LOSS FACTOR OF TAPERED STRUCTURES FOR SHORT BUNCHES*

A. Blednykh# and S. Krinsky
BNL, Photon Sciences, Upton, New York, 11973-5000, U.S.A.

Abstract

Using the electromagnetic simulation code ECHO, we have found [1] a simple phenomenological formula that accurately describes the loss factor for short bunches traversing an axisymmetric tapered collimator. In this paper, we consider tapered collimators with rectangular cross-section and use the GdfidL code to calculate the loss factor dependence on the geometric parameters for short bunches. The results for both axisymmetric and rectangular collimators are discussed.

INTRODUCTION

In the design of the vacuum enclosure for particle accelerators, there is often the need to incorporate a change in cross-sectional geometry. Examples of accelerator components that require changes in cross-section are RF cavities to accelerate the beam and collimators to scrape off the beam halo. When a charged particle beam travels through a vacuum chamber with changes in the cross-section, electromagnetic fields are generated which can act back on the beam causing it to lose energy and even become unstable. To describe this process, we must calculate the beam impedance [2-4]. In recent years there has been significant progress in both the development of numerical [5-7] and analytical [8-11] methods to calculate the beam impedance.

In this paper, we consider structures with axisymmetric and rectangular cross-sections. Two programs, the ECHO code [5,6] and the GdfidL code [7] are used to calculate the loss factor. We consider a bunch of total charge Q having a Gaussian longitudinal profile with rms length σ. The loss factor κ depends on the bunch length σ and is defined to be the energy ΔE lost by the bunch when it traverses the structure divided by the total bunch charge squared,

$$\kappa(\sigma) \equiv \Delta E / Q^2 .$$

The tapered axially symmetric collimator is illustrated in Fig. 1a and the tapered rectangular (flat) collimator in Fig. 1b. We denote the smaller pipe radius in round structure and the smaller half-aperture in flat structure by b and the larger radius and the larger vertical half-aperture by d. The length of the inner section of the collimator is denoted by g. The length of the tapered section is denoted by L. In all cases that we shall consider, we assume the rms bunch length is short compared to the smallest radius, i.e. $\sigma << b$.

We start our analysis with the known expressions for the longitudinal impedance for the round and flat collimators with step transitions. Once we know the impedance, we can determine the loss factor.

For the untapered round collimator with $d-b >> \sigma$ the impedance is approximately given by [2,3,9]

$$Z_{\parallel,step}^{rnd} = \frac{4}{c}\log(d/b) \qquad (1)$$

and the loss factor is

$$\kappa_{step}^{rnd} \approx \frac{c}{2\sqrt{\pi}}\frac{1}{\sigma}Z_{\parallel,step}^{rnd} . \qquad (2)$$

To convert to mks units, multiply by $cZ_0/4\pi$.

For the untapered rectangular collimator, the longitudinal impedance in the "optical regime" for width $w \to \infty$, has been derived by G. Stupakov, K.L.F. Bane and I. Zagorodnov as [10,11]

$$Z_{\parallel,step}^{flt} = \frac{8}{c}\int_0^\infty dx \frac{\sin(\pi\alpha/2)\cdot\cosh(\pi x)}{\cos(\pi\alpha)-\cosh(2\pi x)}\log\left(\frac{\cosh(\pi x)-\cos(\pi\alpha/2)}{\cosh(\pi x)+\cos(\pi\alpha/2)}\right) \qquad (3)$$

where $\alpha = b/d$. The loss factor is given by

$$\kappa_{step}^{flt} \approx \frac{c}{2\sqrt{\pi}}\frac{1}{\sigma}Z_{\parallel,step}^{flt} . \qquad (4)$$

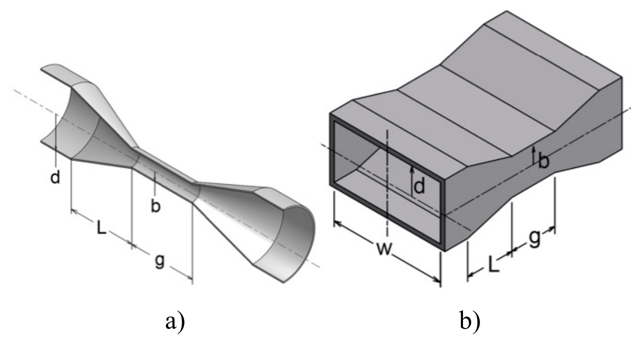

Figure 1: a) Tapered axially symmetric collimator. b) Tapered rectangular collimator.

The results of our numerical calculations using ECHO and GdfidL, can be approximated by the phenomenological formula

$$\kappa_{taper}/\kappa_{step} \approx \left[\frac{2}{\pi}\arctan\left(\frac{\mu\, d^2}{\sigma L}\right)\right]^2 , \qquad (5)$$

where μ=0.2 for the round structures [1] and μ=0.4 for the flat structures with $w >> d$.

AXIALLY SYMMETRIC TAPERED COLLIMATOR

When changes in the vacuum chamber cross-section are necessary, it is sometimes possible to reduce the resulting impedance by providing a tapered region between the two different geometries In the case of axisymmetric

*Work supported by DOE contract No: DE-AC02-98CH10886
#blednykh@bnl.gov

collimators with $d-b>b$ we have found that the effect of tapering on the loss factor is well described by the simple formula

$$\kappa_{taper}^{rnd} \approx \kappa_{step}^{rnd} \cdot \left[\frac{2}{\pi} \arctan\left(\frac{0.2d^2}{\sigma L} \right) \right]^2 \qquad (6)$$

The dependence of the loss factor on taper length L is illustrated in Fig. 2. The good agreement with Eq. (6) is apparent. As can be seen from Fig. 2, Eq. (6) holds for a geometry with zero taper length as well as for large L. For L = 0, Eq. (6) becomes equivalent to the well know equation Eq. (2). On increasing L, the loss factor changes slowly for small L, but it drops dramatically $\propto 1/L^2$ for large L. This means it is preferable to use long tapers for vacuum components in order to reduce the loss factor significantly when necessary. The agreement between the numerical results and the formula of Eq. (6) is very good.

In Figure 2, we plot numerical data for tapered round collimators with different parameters σ, d, b and fixed g. The vertical axis is the numerically obtained loss factor determined from ECHO, divided by the loss factor of Eq. (2). The horizontal axis is the taper length multiplied by a scale factor σ/d^2. Analytical data due to Eq. (6) are plotted as the ratio of the loss factors for the tapered collimator to the step collimator with appropriate bunch lengths (green curve). As one can see, all numerical data lie close to the analytical curve. The agreement is good for all values of L.

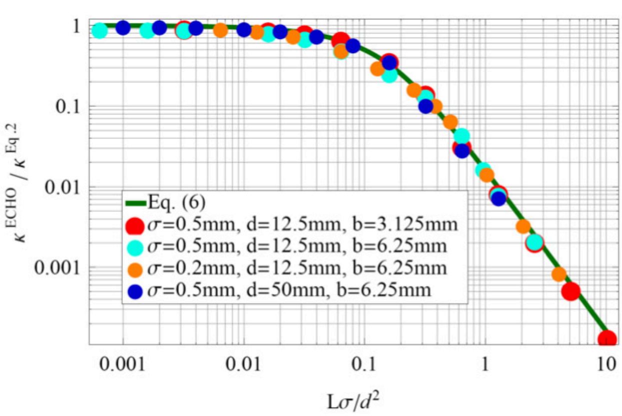

Figure 2: For a variety of tapered round collimators with g = 500mm, we plot the ratio of the simulated loss factor (ECHO) to the loss factor due to Eq. (2) $\kappa^{ECHO}/\kappa^{Eq.2}$ versus the dimensionless scaled length $L\sigma/d^2$. Results of ECHO calculations for the loss factor are shown for parameters: σ=0.5mm, *d*=12.5mm, *b*=3.125mm (red dots); σ=0.5mm, *d*=12.5mm, *b*=6.25mm (cyan dots); σ=0.2mm, *d*=12.5mm, *b*=6.25mm (orange dots); and σ=0.5mm, *d*=50mm, *b*=6.25mm (blue dots). The green curve is given by Eq. (6).

RECTANGULAR TAPERED COLLIMATOR

Based on the results of GdfidL calculations, we have found that for the flat collimators with $d-b>b$ and width $w>>d$, the effect of tapering on the loss factor is well described by

$$\kappa_{taper}^{flt} \approx \kappa_{step}^{flt} \cdot \left[\frac{2}{\pi} \arctan\left(\frac{0.4d^2}{\sigma L} \right) \right]^2 \qquad (7)$$

where κ_{step}^{flt} is the loss factor of the untapered flat collimator given in Eq. (4).

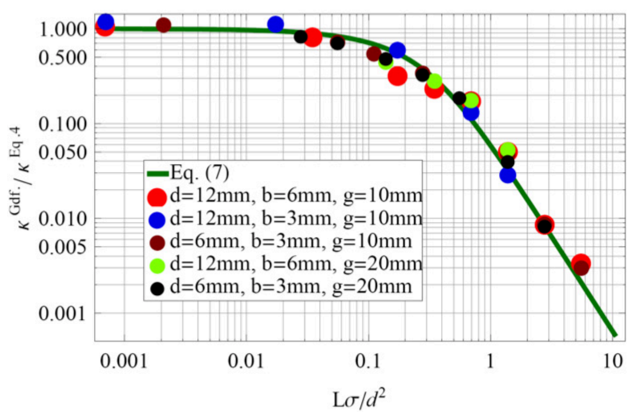

Figure 3: For a variety of tapered rectangular collimators with w = 48mm, we plot the ratio of the simulated loss factor (GdfidL) to the loss factor due to Eq. (4), $\kappa^{Gdf.}/\kappa^{Eq.4}$, versus the dimensionless scaled length $L\sigma/d^2$. Results of GdfidL calculations for the loss factor ratios are shown for parameters: *d*=12mm, *b*=6mm, *g*=10mm (red dots); *d*=12mm, *b*=3mm, *g*=10mm (blue dots); *d*=6mm, *b*=3mm, *g*=10mm (wine dots); *d*=12mm, *b*=6mm, *g*=20mm (light green dots); *d*=6mm, *b*=3mm, *g*=20mm (black dots). The green curve is given by Eq. (7).

For $L \to 0$, Eq. (7) becomes equivalent to Eq. (4) for the step collimator. For $L \to \infty$, the behavior of the loss factor ratio is similar to that for the round collimator $\propto 1/L^2$ except the coefficient in the numerator is a factor of four larger.

In Figure 3, we plot data obtained due to simulations using the GdfidL code for different geometric parameters d, b, L, σ and g of the rectangular collimator. The numerically simulated data agree well with Eq. (7) (green curve).

CONCLUSION

The behaviour of the impedance of tapered structures for very short bunches in the optical regime has been determined in refs. [10,11]. Here, for the loss factors for two particular geometries, we have studied the departure from the optical regime behaviour as bunch length is increased. In both cases, the ratio of the loss factor for the tapered collimator to the loss factor in the optical regime is a function only of the scaling parameter $\sigma L/d^2$. The fact that the bunch length σ and the taper length L appear as a product is consistent with the recent scaling derived by Stupakov in ref. [12], since there is only a weak dependence on g. One noteworthy fact that is not a priori expected is that only the larger radius or vertical half-aperture d appears. The reduction factor is independent of b. Moreover, it is striking that the specific form involving the arctan given in Eq. (5) holds for both geometries, with only the coefficient μ differing by a factor of ≈ 2 for flat vs round. This suggests that there may be a useful phenomenological form for more general geometries which may follow from natural extensions of Eq. (5). This possibility is presently being investigated.

ACKNOWLEDGEMENTS

We thank Dr. I. Zagorodnov for making his ECHO code available for our use and B. Kosciuk for providing us with 3D rendering geometries.

REFERENCES

[1] A. Blednykh and S. Krinsky, Phys. Rev. ST-AB **13**, 064401 (2010).

[2] A. W. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators* (John Wiley & Sons, New York, 1993).

[3] B. W. Zotter and S. A. Kheifets, *Impedances and Wakes in High-Energy Particle Accelerators* (World Scientific, London, 1998).

[4] S. Heifets and S. Kheifets, Rev. Mod Phys. **63**, 631 (1991).

[5] I. Zagorodnov and T. Weiland, Phys. Rev. ST-AB **8**, 042001 (2005).

[6] I. Zagorodnov, Phys. Rev. ST-AB **9**, 102002 (2006).

[7] W. Bruns, GdfidL, http://www.gdfidl.de.

[8] G.V. Stupakov, "Impedance of Small-Angle Collimators in High-Frequency Limit," SLAC – PUB-8857 (2001).

[9] G. Stupakov, "Advances in Impedance Theory," Proc. PAC09, Vancouver, B.C., Canada, 4-8 May 2009.

[10] G. Stupakov, K.L.F. Bane and I Zagorodnov, Phys. Rev. ST Accel. Beams **10**, 054401 (2007).

[11] G. Stupakov, K.L.F. Bane and I Zagorodnov, Phys. Rev. ST Accel. Beams **10**, 074401 (2007).

[12] G. Stupakov, K.L.F. Bane and I Zagorodnov, Phys. Rev. ST-AB **14**, 014402 (2011).