$. For the given initial conditions (3) and (4), the tridimensional plot displays the regular behavior of the corresponding nonlinear pulses. At each $x_\perp$ the laser oscillates, but with a steady phase slippage rate existing along this transverse axis. This is why one observes curved wave fronts as ξ grows.

Keeping the parameters of panel (a), in panel (b) of the same Fig. 2 the transverse term is turned on. We see that the train of pulses starts to develop sharp spikes before

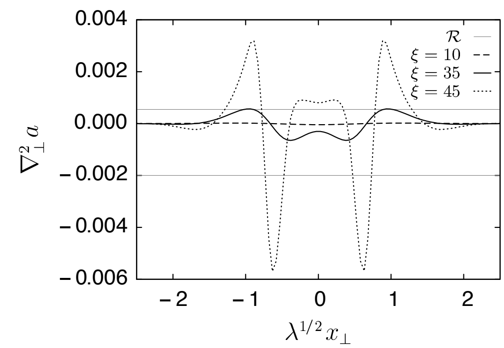

Figure 1: Magnitude of the transverse Laplacian as a function of $x_\perp$ and ξ. At $\xi = 35$ the Laplacian touches the boundary of region $\mathcal{R}$, which signals the instant where its corresponding effects become noticeable.

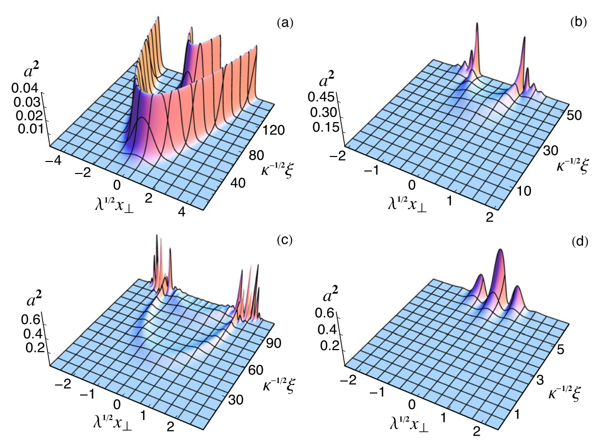

Figure 2: Space-time dynamics of the laser field in the nonresonant regime. In all cases we take $\delta = 10^{-2}$ and $v_g^2 = 0.2$. In panel (a) we consider $\lambda = 10^{-3}$ but keep the transverse Laplacian switched off. In the remaining panels the Laplacian is turned on with $\lambda = 10^{-3}$ in (b), $\lambda = 2.5 \times 10^{-4}$ in (c), and $\lambda = 10^{-1}$ in (d).

$\xi = 50$, which agrees with the previous estimates. The spikes arise from the action of the second derivatives along $x_\perp$, as can be seen again from Fig. 1, and shortly afterwards the space-time dynamics merges into a chaotic pattern. The chaotic pattern cannot be followed for long due to the fact that, in that stage, the fields grow to values beyond our weakly nonlinear approximation. However, the important information to be gathered here is on how far solitary pulses behave as such, before being affected by the transversal effects.

For sake of comparison, in panel (c) we also examine what happens when the initial beam width is extended, with a smaller λ given by $\lambda = 2.5 \times 10^{-4}$ (remaining parameters are kept with the same previous values). In that case our estimate indicates that, as expected, the transverse term takes longer to affect the pulse dynamics. The critical time ξ, once again obtained as the one where $\nabla_\perp^2 a$ escapes region

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$\mathcal{R}$, reads $\xi \sim 70$, which agrees with the curves displayed in the present Fig. 2 (c). The case with large values of λ, say $\lambda = 10^{-1}$ ($\gg \delta$), is also worthy of inspection, as done in Fig. 2(d). In this case the transverse term dominates the dynamics, generating the exponential growth commented earlier [15]. As a matter of fact, in this latter case of large λ, the maximum of $\nabla_\perp^2 a$, here located at $x_\perp = 0$, already starts off from a value beyond the boundaries of $\mathcal{R}$.

Resonant Regimes

Since with transverse derivatives neglected the system already displays a nonintegrable behavior in the resonant regime, the quasi-integrable estimates based on the reduced - and integrable - equation (5) becomes inaccurate. However, one can still be tempted to associate the time scale $(\delta/\kappa)^{-1/2}$ with the longitudinal dynamics, and the time scale $(\lambda/\kappa)^{-1/2}$ with the transverse term, as discussed previously. In panel (a) of Fig. 3 we take $v_g^2 = 0.9$ and $\delta = 10^{-2}$, along with $\lambda = 10^{-4}$. Here λ is much smaller than the mismatch δ and we are therefore under circumstances where pulses have time enough to be formed, even if in an irregular fashion. This is essentially what happens, as can be appreciated from the figure. In panel (b) we analyze the dynamics with the same set of control parameters, but with the transverse term switched off. A comparison between both panels allows us to realize that the presence of the transverse structure, even for λ as small as $\lambda = 10^{-4}$, already has a relatively noticeable effect on the dynamics of the laser amplitude a. In fact, the transverse term shortens somewhat the characteristic oscillatory time scale, what is expected from the larger transverse gradients associated with a chaotic case. The shortening of the time scales becomes gradually more pronounced as λ grows.

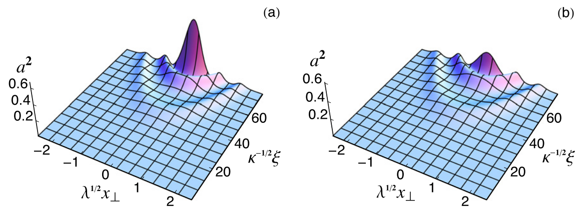

Figure 3: Space-time dynamics of the laser field in the resonant regime. In panel (a) we take $\delta = 10^{-2}$ and $v_g^2 = 0.9$, along with $\lambda = 10^{-4}$. Panel (b) is plotted for the same parameters, but with the transverse term switched off.

CONCLUSIONS

Space-charge waves are excited as the pulses travels into the plasma. Both resonant and nonresonant cases of space-charge plasma wave excitation were examined. In the nonresonant regime, accurate estimate of the transverse effects is possible. One can, for instance, predict, with relatively narrow margins of error, how far does a soliton train move into a plasma before being distorted.

In resonant cases estimates become less accurate due to the nonintegrability of the overall dynamics, but one can still produce reliable predictions on the wave pattern. In the case specifically studied here, the transverse scale $(\lambda/\kappa)^{-1/2}$ is sufficiently larger than the pulse length $(\delta/\kappa)^{-1/2}$ and localized structures can be observed before being affected by the transversal effects.

In general, the number of unperturbed pulses in a train can be roughly estimated as $(\delta/\lambda)^{1/2}$. Since the amplitude a_p of the pulses is proportional to $\delta^{1/2}$, one sees that the number of pulses in a train can be rewritten as $a_p\,\lambda^{-1/2}$. The factor $\lambda^{-1/2}$ itself is the transverse length measured in units of the plasma wavelength. If the laser beam is wide with $\lambda^{-1/2} \gg 1$ the number of pulses can still be appreciable even with a_p small. This is the case of a beam width $50\mu m$ and plasma wavelength $0.5\mu m$, which matches one of the cases studied in the text, where $\lambda = 10^{-4}$. In extreme cases where $\lambda^{-1/2} \sim 1$, only strong lasers can form pulses; otherwise, no pulses are formed as in the cases with $\lambda \sim 10^{-1}$ also investigated in the work.

The present analysis essentially adds transverse effects to the known one-dimensional behavior of train of planar pulses. As an overall conclusion, we see that transversal effects become already important even when the cross section of the radiation beam is relatively larger than the longitudinal pulse size.

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