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Chromaticity Correction for a Muon Collider Optics

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Contents:

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Introduction

Muon Collider is a promising candidate for the next energy frontier machine. First proposed by Budker (1967), the idea of a MC has been re-launched by recent progress on new ideas for small emittance muon beams. The re-newed interest is testified by the large number of papers presented at this conference.

Muons are

- point-like as $e^{\pm}$ $\rightarrow$ the whole beam energy is carried by the interacting particles.
- but 207 times heavier $\rightarrow$ no radiation, in practice ($U_{turn} = q^2\beta^3\gamma^4/3\epsilon_0 R$)

The small lifetime ($\tau = \gamma$ 2.2 μs) requires

- large number of muons be produced
- and 6D cooled and quickly accelerated.

Machine parameters vary depending on the available number of muons and their emittance. Expectations for two possible cooling scenarios:

	high transv. emitt.	low transv. emitt.
$N_b \times N_\mu$	$1 \times 20 \cdot 10^{11}$	$10 \times 1 \cdot 10^{11}$
$\Delta p/p$	0.1%	1%
$\epsilon_N = \epsilon_N^x = \epsilon_N^y$	25 μm	2 μm

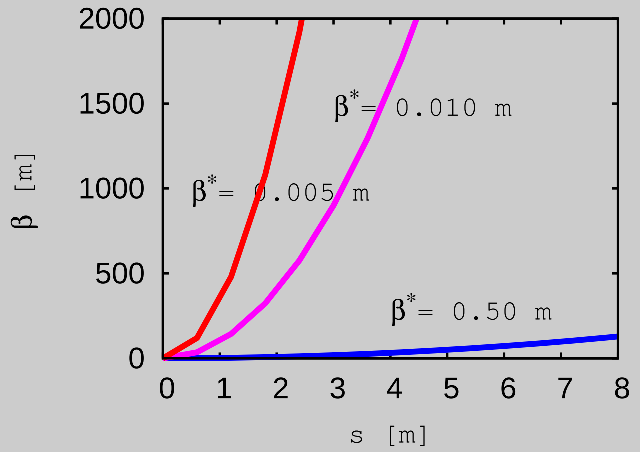
In order to reach a luminosity of $\simeq 10^{34}$ cm^2 s^{-1}, very small β^* is required. As a compromise between luminosity and feasibility $\beta_x^* = \beta_y^* = 1$ cm has been chosen in this design.

For comparison:

	LHC	Tevatron	Hera-p (y)
β^* (m)	0.55	0.29	0.18

Design Issues

$$\beta(s) = \beta^* + s^2/\beta^*$$



Large β at strong quadrupoles:

- large sensitivity to misalignments and field errors
- large chromatic effects limit the momentum acceptance and require strong correction sextupoles
- large non-linearities limit the Dynamic Aperture

Hourglass effect limits $\sigma_\ell \leq 1$ cm. For achieving such short bunches with reasonable voltage, $|\alpha_p|$ must be as small as possible ($\leq 1\times10^{-4}$).
Ring circumference should be as small as possible, luminosity being inversely proportional to the collider length.

Assuming we are able to accelerate enough muons, the design of the collider ring itself is not trivial either..

Constraints for present design

Design constraints	
β_x^*, β_y^* $(\epsilon_x = \epsilon_y)$	10 mm
free space around IP	$\pm$ 6 m
$\lvert\alpha_p\rvert$	$\leq 1 \times 10^{-4}$
$\hat{g}$	$\leq$ 260 Tm^{-1}
$\hat{B}$	10 T
	(8 T in the IR)

Moreover: $\ell_B \leq 6$ m, $\ell_Q \leq 3$ m.

Energy for this design: 750 GeV per beam.

Solutions for 1.5 TeV per beam are under study.

Chromatic correction

To obtain large momentum acceptance it is necessary to correct the dependance on momentum of β-functions and tunes.

Montague chromatic functions

$$B_z \equiv \frac{1}{\beta_z^{(0)}} \frac{\partial \beta_z}{\partial \delta} \qquad A_z \equiv \frac{\partial \alpha_z}{\partial \delta} - \alpha_z^{(0)} B_z \qquad (z = x/y)$$

obey the equations

$$\frac{dB_z}{ds} = -2A_z \frac{d\mu_z}{ds} \qquad \text{and} \qquad \frac{dA_z}{ds} = 2B_z \frac{d\mu_z}{ds} - \beta_z^{(0)} k$$

$$k \equiv \pm(K_{Quad} - D_x K_{Sext}) \qquad (+/- \ \text{ for } x/y)$$

Second order chromaticity

$$\xi_z^{(2)} = \frac{1}{8\pi} \int_0^C ds \left(-k B_z \pm 2 K_{Sext} \frac{dD_x}{d\delta}\right) \beta_z^{(0)} - \xi_z^{(1)}$$

$$\xi_z^{(1)} \equiv \text{linear chromaticity}$$

“Classical” approach

- Chromatic IR beta-wave is corrected with sextupole families in the arcs where $D_x \neq 0$.
- Different families are used to correct linear and second order chromaticity and possibly the first order dependence of $\alpha(s)$ on momentum; non-orthogonality of such corrections result in an increase of the needed sextupole strenght.
- Constraints on the phase advance between sextupoles of the same family allow to make the lowest order driving terms of the 3th order resonances vanish.

This scheme works well in successfully operating colliders as Tevatron and LHC, has been tried for earlier MC versions but led to extremely small values of the momentum stability range.

“Special sections” approach

Owing to the large chromaticity, the IR optics of a high luminosity Muon Collider must be designed having non-linear corrections in mind.

The use of “special sections”, with large beta functions and dispersion, next to the low-β region has been suggested:

- **after the first sextupole located at a knot of the IR chromatic wave, a pseudo $-I$ section is inserted between it and a “twin” sextupole compensating the non-linear kick.**

Two such sections are needed for correcting in both planes. In practice such kind of schemes are prone to focusing errors.

The optics proposed by K. Oide (1996) for a β^*=3 mm 2$\times$2 TeV MC based on this scheme had large momentum range and DA.
$\hat{\beta}_y$=900 km, very strong sextupoles and their large number of families (22) make this (very instructive) design likely un-feasible as it is.

Local chromatic correction

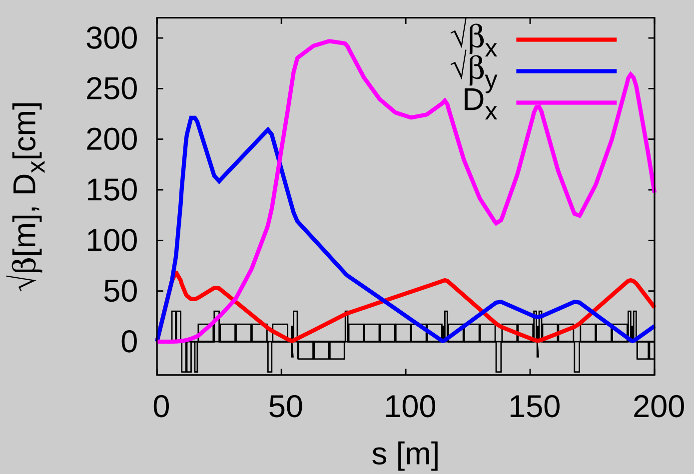
$$\frac{dA_z}{ds} = 2B_z \frac{d\mu_z}{ds} - \beta_z^{(0)} k \qquad \text{and} \qquad \frac{dB_z}{ds} = -2A_z \frac{d\mu_z}{ds}$$

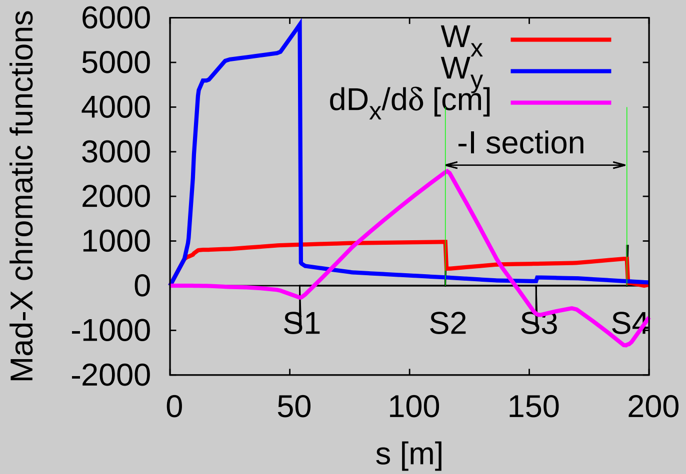
- A_z becomes non zero when the low-β quadrupoles are encountered, but as long as the phase advance does not change, B_z is unchanged.
- At the low-β quadrupoles, the phase advance changes slowly and there is a possibility of correcting the chromatic perturbation before $\beta(\delta)$ and $\mu(\delta)$ start differing from the unperturbed values.

"Local" correction with sextupoles is possible if the IR dispersion is non-vanishing. If $D_x = D'_x(IP) = 0$ the insertion of relatively strong bending magnets in the IR region is necessary. They can help avoiding neutrinos hot spots.

IR optimization for chromatic correction

- Non-symmetric IR design: $\hat{\beta}_y \gg \hat{\beta}_x$ (as in Oide design).
- Local (as explained) chromatic correction for the larger vertical chromatic wave with a single sextupole (S1) located where β_x is small.
- A simultaneous local correction in the horizontal plane being not possible, a $-I$ section is inserted for accommodating a pair of horizontal sextupoles (S2 and S4).
- A 4th sextupole (S3) at small $\beta_{x,y}$ location corrects 2th order dispersion.

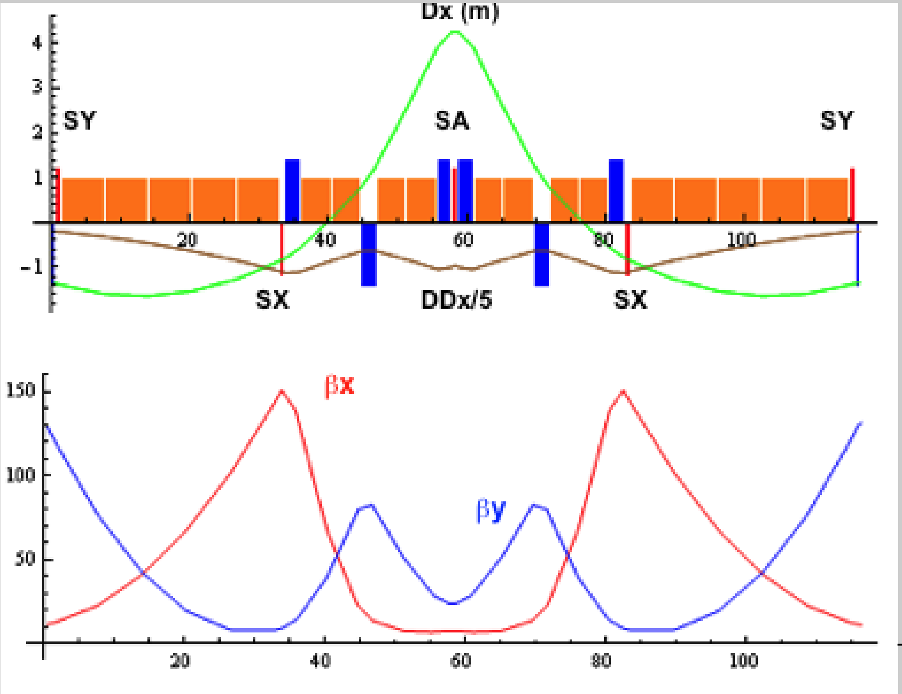




Arc cell

The collider includes two identical IPs (twofold symmetric optics).

The IR having a large positive contribution to α_p, arcs must give a negative contribution so to get $|\alpha_p| \leq 10^{-4}$.



Proposed cell

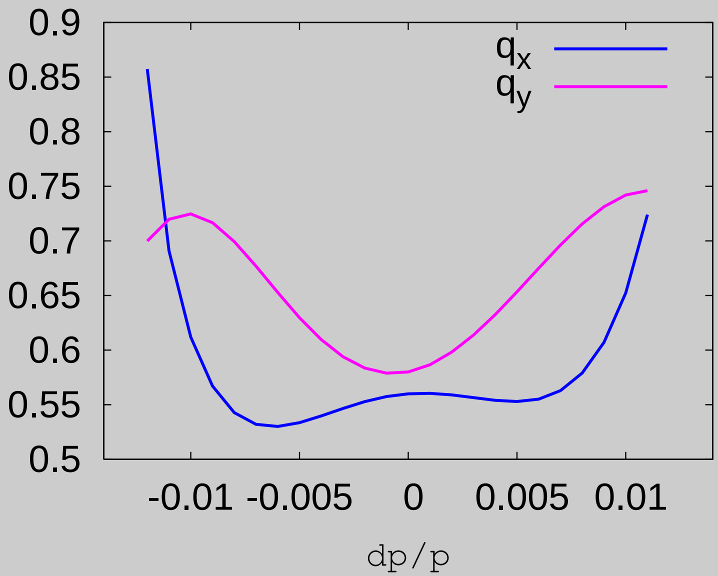
- Almost orthogonal chromaticity correction (one family/plane).
- 300 deg phase advance/cell: cancellation over 6 cells.
- α_p and its dependence on momentum [a] controlled through middle quadrupole and sextupole.

[a] $\frac{1}{\mathcal{L}} \int ds [\frac{1}{\rho} \frac{\partial D_x}{\partial \delta_p} + \frac{1}{2} D'_x]$

Collider circumference (including matching sections): 2727 m.

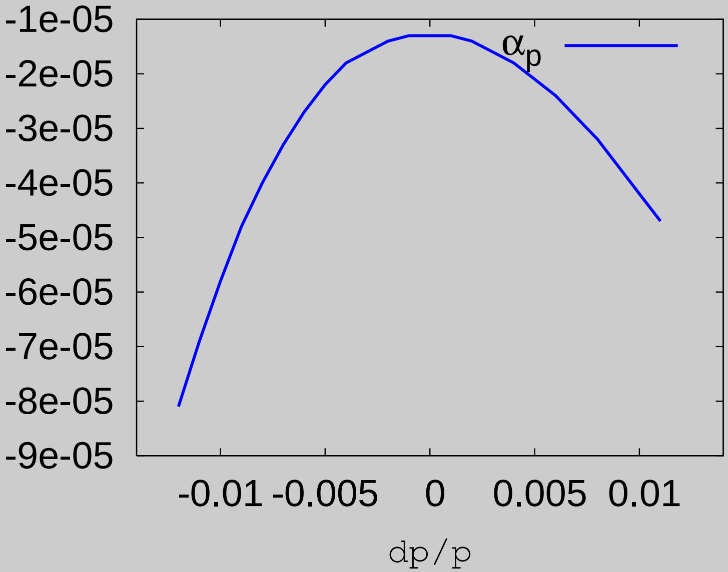
Lattice performance

Fractional tunes just above half-integer chosen for orbit and β-beat at low-β quads considerations.



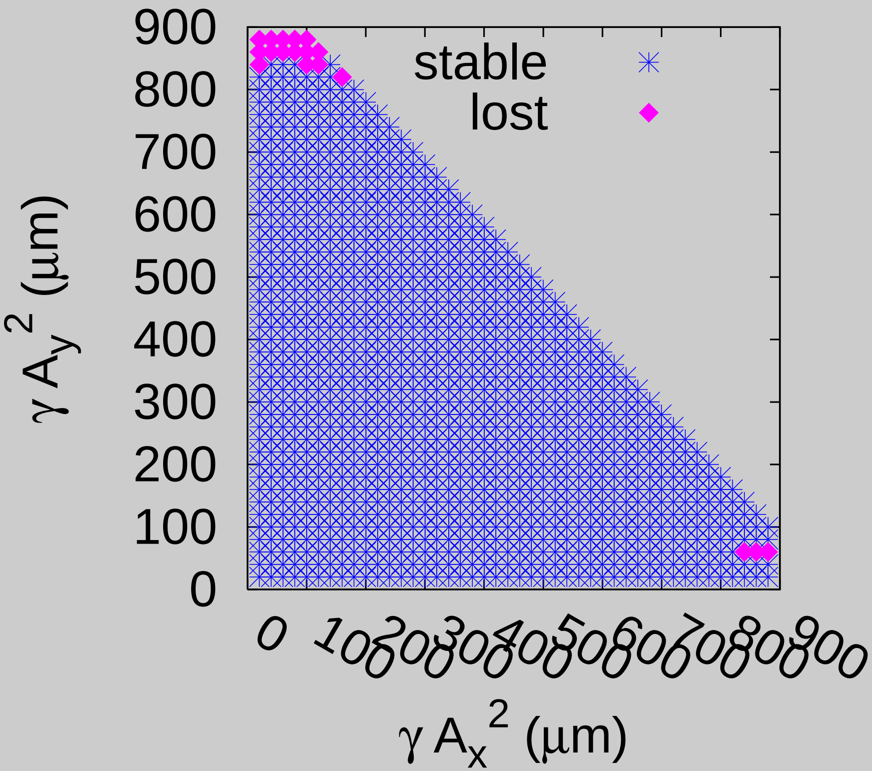
Tunes (fractional part) vs. dp/p

1% required at most



α_p vs. dp/p

Dynamic Aperture determined by tracking particles over 1000 turns (time needed for the beam current to decay by a factor $\simeq$ 2).



MAD8 DA

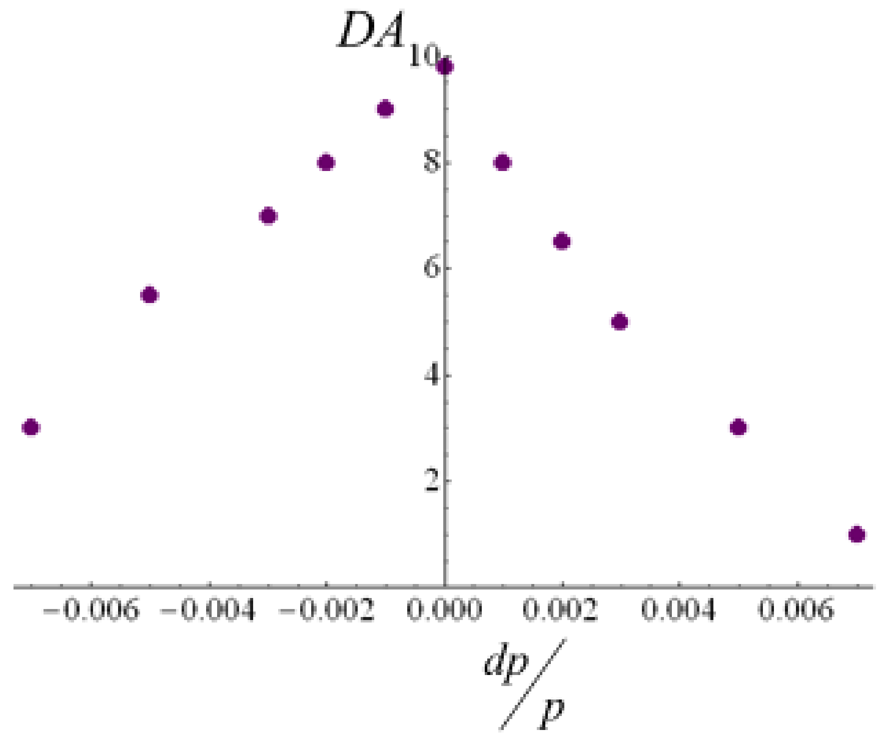
(on energy, w/o synchrotron oscillations)

$A \equiv$ **oscillation amplitude**

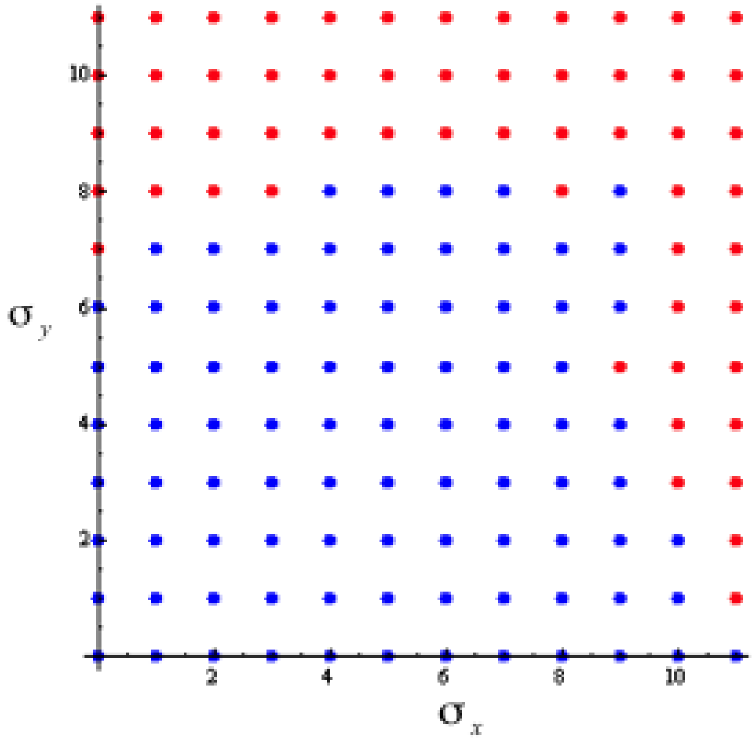
$\left(\#\sigma = \sqrt{\frac{\gamma A^2}{\epsilon_N}}\right)$

DA is 5.7σ for ϵ_N=25 μm (3σ needed)

(A. Netepenko)



Diagonal DA, MAD-8 calculation (4D tracking) for different constant dp/p, BeamBeam included, 1024 turns



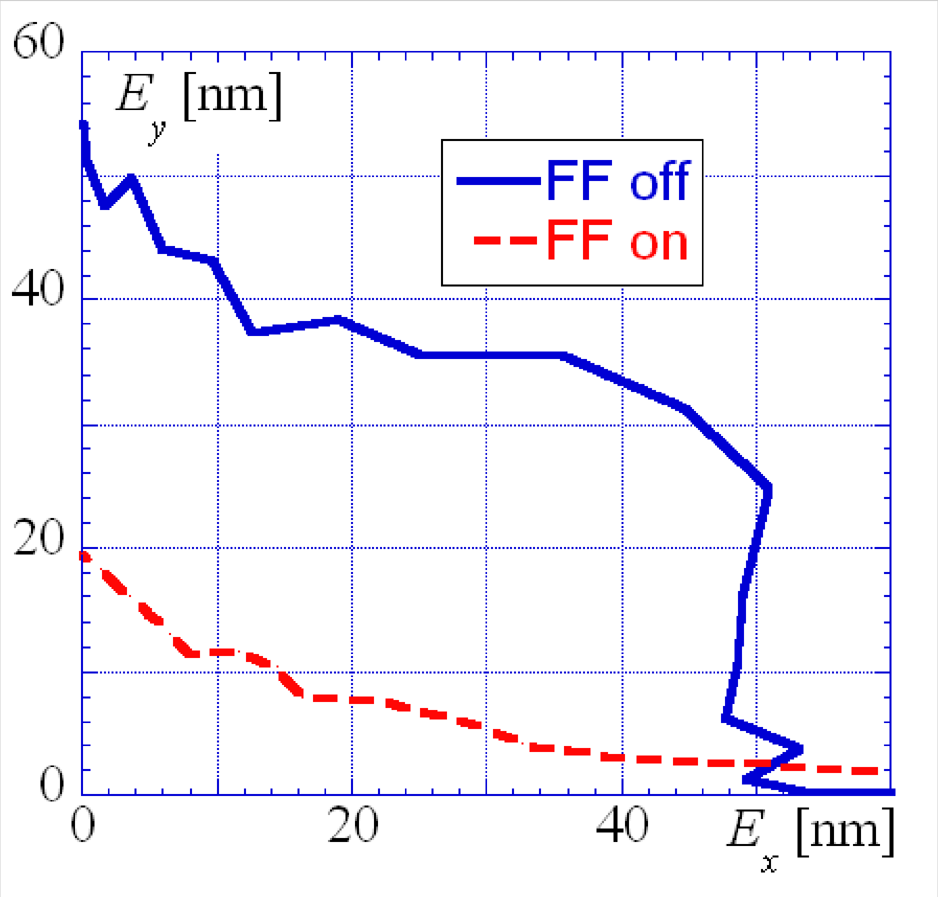
MAD-X calculation 6D tracking with synchrotron oscillations, no BeamBeam, 1024 turns

$f_{RF} = 800 MHz \qquad V_{RF} = 4 \times 16 MV$

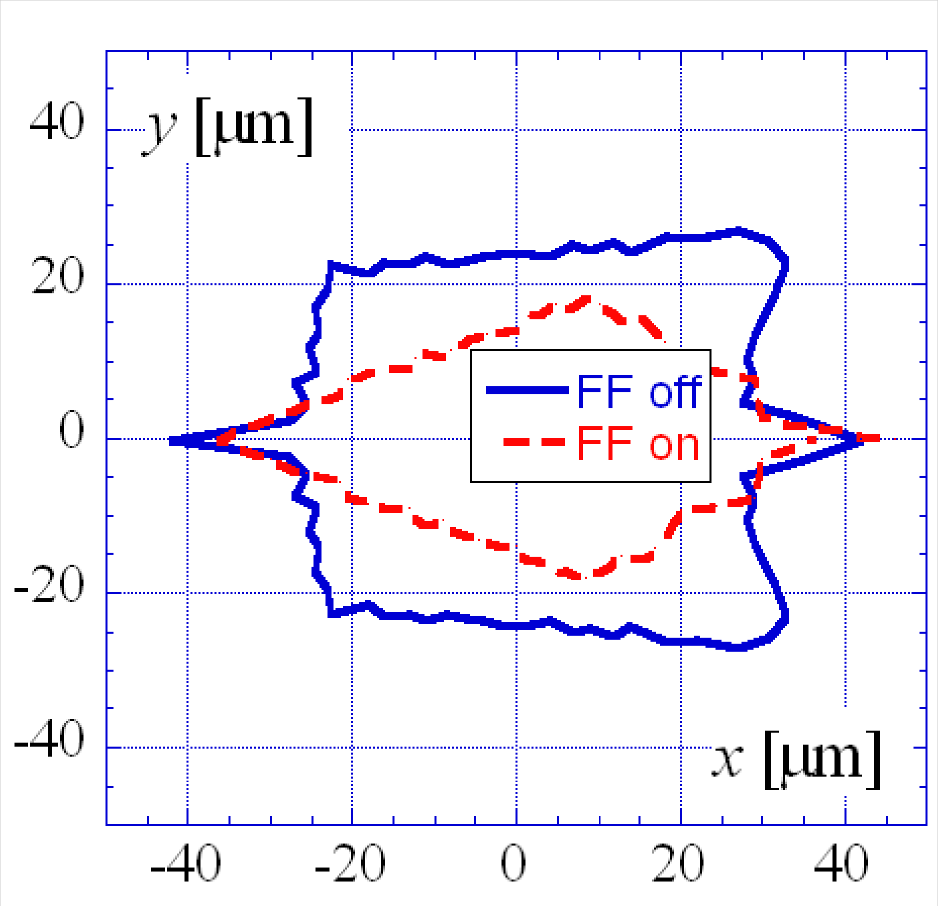
$Q_s \cong 10^{-3}$

$\epsilon_N = 10\ \mu m$

Fringe fields effect

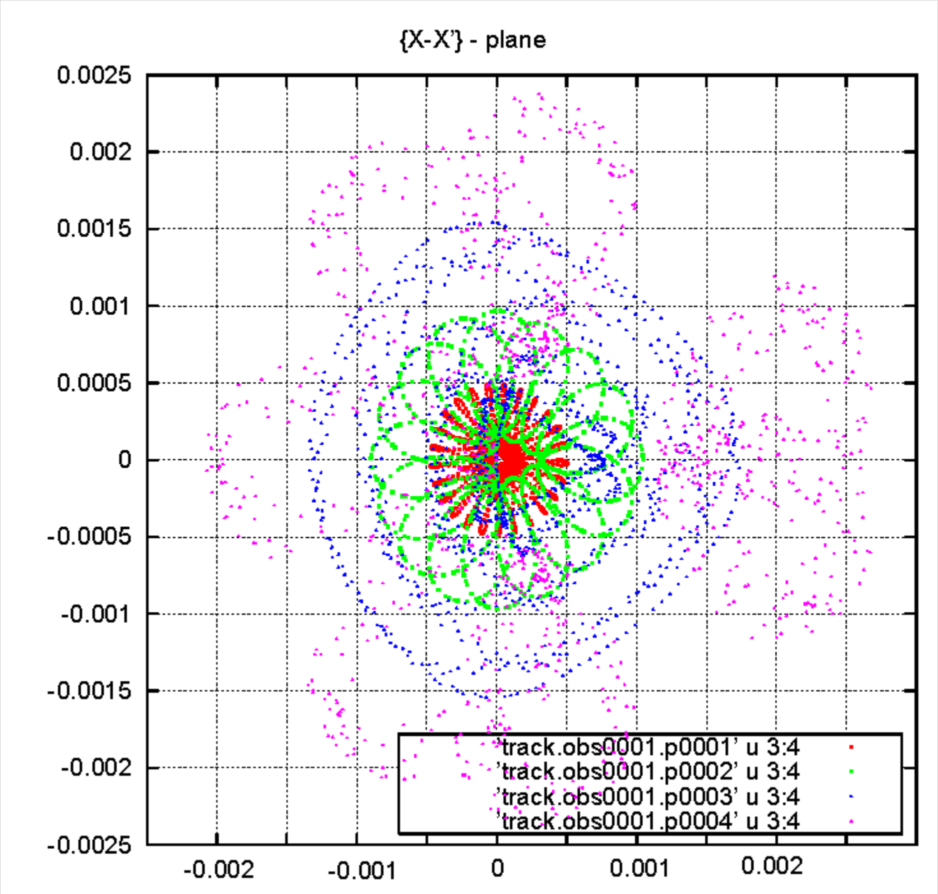


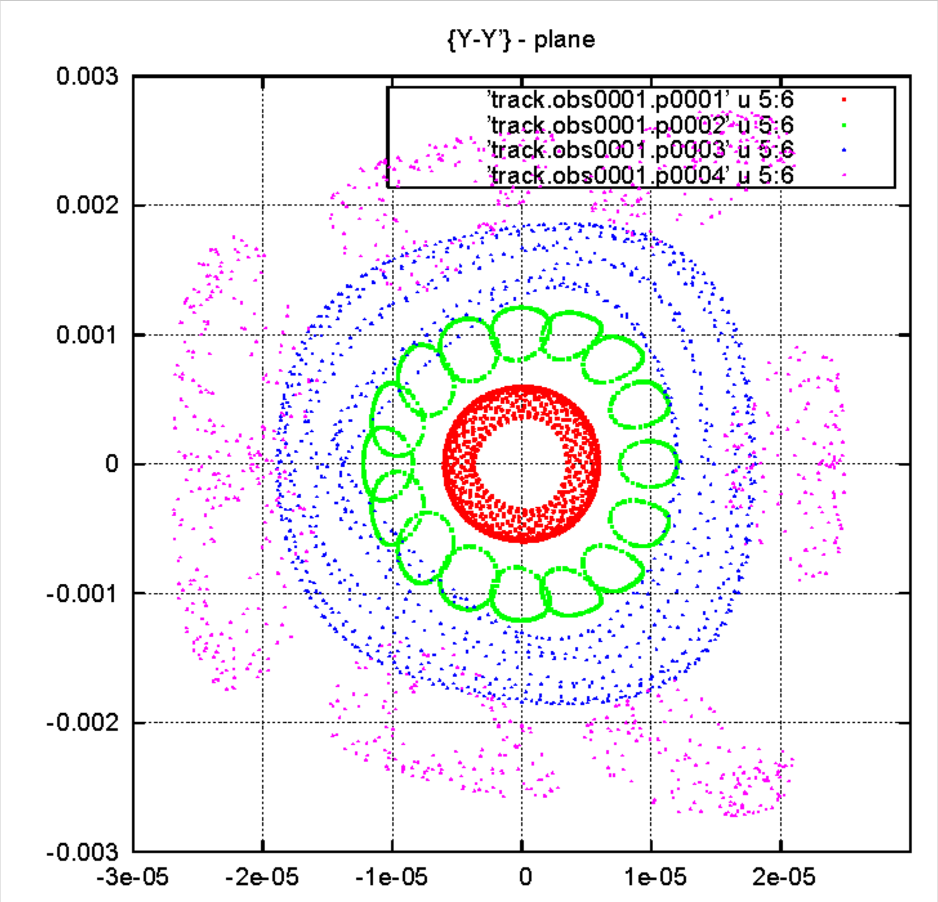
DA in terms of amplitudes



...or starting coordinates at IP

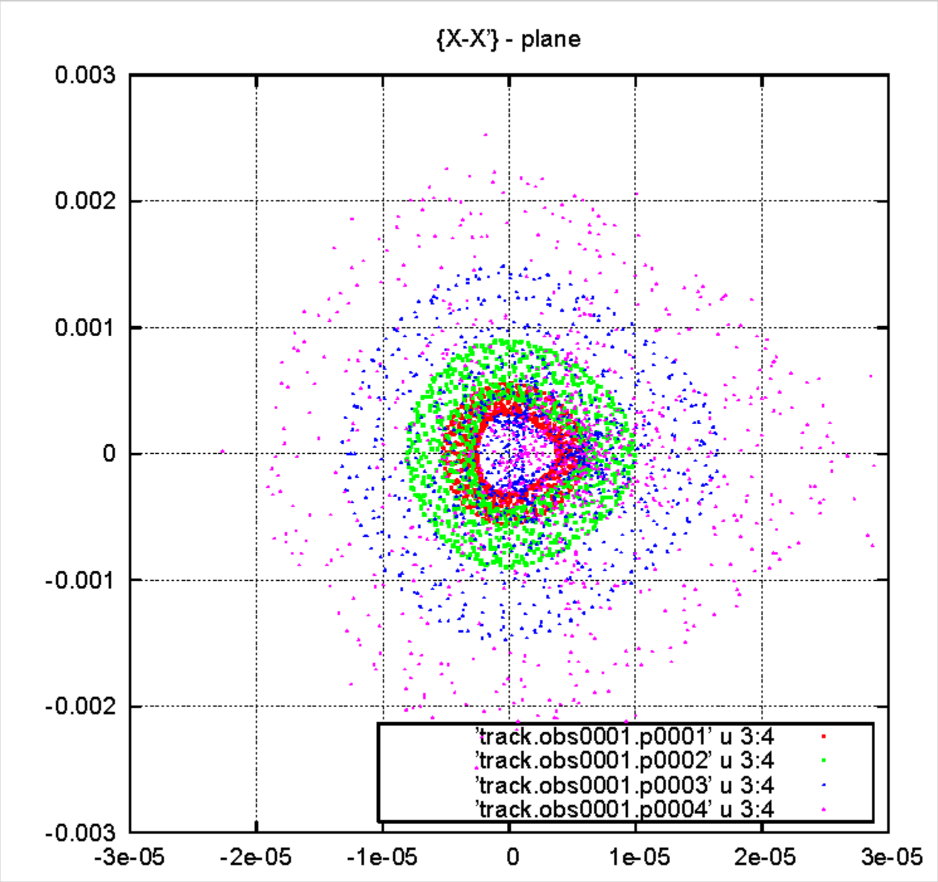
Phase space trajectories (x_0=0)

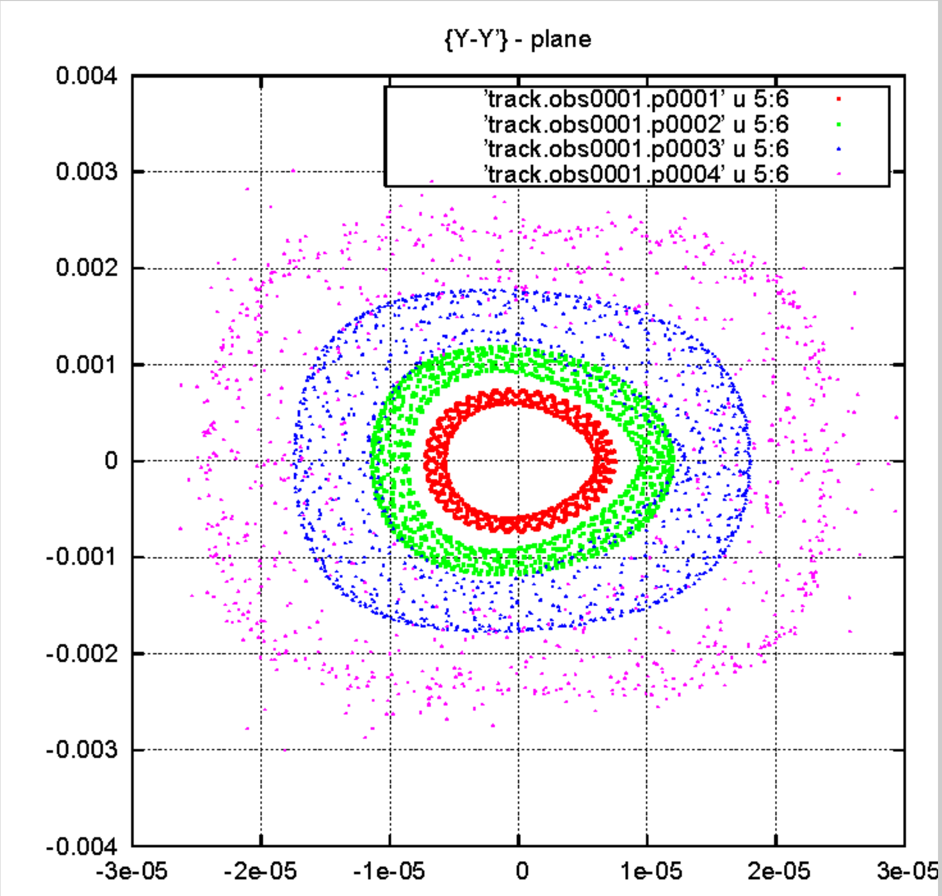




- y_0=6 μm
- y_0=12 μm
- y_0=18 μm
- y_0=24 μm

Phase space trajectories (x_0=6 μm)





- y_0=6 μm
- y_0=12 μm
- y_0=18 μm
- y_0=24 μm

Summary

Owing to the large chromaticity, the IR optics of a high luminosity Muon Collider must be designed having non-linear corrections in mind.

- **Wrt previous designs we have tried to make a feasible design and simplify the non-linear correction.**
- **In the scheme here presented the larger vertical chromatic wave is corrected "in loco"; yet an error-prone pseudo $-I$ insertion is needed for correcting the horizontal chromatic wave.**
- **The stability energy range is about $\pm 1.2\%$ ($\pm 1\%$ required at most).**
- **DA for nominal optics is large: 5.7σ for ϵ_N=25μm (3 required).**
- **Preliminary studies of beam-beam effects indicate a tolerable reduction of DA. Multipole errors can be corrected. Fringe fields have large impact and must be corrected too.**