# LOW-EMITTANCE LATTICE DESIGNS FOR ALS ULTIMATE UPGRADE\*

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## Abstract

A new method is developed to optimize low-emittance and low-beta lattices for ALS ultimate upgrade. The study provides us a different perspective on the lattice design, and confirms results early found using both Global Scan of All Stable Settings (GLASS) and Genetic Algorithms (GA) techniques.

## INTRODUCTION

The Advanced Light Source (ALS) at Lawrence Berkeley National Laboratory is one of the earliest 3rd generation light sources. Since the commissioning in 1993, a series of upgrades have been successfully completed, including the installation of superconducting bend magnets (Superbends) in 2001 [1] and implementation of top-off injection in 2007 [2]. To keep the ALS competitive in the future, it was recognized a few years ago that further upgrades to lower the storage ring emittance will be necessary.

The ALS low-emittance upgrade project has been started since 2009 [3]. After this upgrade is finished, the horizontal emittance is reduced by three factors from current 6.8 nm-rad to about 2 nm-rad. Fig. 1(a) and (b) show optics functions of one ALS sector before and after the upgrade. From the plots we can see that the major quantitative changes of the optics functions are the dispersion functions which are increased in the straights and decreased in the arcs, and the horizontal beta functions which are increased in the straights. This upgrade, so-called baseline upgrade which has been fully funded, will improve the brightness of many beamlines by several factors. It was quickly realized that the baseline upgrade lattice does not provide "ultimate" insertion device brightness due to the phase space mismatch of the electron and photon beams and large dispersion functions in the center of straights. If we could reduce the horizontal beta and dispersion functions to small values at the center of straight, i.e., upgrade the lattice to the one shown in Fig. 1(c) (ultimate upgrade), the insertion device brightness could be improved by another 2 or 3 factors.

We develop a new approach, so-called "inverse" method, to search for low-emittance and low-beta lattices. This approach can provide us a different perspective on the lattice design. The study confirms results early found using both Global Scan of All Stable Settings (GLASS) [4] and Multi Objective Genetic Algorithms (MOGA) [5] techniques.

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Figure 1: Layout of one sector of ALS lattice and its associated optics functions at three different operation modes: (a) Current lattice, (b) Baseline upgrade, (c) Ultimate upgrade.

## THEORY

It is well known that the horizontal natural emittance of a storage ring is given by

$$\epsilon_x = C_q \gamma^2 \frac{\langle \mathcal{H}(s)/|\rho(s)|^3 \rangle}{J_x \langle 1/\rho(s)^2 \rangle},\tag{1}$$

where  $C_q = 3.832 \times 10^{-13}$  m;  $\gamma = E/mc^2$  is the Lorentz factor of the electron beam;  $J_x$  is the horizontal partition factor;  $\rho(s)$  is the bending radius; s represents the longitudinal position along the ring; and the brackets " $\langle \rangle$ " mean averaging over the storage ring; the  $\mathcal{H}(s)$ -function is given by

$$\mathcal{H}(s) = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2, \qquad (2)$$

where  $\beta_x, \alpha_x$  and  $\gamma_x$  are Twiss functions of the beam; and  $\eta_x$  and  $\eta'_x$  are dispersion functions.

For an isomagnetic storage ring which has identical bending magnets, Eq. (1) reduces to

$$\epsilon_x = \frac{C_q}{J_x} \frac{\gamma^2}{\rho} \langle \mathcal{H}(s) \rangle,$$
 (3)

and  $\langle \mathcal{H}(s) \rangle$  is given by

$$\langle \mathcal{H}(s) \rangle = \frac{1}{2\pi\rho} \int_{dipole} (\gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2) \mathrm{ds.} \quad (4)$$

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<sup>30</sup> Tune: (14 25, 9 28) (a) Current 20 3 (b) Baseline Tune: (16.25, 9.30) (cm) 2 (m), ŋ, ( 10 β<sub>x</sub>, 4 (c) Ultimate Tune:(21.67,8.69) 30 20 1 10 8 Position (m)

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Here,  $\int_{dipole} \cdot ds$  represents the integration over the dipoles in the storage ring. For notational simplicity, the subscript x for Twiss and dispersion functions will be suppressed in the following discussions.

For a sector dipole, Twiss functions  $\beta(s), \alpha(s)$  and  $\gamma(s)$ and dispersion functions  $\eta(s)$  and  $\eta'(s)$  along the dipole can be express as functions of their values ( $\beta_0, \alpha_0, \gamma_0, \eta_0$ and  $\eta'_0$ ) at the entrance of the dipole [6], i.e.,

$$\begin{aligned} \beta(s) &= \beta_0 C^2 - 2\alpha_0 \frac{CS}{k} + \gamma_0 \frac{S^2}{k^2}, \\ \alpha(s) &= \beta_0 k CS + \alpha_0 (C^2 - S^2) - \gamma_0 \frac{CS}{k}, \\ \gamma(s) &= \beta_0 k^2 S^2 + 2\alpha_0 k CS + \gamma_0 C^2, \\ \eta(s) &= \eta_0 C + \eta'_0 \frac{S}{k} + \frac{1}{\rho k^2} (1 - C), \\ \eta'(s) &= -\eta_0 k S + \eta'_0 C + \frac{S}{\rho k}, \end{aligned}$$
(5)

where  $C = \cos ks$  and  $S = \sin ks$ ;  $k^2 = \frac{1}{\rho^2} + \frac{1}{B\rho} \frac{\partial B_y}{\partial x}$  is the focusing function of the dipole; if it is a focusing dipole,  $k^2 > 0$  and  $k = \sqrt{k^2}$  is a real number; if it is a defocusing dipole,  $k^2 < 0$  and  $k = i\sqrt{|k^2|}$  is an imaginary number.

Substituting Eq. (5) into Eq. (4), and integrating the results, we can obtain the average  $\mathcal{H}$ -function for the sector dipole as follows [6]

$$\langle \mathcal{H}(s) \rangle_{dipole} = \gamma_0 \eta_0^2 + 2\alpha_0 \eta_0 \eta_0' + \beta_0 \eta_0'^2 + \frac{2l}{\rho} \left\{ -(\gamma_0 \eta_0 + \alpha_0 \eta_0') \frac{kl - \sin kl}{k^3 l^2} + (\alpha_0 \eta_0 + \beta_0 \eta_0') \frac{1 - \cos kl}{k^2 l^2} \right\} + \frac{l^2}{\rho^2} \left\{ \gamma_0 \frac{3kl - 4\sin kl + \sin kl \cos kl}{2k^5 k^3} - \alpha_0 \frac{(1 - \cos kl)^2}{k^4 l^3} + \beta_0 \frac{kl - \cos kl \sin kl}{2k^3 l^3} \right\}.$$
(6)

It is obvious that the average  $\mathcal{H}$ -function  $\langle \mathcal{H}(s) \rangle_{dipole}$  of a sector dipole is determined by the Twiss and dispersion parameters at the entrance of the dipole.

For a rectangular dipole, the design orbit of electron  $\Im$  beam are not normal to the edges of the magnet. It turns out that the rectangular dipole contains an extra focusing/defocusing effects at the edges. In the familiar impulse approximation for edge focusing, we assume that  $\beta$  and  $\eta$ functions are unchanged in going through the edge fields, and  $\alpha_0$  and  $\eta'_0$  are changed by [6]

$$\alpha_1 = \alpha_0 - \frac{\beta_0}{\rho} \tan \phi, \ \eta'_1 = \eta'_0 + \frac{\eta_0}{\rho} \tan \phi,$$
(7)

where  $\phi$  is the angle of the design orbit with respect to the  $\stackrel{\frown}{\sim}$  edges of the dipole. Thus, to calculate  $\langle \mathcal{H}(s) 
angle_{dipole}$  for a rectangular dipole, we need to replace  $\alpha_0$  and  $\eta'_0$  in Eq. (6) using  $\alpha_1$  and  $\eta'_1$  defined in Eq. (7).



Figure 2: Horizontal natural emittance of the ALS storage ring as function of  $\beta_0$  for a given dispersion functions  $\eta_0$ and  $\eta'_0$ . The different curves represent different pairs ( $\eta_0$ ,  $\eta'_{0}$ ).



Figure 3: Local minimum horizontal emittance  $\epsilon_x$  as functions of dispersion functions  $\eta_0$  and  $\eta'_0$ . The emittance value is colored coded.

It can be calculated that for an isomagnetic storage ring the damping partition number is approximated by

$$J_x = 1 - \mathcal{D} \approx 1 + 2\frac{\sin kl - kl}{kl} + \frac{\alpha_c R}{\rho}, \qquad (8)$$

where  $\alpha_c$  is the momentum compaction factor, and R is the average radius of the ring.

Eventually, we can express the horizontal nature emittance of an isomagnetic storage ring as follows

$$\epsilon_x = \frac{C_q}{J_x} \frac{\gamma^2}{\rho} \frac{1}{N} \sum \langle \mathcal{H}(s) \rangle_{dipole}, \qquad (9)$$

where N is the number of the dipole in the ring,  $\langle \mathcal{H}(s) \rangle_{dipole}$  is given by Eq. (6), and " $\sum$ " means summations of all the dipoles.

#### APPLY TO ALS LATTICE

Since the basic ALS sector has a mirror symmetric triple bend structure (Fig. 1), the two outer bends have the same

#### **Beam Dynamics and EM Fields**



Figure 4: Lattice solutions calculated using two different approaches, the "inverse" method represented by dots and the Genetic Algorithms represented by circles.

average  $\mathcal{H}$ -functions. Thus, the horizontal emittance of the ALS storage ring is given by

$$\epsilon_x(\beta_0, \alpha_0, \eta_0, \eta'_0, k_{qfa}) = \frac{C_q \gamma^2}{\rho J_x} \left(\frac{2}{3} \langle \mathcal{H} \rangle_o + \frac{1}{3} \langle \mathcal{H} \rangle_i\right),\tag{10}$$

where the subscripts o and i are used to identify the outer and inner bends. Given the Twiss and dispersion parameters  $\beta_0, \alpha_0, \eta_0$  and  $\eta'_0$  at the entrance of the outer bend, and the strength  $k_{qfa}$  of the quadrupole "QFA" between outer and inner bends,  $\langle \mathcal{H} \rangle_o$  and  $\langle \mathcal{H} \rangle_i$  can be easily evaluated using Eq. (6). Thus, the horizontal emittance  $\epsilon_x$  is the function of 5 parameters  $\beta_0, \alpha_0, \eta_0, \eta'_0$  and  $k_{qfa}$ .

Because of the constraint of the mirror symmetric structure, the Twiss parameter  $\alpha_{ic}$  and the dispersion function  $\eta'_{ic}$  should be equal to zeros at the center of the inner bend, i.e.,  $\alpha_{ic} = 0$  and  $\eta'_{ic} = 0$ . Applying the constraint  $\eta'_{ic} = 0$ , we can solve the quadrupole "QFA" strength  $k_{qfa}$  as functions of  $\eta_0$  and  $\eta'_0$ . Applying the constraint  $\alpha_{ic} = 0$ , we can also solve  $\alpha_0$  as function of  $\beta_0$ . However, to avoid solving  $\alpha_0$ , we use the beta function  $\beta_{ic}$  at the center of the inner bend as the free variable instead of  $\beta_0$ , and express  $\beta_0$  and  $\alpha_0$  as function of  $\beta_{ic}$ ,  $\eta_0$  and  $\eta'_0$ . Thus, the five parameter emittance  $\epsilon_x(\beta_0, \alpha_0, \eta_0, \eta'_0, k_{qfa})$  function reduces to three parameter function  $\epsilon_x(\beta_{ic}, \eta_0, \eta'_0)$ .

The emittance  $\epsilon$  vs the beta function  $\beta_0$  for given dispersion functions  $\eta_0$  and  $\eta'_0$  at the entrance of outer bends are shown in Fig. 2. For each pair of dispersion functions ( $\eta_0$ ,  $\eta'_0$ ), we could find a local minimum emittance. For dispersion free ALS lattice, i.e., $(\eta_0, \eta'_0)=(0,0)$ , the theoretical minimum emittance we can achieve is about 6 nm-rad. Relaxing the constraint of the dispersion function, the emittance can be reduced. The local minimum emittance as functions of dispersion function pair ( $\eta_0, \eta'_0$ ) are shown in Fig. 3. We can see the global minimum emittance we can achieve is about 1.3 nm-rad when ( $\eta_0, \eta'_0$ )=(1.7 cm,-0.065).

Having the desired values of the ring emittance and its associated Twiss parameters and dispersion functions at the entrance of the outer dipole, the strength of quadrupoles

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#### **Dynamics 01: Beam Optics (lattices, correction, transport)**

"QF" and "QD" in the straight are determined using lattice matching technique with the constraints  $\alpha = 0$  and  $\eta' = 0$  at the center of the straight. After obtaining all the quadrupole strengths, the stabilities of the ring are checked, and Twiss and dispersion functions at the center of the straight are calculated. The horizontal emittances vs beta functions at the center of straight are shown in Fig. 4. For comparison, the solutions optimized using Genetic Algorithm are also shown in the plot. A good agreement between them is observed. The "inverse" method gives all the possible solution, where the genetic algorithms only gives the optimal ones. It is obvious that for a given emittance, there are two distinct solution regions: one has a large beta function corresponding the lattice used for the baseline upgrade, and the other has a low beta function which can be used for ALS ultimate upgrade.

It is worth to point out that from Fig. 3 we can see that the theoretical minimum emittance we can achieve for the ALS lattice is about 1.3 nm-rad. However, from Fig. 4, we see that the minimum emittance is about 1.7 nm-rad. This difference is because some solutions shown in Fig. 3 do not exist when determining the quadrupole strengths of "QF" and "QD" using lattice matching technique. If additional quadrupole families are added to the straight we may restore the lost solutions.

### CONCLUSIONS

In this paper, we present a new method to optimize lowemittance and low-beta lattices for ALS ultimate upgrade. It provides us a different perspective on the lattice design, and confirms results early found using both Global Scan of All Stable Settings (GLASS) and Genetic Algorithms (GA) techniques. This method reveals that the horizontal natural emittance of ALS lattice can be further reduced to 1.3 nm-rad if additional quadrupole families are added to the straight sections, which were not found using GLASS and GA techniques.

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