# NUMERICAL STUDIES OF NON-LINEAR DYNAMICS IN BEP 

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#### Abstract

An analysis of the dependence of experimental captured positron current data from the booster storage ring BEP (VEPP-2000 facility, BINP, Russia) on the working point position on the frequency map has uncovered a great number of different non-linear resonances. The number of captured positrons after a single injection is observed to be much less than the expected value. It is anticipated that the high degree of symmetry in the magnet system of BEP, however, should lead to the suppression of such resonances. To study this discrepancy, numerical simulations of positron beam movement under different perturbations to account for potential errors in magnetic field gradient of non-linear elements and errors in their angular location are used. The findings of this research provide qualitative explanations of the experimental work diagram and answers to two main questions, specifically "Why in the absence of skew-sextupoles in structure and small coupling are strong skew-sextupole resonances observed?" and "Why skewsextupole resonances are stronger than sextupole ones of the same harmonic?".


## INTRODUCTION

The VEPP-2000 collider (BINP SB RAS, Russia) with $2 \times 1 \mathrm{GeV}$ design energy and $10^{32} \mathrm{~cm}^{2} \mathrm{sec}^{-1}$ design luminosity has been built for the studying of a hadron production in $\mathbf{e}^{+} \mathbf{e}^{-}$collisions. Besides particle physics experiments, it has examined the round beam concept which ensures the required luminosity.


Figure 1: Scheme of the VEPP-2000 facility.
At this time the beam reproduction is realized by the scheme presented in a Fig. 1. The conversion target is placed in B-3M - BEP bypass channel for positron production. The theoretical value of the captured current of an $\mathbf{e}^{+}$beam should be around $600 \mu \mathrm{~A}$ with 1 A current of the $\mathrm{e}^{-}$beam ejected from B-3M. In reality the single capture doesn't exceed $100 \mu \mathrm{~A}$, which makes the operation of whole facility more complicated. In the future the injection will be from VEPP-5 facility which is under construction.

[^0]Research of the positron capture efficiency depending on beam's position on a tune diagram shows the presence of a large number of non-linear resonances the source of which is not clear (e.g. skew-sextupole resonances). The results of this paper explain the source of this resonances and their strength.

## BEP OPTICS

The booster synchrotron BEP consists of 12 the same identical periods composed of $30^{\circ}$ bending magnet and doublet of D- and F-lens. Structural functions and the arrangement of magnet optics are shown in the Fig. 2.


Figure 2: Structural functions and the magnet optics arrangement of one period of BEP.

The main parameters of the BEP estimated for injection energy are presented in Table 1 [1].

Table 1: BEP Parameters.

| Physical Quantity | Value |
| :--- | :---: |
| $\Pi$, perimeter | 22.35 m |
| $H$, field in bending magnet | 0.313 T |
| $G_{D}$, gradient in D-lens | $-63 \mathrm{mT} / \mathrm{cm}$ |
| $G_{F}$, gradient in F-lens | $+41 \mathrm{mT} / \mathrm{cm}$ |
| $U_{0}$, RF voltage | 30 kV |
| $\omega_{0}$, RF frequency | 26.83 MGHz |
| $q$, multiplicity of RF harmonic | 2 |
| $\alpha$, orbit compaction factor | 0.05 |
| $E$, injection energy | 120 MeV |
| $\nu_{x}$, x-betatron frequency | 3.46 |
| $\nu_{z}$, z-betatron frequency | 3.21 |
| $\nu_{s}$, synchrotron frequency | 0.001 |
| $W$, radiation losses | $15 \mathrm{eV} / \mathrm{turn}$ |
| $\tau_{x}$, time of damping | 1.3 sec |
| $\tau_{z}$, time of damping | 1.2 sec |
| $\delta E / E$, energy deviation | $\pm 3 \%$ |

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Further to main optics the following corrections are added to the structure of BEP:

- Additional winding in D - and F-lenses for the quadrupole gradient correction of betatron frequencies variation.
- Dipole correctors in D-lenses and bending magnet for the orbit correction.
- Sextupole corrections for chromatism compensation; main sextupoles $S_{D}$ and $S_{F}$ are contained in quadrupole lenses, and additional sextupoles $S_{X}$ and $S_{Z}$ are located after F- and D-lenses respectively.


## EXPERIMENTAL DATA

## Tune Diagram and Resonances

In addition to integer and linear various non-linear resonances can occur in a system due to multipole fields. In general they are given by:

$$
\begin{equation*}
m_{x} \nu_{x}+m_{z} \nu_{z}=n \tag{1}
\end{equation*}
$$

where $m=\left|m_{x}\right|+\left|m_{z}\right|$ - resonance order and $n$ - perturbation harmonic \#.


Figure 3: Tune diagram and resonance lines.
The tune diagram with resonance lines up to the fourth order is shown in a Fig. 3. Main resonances due to straightand skew-sextupoles are listed in the Table 2 where $r$ - the order of perturbation theory where a resonance appears.

Table 2: Resonances Due to Straight- and Skew-Sextupoles

| $\mathbf{r}$ | Sextupole | Skew-sext. |
| :---: | :---: | :---: |
| 1 | $\nu_{x}=n$ | $\nu_{z}=n$ |
|  | $3 \nu_{x}=n$ | $3 \nu_{z}=n$ |
|  | $\nu_{x} \pm 2 \nu_{z}=n$ | $2 \nu_{x} \pm \nu_{z}=n$ |
| 2 | $2 \nu_{x, z}=n$ | $\nu_{x} \pm \nu_{z}=n$ |
|  | $4 \nu_{x, z}=n$ | $3 \nu_{x} \pm \nu_{z}=n$ |
|  | $2 \nu_{x} \pm 2 \nu_{z}=n$ | $\nu_{x} \pm 3 \nu_{z}=n$ |

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Figure 5: Structure without perturbation.


Figure 6: Errors in sextupole angular location $\sigma_{\phi_{\mathbf{3}}}=\mathbf{0 . 6}{ }^{\circ}$.
where $\theta_{F D}$ is an angular distance between S- and Dsextupoles. Fourier series expansion of $G$ shows that only the harmonics with \# multiple of 12 are present. The result of simulation for this case is shown in Fig. 5; as predicted only integer and weak linear coupling resonances appear.

The errors in angular location of sextupoles, $\varphi_{3}$, were introduced to simulation in the effort to explain the appearance of skew-sextupole resonances (Fig. 6). Assuming that $\varphi_{3}$ is a small random variable with zero mean-value and variance $\sigma_{\varphi_{3}}^{2}$, the averaged root-mean-square amplitudes of distributed straight- and skew-sextupole gradients are

$$
\begin{align*}
\left\langle c_{n}^{S t S e x t} c_{n}^{S t S e x t^{*}}\right\rangle & =\left(\frac{1}{2 \pi} \frac{9 \sigma_{\varphi_{3}}^{2}}{2}\right)^{2} 12\left(G_{D}^{2}+G_{F}^{2}\right)  \tag{2}\\
\left\langle c_{n}^{S q S e x t} c_{n}^{S q S e x t^{*}}\right\rangle & =\left(\frac{1}{2 \pi} 3 \sigma_{\varphi_{3}}\right)^{2} 12\left(G_{D}^{2}+G_{F}^{2}\right) \tag{3}
\end{align*}
$$

for $n \in(\mathbb{N} \backslash 12 k) ; k=0,1,2, \ldots$. This fact gives answers to both formulated questions and shows that the strength


Figure 7: $\sigma_{\phi_{\mathbf{3}}}=1 . \mathbf{0}^{\circ}$ and $\sigma_{\mathbf{F}, \mathbf{D}}=\mathbf{2} \%$.


Figure 8: Coupling inclusion ( $\sigma_{\phi_{\mathbf{3}}}=\mathbf{0 . 6}{ }^{\circ}, \sigma_{\mathbf{F}, \mathbf{D}}=\mathbf{3} \%$ ).
of skew-sextupole resonances should be greater than sextupole ones for the harmonics of the perturbation with \# different from $12 k$.

The only errors in magnetic field gradient $\sigma_{F, D}$ can not produce skew-sextupole resonances, but they can contribute to them due to additional gradient variation in a case with $\sigma_{\varphi} \neq 0$ (Fig. 7).

The coupling inclusion leads to the amplification of straight- and skew-sextupole resonances, parametric halfinteger resonance $2 \nu_{x}=7$ appearance (which presents in experimental data) and several stop-bands formation (Fig. 8).

## REFERENCES

[1] "BEP storage ring", BINP preprint 83-98, Novosibirsk, 1983.
[2] V. Ptitsyn, "Research of the dependence of $e^{+}$capture on the working point position in BEP booster", degree work, NSU, 1991.


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