IMPROVED ALGORITHMS FOR MULTIPACTING SIMULATION IN THE ANALYST CODE

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Abstract

Electron multipacting is often deleterious in RF structures and must be controlled via modifications to the geometry, materials, or external fields. Recent improvements to the capabilities for modelling multipacting in the Analyst software package are presented in this paper. A 4th order Runge-Kutta scheme [1], coupled with Newton-Raphson iteration, is used to integrate particle position/momentum, with integrations interrupted at element faces to minimize errors and lost particles. Support for the Furman-Pivi secondary emission model [2] has been implemented, with separate representations for low energy, re-diffused, and backscattered secondary particles, and multiple emissions per impact based upon a probability distribution. We have also developed a method to prune the tree of secondary particles resulting from an impact that minimizes particle count growth while maintaining important statistical information about the resonance. Finally, we have added support for volumetric sourcing of primaries, wherein the model volume is seeded with a population of particles with random positions and initial velocities. These improvements have been validated using benchmark calculations.

TRACKING ALGORITHM

The Analyst frequency-domain solvers use the finite-element method to determine the electric fields on an unstructured mesh of triangles for two-dimensional problems, and tetrahedrons for three-dimensional problems. Within each element the field is given by a sum over vector basis functions that have local support. The basis functions enforce continuity of the tangential field components across element boundaries, but the normal components may be discontinuous. Although smoothing can be used to enforce total field continuity, we instead integrate the particles in the exact finite-element fields within each element. This requires the determination of the element exit point for a particle, so that the field representation can be switched to that of the next element along its trajectory. Although the identification of the exit point is computationally expensive, it has the benefit that no searching is required to determine which element contains the particle at each time-step.

Within an element the 4th-order Runge-Kutta method is used to integrate the particle trajectory. Newton-Raphson iteration is used to determine a consistent solution to the dynamical equations at each time-step, which are coupled by the electromagnetic field. The method is not symplectic. However, numerical tests have shown high-accuracy results can be obtained with a modest number of time-steps per element, with total positional errors less than 1 part in 100000 per orbit for cyclotron orbits in uniform magnetic fields (see Figure 1).

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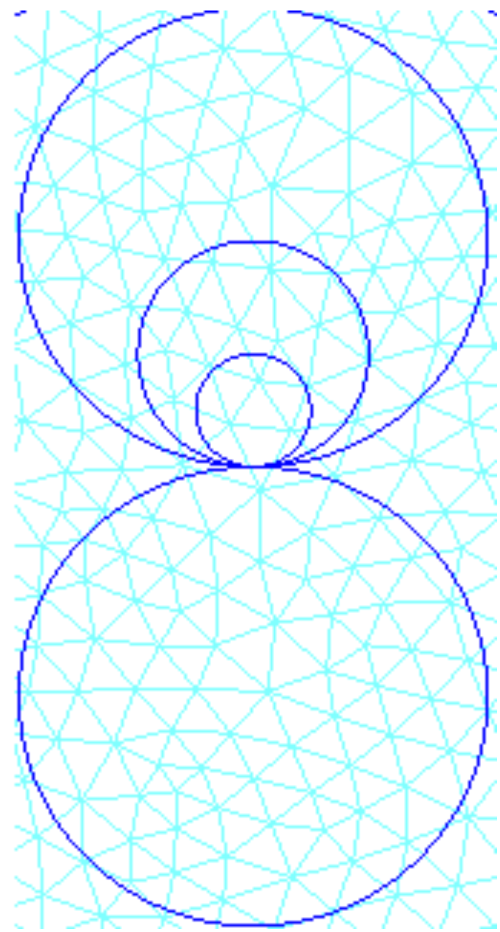

Figure 1: 500 orbits in uniform B-field for 4 different initial particle velocities. Positional error was 0.5% of the orbit radius after 500 orbits, in the form of a lag along the particle trajectory.

SECONDARY EMISSION MODEL

The secondary emission model described by Furman [2] has been implemented. This stochastic model has separate representations for low-energy, backscattered, and re-diffused secondary particles. The model for low-energy, or "true", secondary emission yields a variable number of emitted particles per primary impact, each with an initial energy and direction obtained by sampling a particular distribution function. The true secondary yield curve uses a common parameterization that allows modelling of a variety of materials.

MAGNETIC FIELD REPRESENTATION

Analyst uses an electric-field finite element formulation. Normally the magnetic field is obtained from the electric field using the curl operator, but this results in a one-order reduction in accuracy, e.g., if quadratic basis functions were used in the finite-element model, then the electric field will be known to 2nd order, and the magnetic field will be known only to 1st order.

An alternative approach is to enforce the curl relationship in the weak sense, which leads to a positive definite matrix equation of the form $j\omega\mu_0\bar{\bar{B}}\bar{h} = -\bar{\bar{C}}\bar{e}$ that relates the magnetic field coefficients to the electric field solution vector $\bar{e}$. The matrix equation can be solved using the pre-conditioned conjugate gradient to obtain the magnetic field projected onto the same basis set that is used for the electric field. The operation does not recover the magnetic field to full accuracy, as this would require use of the magnetic field coefficients in the original matrix equation. Nevertheless, the projected magnetic field shares the same properties as the electric field, i.e., continuous tangential components across element boundaries, which is expected to improve particle tracking accuracy.

MULTIPACTING CALCULATION IN LOW-BETA CAVITY

We used Analyst to study a prospective cavity under development at Fermi National Accelerator Laboratory (FNAL); see Figure 2. In operation the peak surface fields are expected to be less than about 50 MV/m, and the fundamental mode is resonant at 650 MHz. Because of the geometric and electromagnetic symmetry, any potential multipactors are expected to have orbits in the r-z plane, so the problem can be analysed in two dimensions.

Figure 3 shows the mesh used in our analysis. Multipactors are possible near the “equator” of the cavity, that is, at the top near the axial symmetry plane, so the mesh was constrained in this region to allow more accurate computation of particle orbits. The element size in the bulk of the cavity was 5 mm, with 0.5 mm elements in the upper region, and 0.1 mm elements close to the equator. The Analyst two-dimensional eigensolver OM2P was used to extract the fundamental mode (Figure 4), and the mode fields were then used in the particle tracking calculation.

The calculation showed a broad multipacting band from 20-35 MV/m peak surface field, and also a less pronounced band in the 45-50 MV/m range, as seen in Figure 5. Figure 6 shows the predicted particle count growth rates for the same field range, with positive values indicating the potential for sustained multipacting.

Orbits in the lower field band are classic two-point orbits, with the particles transiting the cavity equator every half RF period. The orbits in the high-field band are similar except that additional small loops are traversed after a larger loop transits the equator (Figure 7).

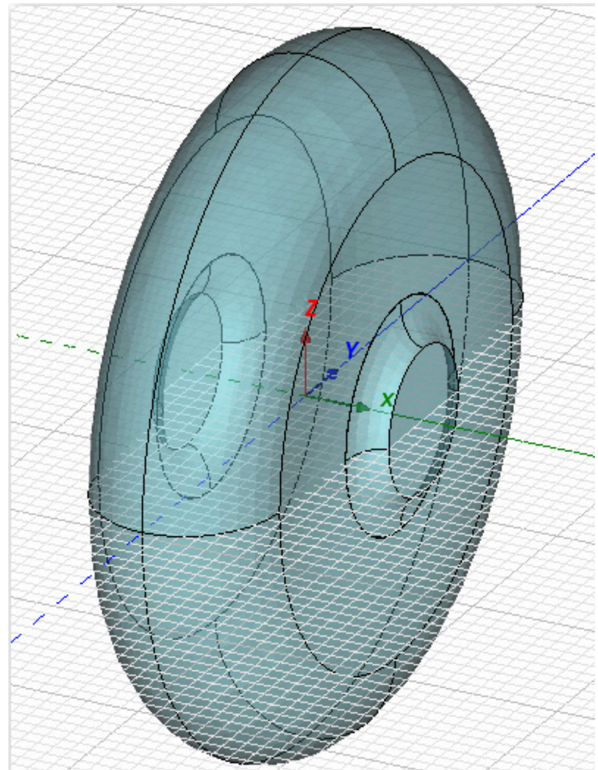

Figure 2: FNAL Beta=0.61 Project X cavity.

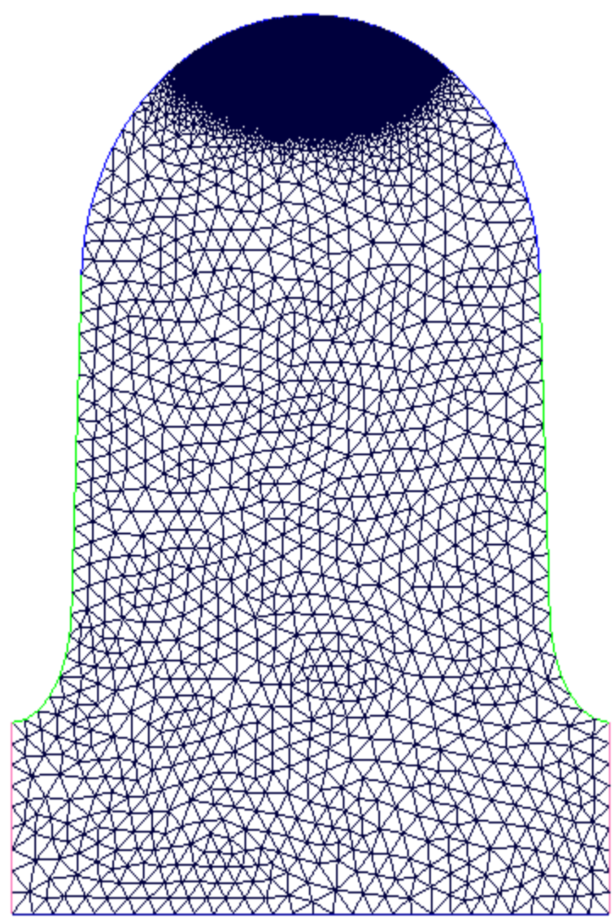

Figure 3: Mesh of beta=0.61 Project X cavity used in multipacting calculation.

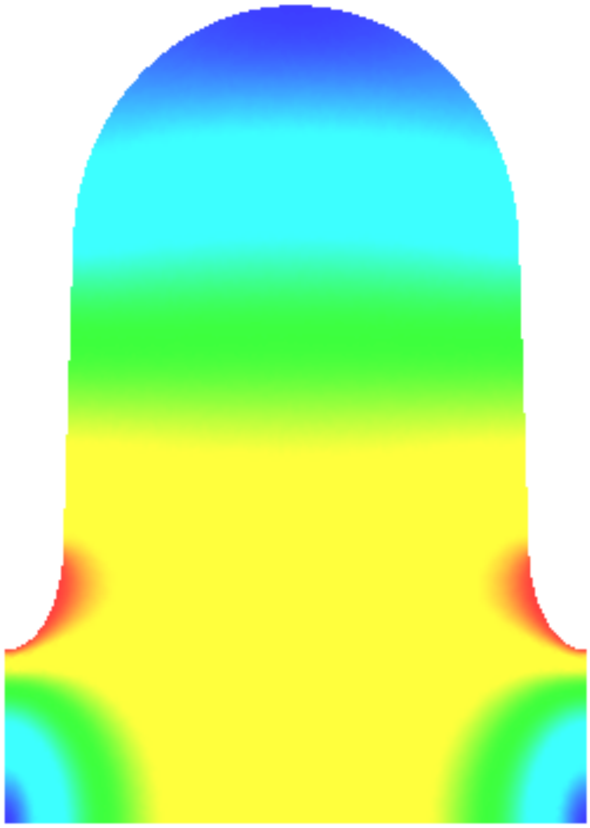

Figure 4: Electric field magnitude in cavity.

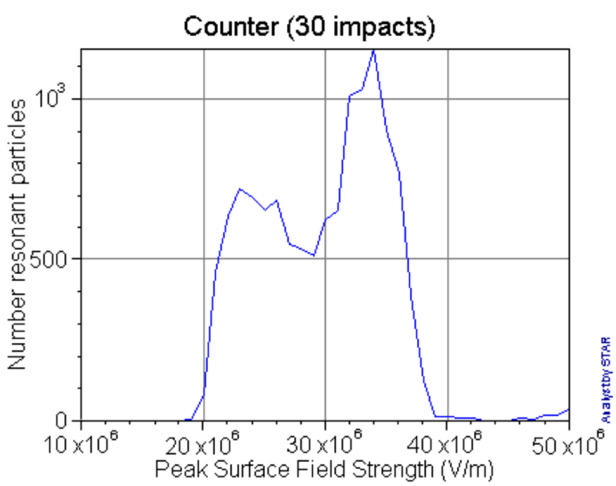

Figure 5: Counter function for low-beta cavity in the range of 10-50 MV/m. The 20-35 MV/m band exhibits classic multipactor orbits, while small rise at 50 MV/m comes from a different type of orbit.

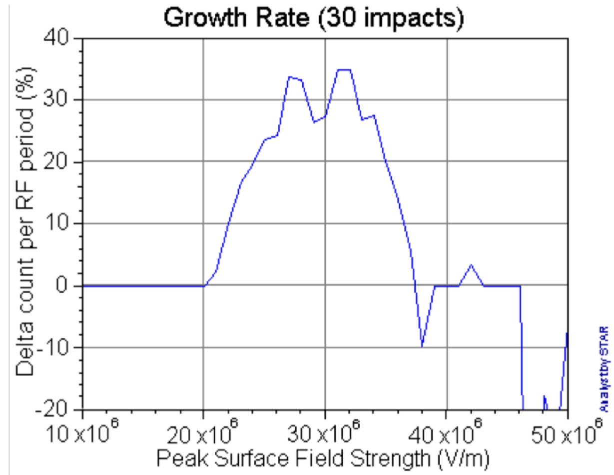

Figure 6: Overall particle count growth rates, shown in terms of the percentage change in count per RF period.

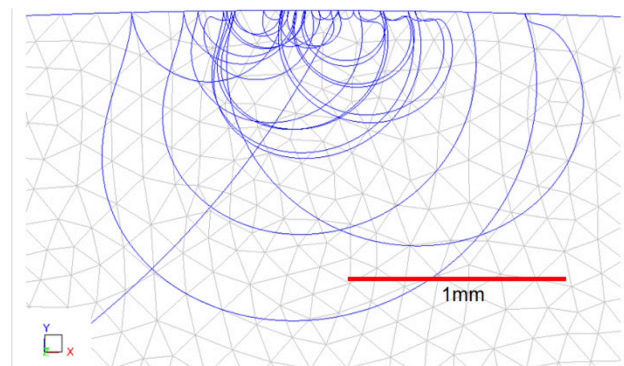

Figure 7: Multipacting orbit at 50 MV/m peak surface field. Variability stems from stochastic nature of emission model.

CONCLUSION

A new particle tracking algorithm and a new secondary emission model have been implemented in the Analyst particle simulation module. A more accurate method of obtaining the magnetic field from electric field finite element solutions has also been implemented. The updated software has been used to study multipacting in a proposed cavity for Project X at FNAL, finding potential bands at 20-35 MV/m and 45-50 MV/m.

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REFERENCES

[1] D. Darmofal, et al., Jour. Comp. Phys., 123, 1996, pp. 182-195.
[2] M. Furman, et al., LBNL-52807, June, 2003.