### **Tutorial on Plasma-Based Acceleration**

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### Large Hadron Collider (LHC) at CERN Terascale Physics

14 Trillion Volts (CM) pp 27 km circumference \$6 Billion+? What is next?

> Thinking big Accelerators like the future LHC

the future LHC require long tunnels and powerful bending magnets.

### **Particle Accelerators**

Requirements for a High Energy Physics Collider

- High Energy
  - ~TeV
- High Luminosity (event rate)
  - L=fN<sup>2</sup>/ $4\pi\sigma_x\sigma_y$
  - ~nC of charge at 10 kHz: ~kJ per bunch and 20 MW of average power in beam
- High Beam Quality
  - Energy spread  $\delta\gamma/\gamma \sim .1 10\%$
  - Low emittance:  $\varepsilon_n \sim \gamma \sigma_y \theta_y < 1 \text{ mm-mrad}$
- Low Cost (one-tenth of \$10B/TeV)
  - Gradients > 100 MeV/m
  - Efficiency > few %

### **Particle Accelerators**

Requirements for an X-RAY FEL

- Moderate Energy
- High peak current ( $N/\sigma_z \sim 10$  kA)
- High Beam Quality
  - Energy spread  $\delta\gamma/\gamma \sim .1 1\%$
  - Low emittance:  $\varepsilon_n \sim \gamma \sigma_y \theta_y < 1$  mm-mrad

### Particle Accelerators Why Plasmas?

### **Conventional Accelerators**

- Limited by peak power and breakdown
- 20-100 MeV/m
   20km /0.8 TeV

### <u>Plasma</u>

No breakdown limit

• 10-100 GeV/m

### Plasma Accelerator Progress "Accelerator Moore's Law"



### Plasma-based acceleration is rich in science: Today I will talk about some of the issues for designing an accelerator



### **Simple Wave Amplitude Estimate**



$$\nabla \bullet E \sim ik_p E = -4\pi e n_1$$

$$k_p = \omega_p / V_{ph} \approx \omega_p / c$$

$$n_1 \sim n_o$$

$$\Rightarrow eE \sim 4\pi e n_o e^2 c / \omega_p = m c \omega_p$$

$$or \quad eE \sim \sqrt{\frac{n_o}{10^{16} c m^{-3}}} \frac{10 \, GeV / m}{c^{10}}$$

**Gauss' Law** 

## Create a plasma wave wake: Wake Behind a Motor Boat



# How does one excite a relativistic plasma wave wake:Concepts for plasma based accelerators\*



### Quasi-static approximation Sprangle, Esarey, and Ting 1990



For a fixed driver shape the wake can be calculated. The wake only changes if the driver shape changes. The driver's shape changes very slowly.

#### Use appropriate variables

• Transform from:

• Transform to:

$$(\xi = z - v_{\phi}t, x, y; s = z)$$

#### Meaning of new variables

- .  $\xi=z-v_\phi t_{\rm \ is\ the\ distance}$  from front of the driver
- $s = z_{\rm is\ the\ distance\ the\ driver\ has\ propagated\ into\ the\ plasma}$

### Mathematical meaning of quasi-static approximation

 $\partial_s << \partial_{\mathcal{E}}$ 

Let the wake move at c and make the quasi-static approximation

$$E_{z} = -\partial_{z}\phi - 1/c \ \partial_{t}A_{z}$$

$$F_{z} \approx -q\partial_{\xi}(\phi - A_{z})$$

$$\vec{F}_{\perp} = q\left(\vec{E}_{\perp} + (\vec{v}_{b} \times \vec{B})_{\perp}\right)$$

$$\vec{v}_{b} = \hat{z}c$$

$$F_{\perp} \approx q(-\nabla_{\perp}(\phi - A_{z}))$$

#### **Psuedo-potenial**

$$\psi = (\phi - A_z)$$

Don't choose a gauge where

$$\phi = A_z$$

Forces on relativistic particle

$$F_z = -\partial_\xi \psi$$

$$F_{\perp} = -\nabla_{\perp}\psi$$

#### Relationship between accelerating and focusing forces

### Forces come from a single potential:

$$F_{\perp} = -\nabla_{\perp}\psi$$

$$F_z = -\partial_\xi \psi$$

From which it follows (and vice versa):

$$\nabla_{\perp}F_z = \partial_{\xi}F_{\perp}$$

- Ideal properties for an accelerating structure is that:
  - The accelerating fields do not depend on the transverse coordinate: Low energy spread
  - The focusing fields to not depend on the axial coordinate: emittance preservation
- If one is met so is the other
- THEOREM IS TRUE FOR LINEAR AND NONLINEAR WAKES

### Linear theory for **particle beam** and **laser** drivers

What are the wakefields produced by a laser or a particle beam? For wide beams (spot sizes comparable or larger to the wavelength of the wake) there are no differences!

#### Linearized equation for plasma density (valid for wide and narrow beams):

$$\left(\partial_t^2 + \omega_p^2\right) \frac{n_1}{n_0} = \omega_p^2 \left(\frac{n_b}{n_0} - k_p^2 \nabla^2 \frac{a_0^2}{4}\right)$$

Normalized vector potential of the laser:

$$a_0 \equiv \frac{eA_0}{mc^2} = .85 \times 10^{-9} \sqrt{I_{W/cm^2}} \lambda_{\mu m}$$

Normalized electron plasma and beam (it can be + or -) densities:

$$\frac{n_1}{n_0} = \frac{n_b}{n_0}$$

#### For wide beams the equation for the wake potential is simple:

$$\frac{\partial^2}{\partial\xi^2}\overline{\psi} + k_p^2\overline{\psi} = -k_p^2\frac{n_b}{n_0} + k_p^2\overline{\phi}_p$$

Ponderomotive potential or radiation pressure:

$$\overline{\phi}_p \equiv \frac{a_0^2}{4} \qquad \overline{\psi} \equiv \frac{e\psi}{mc^2}$$

Note: Transverse profile of the wake follows that of the driver!

### Solution for wake is easily calculated via Greens functions

$$\bar{\psi}(x_{\perp},\xi) = k_p \int_{\xi}^{\infty} d\xi' \sin\left[k_p(\xi - \xi')\right] \left(\frac{-n_b(x_{\perp},\xi)}{n_0} + \bar{\phi_p}(x_{\perp},\xi)\right)$$

### Accelerating field



 $F_z = -\partial_\xi \psi$ 

### Focusing field









Only half of the accelerating phase can be used.

Formalism for electrons and positrons is the "same": Just switch the sign of nb.

# The transverse profile of wakefield potential is the same as that of a wide particle beam or laser



### Linear theory tells us we need a poweful laser or particle beam to make large wakes

For example, if  $a0 \sim l \text{ or nb/n0} \sim .25$ , if the pulse length is matched to the plasma frequency, and if the spot size is ~half the wakes wavelength then:

For a laser this corresponds to:

For a particle beam: this corresponds to:

 $P\approx 110TW(\frac{\tau_{pulse}}{100\,fs})^2$ 

 $\frac{Q}{\tau_{pulse}}\approx \frac{50nCoul}{100\,fs}(~50kA)$ 

Linear theory for arbitrary transverse profiles is still straightforward for both a particle beam and a laser driver. The wake potential is now given by:

$$\overline{\psi}(\xi, r, \phi) = -\int_{\xi}^{\infty} k_p d\xi' \sin(k_p \left[\xi - \xi'\right]) \left[\overline{\phi}_p(\xi'\phi', r') + \int_0^{2\pi} \frac{d\phi'}{2\pi} \int_0^{\infty} dr' r' K_0 \left(k_p |\bar{r} - \bar{r'}|\right) \frac{n_b(\xi', \phi', r')}{n_0}\right]$$

However, there is a very important difference between the wake of a narrow particle beam and that of a laser: The wake of a very narrow particle beam still extends out to a skin depth. (There is no reason to use a very narrow laser!)



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# For a fixed amount of charge you can always make nb/n0 large by decreasing the spot size of a beam

The wake amplitude for a narrow beam depends on the normalized charge per unit length:

$$\Lambda \equiv \frac{n_b}{n_0} k_p^2 \sigma_r^2 \approx 2 \frac{N}{2 \times 10^{10}} \frac{20_{\mu m}}{\sigma_z} = \frac{I}{24kA}$$

Linear theory works when  $\Lambda < 1$  and  $\frac{n_b}{n_0} < 10$  for e- beams and when  $\Lambda < 1$  and  $\frac{n_b}{n_0} < 1$  for e+ beams.



### Very intense lasers also create similar looking nonlinear wakes

#### Driven by an electron beam



Driven by a laser pulse



Called blowout or bubble

Need a nonlinear description of these wakes

Very stable wakes!

# Field structure in Blowout/Bubble regime is described by the wake potential.

Just need an understanding of electrostatic for infinitely long systems

Field equations in Lorentz gauge 
$$(\frac{1}{c^2}\partial_t^2 - \nabla^2)\vec{A} = \frac{4\pi}{c}\vec{J}$$
$$(\frac{1}{c^2}\partial_t^2 - \nabla^2)\phi = 4\pi\rho$$

Make quasi-static approximation

$$-\nabla_{\perp}^{2}\vec{A} = \frac{4\pi}{c}\vec{J}$$
$$-\nabla_{\perp}^{2}\phi = 4\pi\rho$$

Wake potential follow "2D electrostatic" equation

$$-\nabla_{\perp}^2 \psi = 4\pi (\rho - \frac{J_z}{c})$$

#### Acceleration and focusing fields:

$$F_z = -\partial_\xi \psi$$

$$F_{\perp} = -\nabla_{\perp}\psi$$

Inside bubble there only ions so use Gauss's Law for cylinders

$$F_{\perp} = -2\pi e^2 n_0 x_{\perp}$$

### Need an equation for the radius of the bubble: $\frac{dr_b}{dr_b}$

Relativistic blowout regime for blowout radius and for large maximum radius the trajectory of rb is a circle: Bubble Lu et al.PRL 16, 16500 [2006]



### **Transverse Dynamics and Beam Quality**

• Emittance  $\mathcal{E}_n$  = phase space area and a measure of its ability to get focused:

Inside a plasma wake a single particle oscillates as:

 $\overrightarrow{\mathbf{X}} \quad \sigma_r = \sqrt{\left(1 + (\frac{z}{\beta^*})^2\right)} \quad \text{where} \quad \beta^* = \frac{\sigma_r^2}{\epsilon_n} \gamma$ 

• The spot size of a beam in vacuum evolves as:

$$\frac{dP_{\perp}}{dt} = q(-\nabla_{\perp}\psi)$$

• If the focusing force is "linear" in the transverse coordinate then

p<sub>x</sub>

σ

$$\frac{d^2 x_{\perp}}{dt^2} + \omega_{\beta}^2 x_{\perp} = 0$$
$$k_{\beta} \equiv \frac{\omega_{\beta}}{c} \qquad k_{\beta} = \alpha \frac{k_p}{\sqrt{2\gamma}}$$

### **Transverse Dynamics and Beam Quality**

• Emittance  $\mathcal{E}_n$  = phase space area and a measure of its ability to get focused:



• Matching: Focusing length 
$$k_{\beta}^{-1}$$
 = Beam diffraction length  $\beta_*$   

$$\sigma^2 = \varepsilon_n \sqrt{\frac{2}{\gamma}} \frac{c}{\omega_p}$$

- And a Gaussian beam remains Gaussian with a fixed spot size
- No emittance growth
- A nonlinear Focusing force has a matched profile that is different than Gaussian

### Now we are ready to analyze beam loading: What is beam loading?

• The placing of a bunch charge on an accelerating structure to extract energy with high efficiency, small energy spread, and emittance preservation.

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- This involves understanding:
  - how much charge can be loaded.
  - where to place the charge.
  - and how to shape the charge.
- The best properties for the LOADED wake are (while the beam is matched):

$$\partial_{\xi} F_z = 0 \qquad \qquad \partial_{\xi} F_{\perp} = 0$$

$$\nabla_{\perp} F_{\perp} = C_{onstant} \qquad \qquad \nabla_{\perp} F_z = 0$$

### We now have all the tools needed to investigate beam loading in linear regime

$$\overline{\psi}(\xi,r,\phi) = -\int_{\xi}^{\infty} k_p d\xi' \sin(k_p \left[\xi - \xi'\right]) \left[\overline{\phi}_p(\xi'\phi',r') + \int_0^{2\pi} \frac{d\phi'}{2\pi} \int_0^{\infty} dr' r' K_0 \left(k_p \left|\overline{r} - \overline{r'}\right|\right) \frac{n_b(\xi',\phi',r')}{n_0}\right]$$

 $n_b = n_d + n_t$  where **d** stands for driver and **t** stands for trailing. If there is a laser driver then  $n_d = 0$ . The challenge is to get the wake fields inside the trailing beam that provide good beam quality while at the same time get the trailing beam to absorb the wakes energy.



The wake potential (and forces) inside the trailing beam depend on its longitudinal and transverse profiles. The profiles you want depend on the relative position: Phase slippage, driver distortion, and trailing bunch distortion make this challenging.

# Beam Loading linear regime

- In the linear regime one superimposes the wake by trailer to the wake by the driver to find the total wakefield [1].
- Bunch shaping for no energy spread: Triangular.
- The total charge can be found by requiring that all of the energy in the wake is absorbed. You can look at the wake left behind or you can look at the force on the particle.
- Works for electron and positron loads.



[1] T. Katsouleas, S. Wilks, P. Chen, J. M. Dawson, J. J. Su - Part. Accel, 1987

### Original simulation results: 1987



### Efficiency: Linear theory

-20

-10

-5

 $\xi \left[ c / \omega_{p} \right]$ 

5

 Need to absorb the wake including transversely as well : Energy in wake behind the trailing bunch must be a large fraction of that in front of the bunch

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-0.08

-20

-10

r (k<sub>p</sub><sup>-1</sup>)

10

20

- If the trailing bunch is wide then its spot size needs to be equal to the drivers spot size.
- Recall a narrow trailing bunch can absorb energy out to a radius of c/wp. So
  one can use a narrow trailing bunch and a driver with a spot size ~c/wp.
- Note: You can get good efficiency at the expense of the gradient: You make the wake small after (and hence inside) the bunch.

### What did linear theory tell us?

- Emittance preservation (spot size matching) can be achieved by using:
  - a. very narrow beam loads

b. very wide drive beams with tailored transverse profiles to make the focusing forces small

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High efficiency can be achieved by using:

a. Narrow witness beams that absorb energy of the wake out to a c/wp and a driver with a spot size of  $\sim$  c/wp.

b. A matched spot size that equals that of a wide driver.

- Low energy spread can be achieved by using:
  - a. Narrow witness beams or flat and wide beams.
  - b. And by longitudinally and transversely shaping the witness beams.
- The accelerating and transverse fields within the trailing beam depend on the witness beam profiles
  - Need to shape the witness beam
  - Phase slippage is an issue
- Narrow beams with high charge themselves make nonlinear wakes.

# Efficient beam loading requires nC of charge which create nonlinear wakes they are narrow

Trailing beam density:

$$n_b = \frac{N}{(2\pi)^{3/2} \sigma_r^2 \sigma_z}$$

$$N = 1 \times 10^{10}$$

Matching:

$$\sigma_r^2 = \sqrt{\frac{2}{\gamma}} k_p^{-1} \epsilon_N$$

 $\frac{n_b}{-} \approx$ 

 $n_0$ 

Energy spread:

$$\sigma_z = \alpha \frac{c}{\omega_p} \qquad (\Lambda > 1)$$

Leads to:

$$\frac{n_b}{n_0} = 1.4 \times 10^4 \frac{N}{1 \times 10^{10}} \frac{\mu m - rad}{\sqrt{\epsilon_{Nz} \epsilon_{Ny}}} \sqrt{\frac{Energy}{250 GeV}} \frac{1}{\alpha}$$

For collider parameters:

$$10^4 - 10^5$$
 degrade the accelerating and focusing fields, occurs when nb/no~M/m

lon motion which con

### Nonlinear wakefield is IDEAL for accelerating/focusing electrons Trailing beam does not modify focusing fields of wake





For 
$$\frac{r_b}{R_b} \ll 1 \rightarrow \begin{cases} r_b \frac{d^2 r_b}{d\xi^2} + 2\left[\frac{dr_b}{d\xi}\right]^2 + 1 = \frac{4\lambda(\xi)}{r_b^2} \\ E_z = \frac{1}{2}r_b \frac{dr_b}{d\xi} \end{cases}$$

These equations are integrated for a trapezoidal  $\lambda(\xi)$  to obtain  $E_z(\xi)$  and  $r_b(\xi)$ . This allows us to design accelerators with 100% beam-loading efficiency that conserve the energy spread.

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### Efficiency

Very high efficiency can be achieved in the nonlinear regime because the spot size also is reduced when energy is absorbed!

 There is a tradeoff between maximum charge and maximum energy

$$\frac{Q_s \times E_s}{mc^2/r_e} = \frac{1}{4^3} \times (k_p R_b)^4$$
$$\frac{Q_l \times E_l}{mc^2/r_e} = \frac{1}{8\pi} \left(\frac{n_1}{n_0}\right)^2 \times \sin^2(k_p \zeta_0)$$

For nonlinear beam loading the efficiency approaches 100%, while E<sub>z</sub> is constant in z and r.

$$\eta_b = 1 - (\widetilde{R}_b/R_b)^4 = \frac{\widetilde{Q}_s}{Q_s}$$



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# Let me switch gears as we near the end and discuss some basic concepts and terms

### Dephasing



$$(c - v_g)L_{dph} = \lambda_p / 4$$
$$v_g \approx c(1 - \frac{1}{2}\frac{\omega_p^2}{\omega_o^2})$$

$$\Rightarrow L_{dph} \frac{\omega_p}{c} = \pi \frac{\omega_0^2}{\omega_p^2}$$

order 10 cm x  $10^{16}/n_o$ 

or

$$\Rightarrow L_{dph} \frac{\omega_p}{c} = \pi \gamma_b^2$$

Generally not a problem for a particle beam

### Diffraction $L_{dif} \approx \pi L_R = \pi^2 w_0^2 / \lambda = \frac{\pi}{2} \frac{\omega_o}{\omega_p} (w \frac{\omega_p}{c})^2 \frac{c}{\omega_p}$ order mm! • Laser: (but overcome w/ channels or relativistic self-focusing) **Optical diffraction** $\Theta_{diff} = \frac{\lambda}{\pi_{W_0}}$ $Z_{R} = \pi_{W_{0}}^{2} /$ • Particle beam: $L_{dif} \cong \pi \beta^* = \pi^2 w_0^2 / \varepsilon$ where $\varepsilon$ is the emittance (unnormalized) of the beam order meter So not an issue

### Pump depletion: Transformer ratio

$$E_{-} \longrightarrow E_{+} = \gamma_{b}mc^{2} \qquad E_{-} = \partial_{\xi}\psi_{-}$$

$$E_{+} = eE_{+}L_{pd}$$

$$\Rightarrow \Delta W = \frac{E_{+}}{E_{-}}\gamma_{b}mc^{2}$$

$$\frac{E_{+}}{E_{-}} = \text{Transformer ratio} = R$$
Linear theory : For a symmetric bunch R = 2

You want particles in bunch to slow down together:

$$E_{-} = \partial_{\xi}\psi_{-} = Constant$$



High transformer ratio also leads to constant decelerating field



You want photons in bunch to slow down together:

 $E_{-} = \partial_{\xi}\psi_{-} = Constant$ 

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# You want pump depletion to be the limiting factor to the acceleration length for high efficiency.





### SciDAC plasma-based accelerator codes

### **OSIRIS**

**VORPAL** 

**WARP** 



Quasi-static

**Optimized and scale well** 









Scientific Discovery through Advanced Computing



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### Some closing thoughts

- $\checkmark$  This is a very exciting time for the field of plasma based accelerators.
- Both PWFA and LWFA research is now focused on collider concepts that have multiple stages (10-100) that are each ~1meter in length.
- Much of the theoretical and computational tools are in place to study complete concepts for such stages including beam loading scenarios.
- There are many ideas being investigated:
  - Nonlinear scenarios for accelerating electrons (for both collider and light sources (narrow bunches)
  - Linear and weakly nonlinear scenarios (wider bunches) for accelerating e- and e+ beams.
- I did not have time to discuss the evolution and stability of either the driver or trailing beam. But theoretical and computational tools are available to study this as well.
- I also did not have time to discuss ideas for injecting particles into the wakes. This includes self and external injection methods.

### Synchrotron Radiation

$$P_{loss} = \frac{2}{3} \frac{e^2}{c} \gamma^6 \left( \left[ \vec{\beta} \right]^2 - \left[ \vec{\beta} \times \vec{\beta} \right]^2 \right)$$
$$\frac{\varepsilon_r}{\varepsilon_{loaded}} = \frac{1}{\varepsilon_{loaded}} 1.5 x 10^{-5} \left( \frac{E}{50 GeV} \right)^2 \left( \frac{n}{10^{16}} \right)^{3/2} \left( \frac{r}{1\mu m} \right)^2$$
For a matched beam this can be rewritten as:
$$\frac{\varepsilon_r}{\varepsilon_{loaded}} = \frac{1}{\varepsilon_{loaded}} 3.75 x 10^{-3} \left( \frac{E}{250 GeV} \right)^{3/2} \left( \frac{n}{10^{16}} \right) \left( \frac{\varepsilon_n}{10^{-6} m} \right)^2$$

### Hosing

Whittum et al. PRL 67 991 (1991) and Huang et al. PRL 99 225001 (2007)

### Hosing for an intense beam



 $I_{peak} = 7.7 \text{ kA}$