NUMERICAL CALCULATIONS FOR THE SYNCHROTRON RADIATION CHARACTERISTICS DESCRIBED IN TERMS OF QUANTUM THEORY: THE CASE OF WEAKLY EXCITED PARTICLES

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Abstract

We present the numerical calculations based on the theoretical research of synchrotron radiation characteristics for the weakly excited particles. For a spinless and spinor particle the exploration of effective angles and deviation angles is to be conducted. Comparing the obtained data with its classical analogue, one shows that the quantum theory gives a number of unpredictable results.

INTRODUCTION

Once discovered experimentally, the synchrotron radiation (SR) is a phenomenon with a special sort of relations between its theory and corresponding experiments. What we can call a tradition, is a theory always being step by step followed by its experimental confirmation [1, 2]. Thus, continuing the theoretical exploration of SR properties one can discover new horizons for experimental work. We give this short excuse to present a number of results concerning SR obtained with the use of quantum theory.

To investigate the character of radiation the effective angle concept [3, 4] is used. In terms of quantum theory we consider the effective angle of a scalar particle (a boson) and a spin one-half particle with the direction of spin opposite to the external magnetic field (electron). Only the first excited state particles are under investigation mainly because at low levels one can do calculations for all energy values. Moreover, such choice is convenient to give a comparative analysis of the results obtained within classical and quantum approach.

In the framework of classical SR theory one interprets the phenomenon as a radiation of a particle moving around circular orbit in the plane straight perpendicular to the external magnetic field of strengths $H > 0$. The quantum picture differs completely, still we can consider the motion in a similar magnetic field. In this case the energy of a first excited state boson and electron respectively:

$$E = m_0 c^2 \gamma, \text{ where } \gamma^2 = (1-\beta^2)^{-1} =$$

$$= \begin{cases} 1+2B & \text{for an electron} \\ 1+3B & \text{for a boson} \end{cases}, \qquad (1)$$

$$B = \frac{H}{H_0}, \quad H_0 = \frac{m_0^2 c^3}{|e|\hbar}.$$

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Here m_0 is the rest mass of the particle, c is the velocity of light, γ is the relativistic factor, e - the algebraic charge value of the particle, $\hbar$ - the Plank constant.

We use index s to describe the polarization components as follows: if $s = 2, 3$ then it is σ- or π- component of linear polarized radiation respectively, $s = 1$ for right circular polarization (because of the symmetry, there is no need to separately describe left circular polarization component), $s = 0$ indicates the summed over polarization components or so-called total radiation. The direction of photon escape is described by θ.

THE EFFECTIVE ANGLE OF RADIATION

The definition of an effective angle was given in [3]. An effective angle is an opening angle into which the main part of power is radiated. Mathematically we can define an effective angle with the use of the following equations:

$$\begin{cases} \int_{\theta_1}^{\theta_2} \Phi_s(\beta,\theta)\sin\theta d\theta = \frac{1}{2}\int_0^{\pi} \Phi_s(\beta,\theta)\sin\theta d\theta, \\ \Phi_s(\beta,\theta_1)\sin\theta_1 = \Phi_s(\beta,\theta_2)\sin\theta_2. \end{cases} \qquad (2)$$

Here the functions $\Phi_s(\beta,\theta)$ define the character of the angular distribution for SR power. The effective angle is

$$\Delta_s(\beta) = \theta_2(\beta) - \theta_1(\beta)$$

We consider θ_2 instead of the formal deviation angle given in [3,4]. Now, let us write the expressions for $\Phi_s(\beta,\theta)$. In terms of classical theory they could be represented

$$x = x(\beta,\theta) = \beta\sin\theta,$$

$$\Phi_2(\beta,\theta) = \Phi_2(x) = \frac{4+3x^2}{(1-x^2)^{5/2}},$$

$$\Phi_3(\beta,\theta) = \Phi_3(x) = \frac{(4+x^2)\cos^2\theta}{(1-x^2)^{7/2}},$$

$$\Phi_0(\beta,\theta) = \Phi_2(\beta,\theta) + \Phi_3(\beta,\theta)$$

$$\Phi_1(\beta,\theta) = \sum_{n=1}^{\infty} \frac{n^2}{2}[J_n'(n\beta\sin\theta) + \frac{\cos\theta}{\beta\sin\theta}J_n(n\beta\sin\theta)]^2.$$

For the first radiated harmonic one can write

$$\Phi_2(\beta,\theta) = J_1'^2(x),$$

$$\Phi_3(\beta,\theta) = \frac{\cos^2\theta}{x^2}J_1^2(x),$$

$$\Phi_0(\beta,\theta) = \Phi_2(\beta,\theta) + \Phi_3(\beta,\theta),$$

$$\Phi_1(\beta,\theta) = \frac{1}{2}[[J_1'(x) + \frac{\cos\theta}{x} J_1(x)]^2.$$

Here $J_n,\ J_n'$ - are the Bessel function and its derivative respectively.

The quantum theory allows to obtain similar formula separately for a boson

$$x_0 = x_0(\beta) = \frac{\sqrt{3} - \sqrt{3 - 2\beta^2}}{\sqrt{3} + \sqrt{3 - 2\beta^2}},$$

$$x = x(\beta,\,\theta) = \frac{\sqrt{3} - \sqrt{3 - 2\beta^2\sin^2\theta}}{\sqrt{3} + \sqrt{3 - 2\beta^2\sin^2\theta}},$$

$$\Phi_2(\beta,\theta) = \Phi_2(x) = (1+x)^3(1-x)\,e^{-x},$$

$$\Phi_3(\beta,\theta) = \frac{(1+x)^5}{(1-x)}\cos^2\theta\,e^{-x},$$

$$\Phi_0(\beta,\theta) = \Phi_2(\beta,\theta) + \Phi_3(\beta,\theta),$$

$$\Phi_1(\beta,\theta) = \frac{\Phi_0}{2} + (1+x)^4\cos\theta\,e^{-x},$$

and an electron

$$x_0 = x_0(\beta) = \frac{1 - \sqrt{1-\beta^2}}{1 + \sqrt{1-\beta^2}},$$

$$x = x(\beta,\,\theta) = \frac{1 - \sqrt{1-\beta^2\sin^2\theta}}{1 + \sqrt{1-\beta^2\sin^2\theta}},$$

$$f(x) = \frac{(1+x)^3}{(1-x)}\,e^{-x},$$

$$\Phi_2(\beta,\theta) = \Phi_2(x) = (1 - x_0 x) f(x),$$

$$\Phi_3(\beta,\theta) = \frac{(1+x)^2}{(1-x_0 x)}\cos^2\theta\, f(x),$$

$$\Phi_0(\beta,\theta) = \frac{2x_0 - x(1+x_0^2)}{x_0} f(x),$$

$$\Phi_1(\beta,\theta) = \frac{2x_0(1+\cos\theta) - x(1 - 2x_0\cos\theta + x_0^2)}{2x_0} f(x).$$

THE ASPECTS OF CALCULATION

It is clear, that the calculations depend on the behavior of integrands, i. e. of functions $\Phi_s(\beta,\theta)$. If the function $\Phi_s(\beta,\theta)\sin\theta$ at fixed β is symmetrical with respect to $\theta = \frac{\pi}{2}$ then we can reduce the interval and search for θ_1 and θ_2 on $[0,\pi/2]$. In this case we put $\Delta_s(\beta) = 2(\theta_2 - \theta_1)$. Obviously, if $\Phi_s(\beta,\theta)\sin\theta$ is symmetrical with respect to $\theta = \frac{\pi}{2}$ and monotone increasing on $[0,\pi/2]$ at any value of β (thus has its maximum at $\theta = \frac{\pi}{2}$) it becomes necessary to consider only one equation for θ_1 instead of the system (2). That is the case when we deal with σ - component of boson, electron and total classical radiation. But the behavior of first harmonic is different: there is a value of β at which the functions $\Phi_2(\beta,\theta)\sin\theta$ cease being monotone and we need to solve the system of equations. The angular distribution of total radiation demonstrates exactly the same character (the one associated with boson is an exception). In ultrarelativistic limit ($\beta \to 1$) we can use the approximations obtained in [3, 5]

$$\Delta_s(\beta \to 1) = a_s\sqrt{1-\beta^2},$$

$$a_0 = 0.8414,\quad a_1 = 0.6765,$$

$$a_2 = 0.7407,\quad a_3 = 1.1950.$$

THE BEHAVIOR OF CLASSICAL AND QUANTUM EFFECTIVE ANGLES

Using the definition one can built the graphs which demonstrate the behavior of functions $\Delta_s(\beta)$ for all types of polarization components. Now it becomes easier to follow the character of $\Delta_s(\beta)$ and to compare the results provided by classical and quantum theories. Giving a brief regard to the figures below, one can be surprised with the fact that the curve corresponding with an electron behaves more similarly to the classical curve then the one related to boson. Of course, this is an unexpected result.

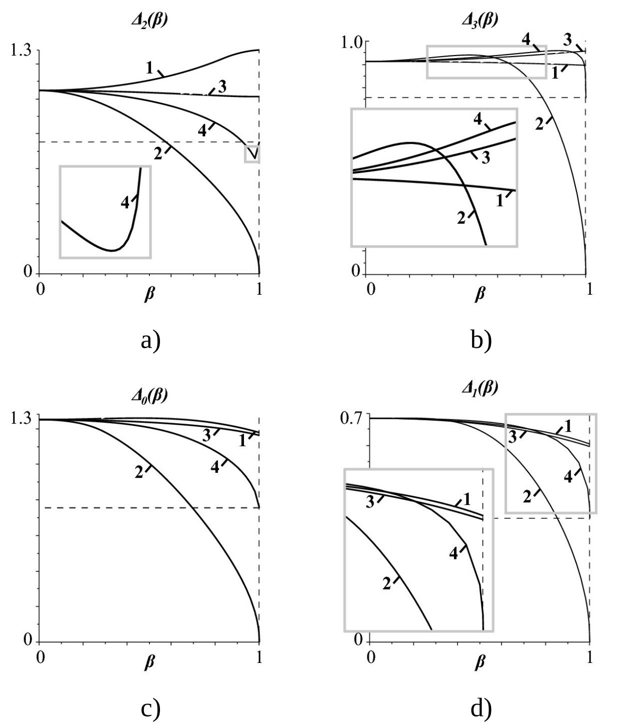

Figure 1: The behavior of $\Delta_s(\beta)$: σ-polarization component (a); π-polarization component (b); total radiation (c) and right circular polarization component (d). Label '1' is for first harmonic, '2' for classical curve, '3' for bosonic curve, '4' for electronic curve.

Considering the ultrarelativistic limit, the quantum theory says (in contrast to classical approach) that no radiation concentration can be observed near the orbits plane. At $\beta \to 1$ bosonic and electronic curves tend to fixed numbers differing from zero. However, in non-relativistic case

($\beta \to 0$) all the curves are very close, they start at the same point at $\beta = 0$ for every s, as it was expected. The values of Δ_s at $\beta = 0$ are as follows

$$\Delta_0 = 1.251540130, \quad \Delta_1 = 0.6823511020,$$

$$\Delta_2 = \pi/3 = 1.047197551, \quad \Delta_3 = 0.9123454428.$$

CONCLUSION

It is shown, that the radiation angular distribution of an electron behaves more similarly to the classical theory predictions, then the one associated with boson. Touching boson, one can observe its effective angles (Δ_1, Δ_0) having the same character as the first harmonic does. Obviously, these results are unexpected.

Considering the ultrarelativistic limit one can find the difference between the values of SR characteristics calculated with the use of classical and quantum approaches. Namely, according to quantum theory no radiation concentration can be observed near the orbit's plane.

REFERENCES

[1] A.A. Sokolov, I.M. Ternov, "Synchrotron Radiation," Berlin, 1968

[2] I.M. Ternov, V.V. Mikhailin , "Synchrotron Radiation. Theory and Experiment," 1986

[3] V.G. Bagrov, "Special Features of the Angular Distribution of Synchrotron Radiation," Tomsk State University Physics Journal, Tomsk, 51, p.335-352 (2008)

[4] V.G. Bagrov, D.M. Guitman, "New Problems of Field Theory Kazan Summer School Proceedings," Kazan, 2004

[5] V.B. Tlyachev, " Special Features of the Angular Distribution of Synchrotron Radiation for Circular Polarization," Tomsk, 2005