# A NEW CORRECTION SCHEME TO COMPENSATE DEPOLARIZING INTEGER RESONANCES AT ELSA 

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## Abstract

For more than four decades, the University of Bonn has supported research at the in house electron accelerator ELSA. Currently, polarized electrons gained from an inverted source are accumulated in a stretcher ring and accelerated within a fraction of a second up to 2.4 GeV . During the fast ramping various depolarizing resonances are crossed. By taking several suitable measures (closed orbit correction, tune jumping, etc.), a high polarization degree of up to $65 \%$ is conserved. One important part of these measures is the correction of integer resonances. Those resonances are compensated by applying additional horizontal fields, distributed sinusoidally along an one-turn orbit length. In case of an appropriate setting of amplitude and phase, all resonance driving effects should be neutralized completely. Detailed studies have shown that vertical displacements and resulting horizontal fields in the quadrupole magnets, caused by the resonance correction, have to be taken into account as well. With regard to a new correction scheme, the first experimental results confirmed by theoretical studies will be presented.

## MOTIVATION

The ELectron Stretcher FAcility (ELSA) features appropriate conditions ${ }^{1}$ to accelerate a polarized electron beam to an energy of typically 2.4 GeV (see Figure 1). During the acceleration we are faced with various depolarizing effects. Nevertheless, under consideration of all optimizations a mean degree of polarization higher than $65 \%$ at the extraction can be obtained.

This optimization process has to avoid resonances with high strength as well as to compensate for resonance driving fields. For instance, due to the fast energy ramp higher order resonances are negligible and thus only the first order resonances, namely intrinsic and integer resonances, have to be suppressed.

Taking into account only flat circular accelerators without solenoids and neglecting longitudinal fringe fields, the polarization vector precesses according to the ThomasBMT equation [1] $(1+a \gamma)$ times per revolution, where $a=0.00115967$ is the gyromagnetic anomaly and $\gamma$ is the Lorentz factor. Along an one-revolution-orbit the acting magnetic fields oscillate in the rest frame of the electrons. Neglecting longitudinal fields, only horizontal fields depolarize the beam, when they appear in phase with the precession of the polarization.

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Figure 1: ELSA overview. Shown are the main components of the ELSA-accelerator facility. Furthermore, the notation for the appearing kick angles as well as the spin phase advance for the third integer resonance $\gamma a=3$ is shown (description below).

For the intrinsic resonance condition, the polarization precesses in phase with the vertical betatron oscillation. This kind of depolarizing effect is suppressed via a so called tune jump correction [2].


Figure 2: The vertical traces of the beam position monitors are shown (colored for each bpm and in black for RMS) during the energy ramp in the stretcher ring $(1.2-2.4 \mathrm{GeV}$ with $4 \mathrm{GeV} / \mathrm{s})$. The correction for the integer resonances causes additional vertical displacements.

Even though the polarization commonly does not precess in phase with the revolution frequency $\omega_{L}$, for multiples of approx. 440 MeV the factor $(1+a \gamma)$ becomes an integer and the integer resonance can be excited (see Figure 2). In this case, the resonance driving fields are caused by misalignments of the magnetic elements and field errors. The resonance driving part of the mentioned fields should
be compensated for in order to achieve a high degree of polarization. Currently, the latter is executed empirically by applying sinusoidally ${ }^{2}$ distributed fields along the orbit using vertical corrector dipoles. Due to these fields the orbit is shifted and therefore additional horizontal fields act inside the quadrupoles on the electrons. The resulting field distribution differs in this case from the desired one.

Via a new formalism a current scheme for the vertical correctors is generated in order to gain the desired field confi guration. The present contribution is dedicated to the first studies and empirical results regarding the mentioned formalism.

## THE NEW INTEGER RESONANCES CORRECTION SCHEME

Only the part of the field distribution which acts in phase with the spin precession influences the strength of the integer resonance significantly. Thus, we have to focus on the fraction of the field distribution that oscillates in the rest frame of the electrons in the horizontal plane with $\left(a \gamma \omega_{\mathrm{L}}\right)$. In the following we simply call this depolarizing component the disturbing fields.

The degree of polarization should be optimized by superposing the disturbing fields destructively. Since misalignments as well as field errors are not known precisely enough, the degree of polarization is maximized empirically. For that, the polarization is measured via Moeller polarimetry at the experiment while changing both amplitude and phase of the applied field distribution.

We focus in the following on the third integer resonance to get a clear arrangement, but the presented studies hold for other integer resonances as well.

## Previous Correction Scheme

First, the so called spin phase advance will be introduced. It is an appropriate quantity to describe the horizontal field distribution along the orbit.

For $a \gamma=3$ the polarization precess $3 \cdot 360^{\circ}$ around the leading dipole fields per turn. By means of a number of 24 dipole magnets, the polarization precesses with a spin phase advance of $\theta=3 \cdot 360^{\circ} / 24=45^{\circ}$ per dipole.

In between two adjacent dipoles all horizontal fields (usually normalized given as kick angles $\alpha$ (see Figure 1)) must be summarized to get the integral kick angle for the $n$-th segment. If at least one corrector per segment is available and if one neglects the additional horizontal fields inside the quadrupoles, a smooth sinusoidal field distribution with defined amplitude and phase can be applied (see green dots in Figure 3 compared to the intended field distribution indicated by the red line). However, if the additional fields inside the quadrupoles are taken into account, the originally sinusoidal field distribution is disturbed and amplitude and phase differ in general from the expected distri-

[^1]bution. From Figure 3 it can be seen that the integral kick angles differ in this case strongly from the intended ones. A least square fit and the large errors demonstrate the discrepance between desired field configuration and the actually acting fields. In spite of the explained systematic complications, the degree of polarization can still be optimized empirically via this kind of correction.


Figure 3: Correction of third integer resonances according to the previous scheme. Each set of samples (green, blue and black dot) represent one dipole segment. The green dots fit smoothly into the desired field distribution (red line). In contrast, the integral kick angles vary strongly from the desired field and a $\chi^{2} /$ dof $=24.203$ is obtained.

## Formalism Regarding Additional Horizontal Fields Inside the Quadrupoles

It takes two general tasks to account for the additional displacements inside the quadrupoles. First, the integral kick angle for each dipole segment has to be calculated. Second, the resulting equation system has to be solved.

For the $n$-th segment between dipole $n$ and $n+1$ all included horizontally acting fields ( $\alpha_{\text {corr }, m}$ for the kick angle of the $m$-th corrector and $\alpha_{\text {quad }, l}$ for the $l$-th quadrupole) must be summed to obtain the integral kick angle $\alpha_{n}$ :

$$
\begin{equation*}
\alpha_{n}=\sum_{m \in \operatorname{segm} . n} \alpha_{\mathrm{corr}, m}+\sum_{l \in \operatorname{segm} . n} \alpha_{\text {quad }, l} \tag{1}
\end{equation*}
$$

The sum of the corrector kick angles can be easily calculated, since only those correctors contribute, which are located inside each segment.

The acting field inside each quadrupole depends on the whole set of the kicks of the correctors. The kick angle can be calculated as follows. A dipole approximation for the quadrupole field yields:

$$
\begin{align*}
\alpha_{\text {quad }, l} & =\arctan \left(\frac{L}{\frac{1}{k_{l} \Delta z_{l}}-\Delta z_{l}}\right) \\
& \approx k_{l} \cdot \Delta z_{l} \cdot L \tag{2}
\end{align*}
$$

for small displacements $\Delta z \ll 1 \mathrm{~m}$, quadrupoles of strength $k<1 \mathrm{~m}^{-2}$ and lengths $L=0.5 \mathrm{~m}$.

## Beam Dynamics and EM Fields

The displacements $\Delta z_{l}$ can be measured via beam position monitors ${ }^{3}$ and depend again on each corrector kick angle. If linear optic is assumed, a quotient

$$
\begin{equation*}
\mathrm{ORM}_{j k}=\frac{\Delta z_{j}}{\Delta \alpha_{k}} \tag{3}
\end{equation*}
$$

of applied kick angle $\Delta \alpha_{k}$ and resulting displacement $\Delta z_{j}$ inside the monitor is given either by simulation or can be measured. The dependence of the displacement inside the quadrupoles on each corrector kick angle is contained by the so called orbit response matrix ORM.

Using equation (1), (2) and (3), the complete kick angle in the $n$-th segment can be given depending on all $N_{\text {corr }}$ corrector kick angles:

$$
\begin{aligned}
& \alpha_{n}=\sum_{\substack{m \\
\in \operatorname{segm} . n}} \alpha_{\mathrm{corr}, m}+L \cdot \sum_{\substack{l \\
\in \operatorname{segm} . n}} k_{l} \cdot\left(\mathbf{O R M} \vec{\alpha}_{\mathrm{corr}}\right)_{l} \\
& =\sum_{\substack{m \\
\in \operatorname{seg} . n}} \alpha_{\mathrm{corr}, m}+L \cdot \sum_{\substack{l \\
\operatorname{segm} . n}} \sum_{j=1}^{N_{\text {corr }}} k_{l} \cdot \mathrm{ORM}_{l j} \alpha_{\mathrm{corr}, j} \cdot
\end{aligned}
$$

This yields an equation system for all 24 integral kick angles which can be written vectorially:

$$
\begin{align*}
\vec{\alpha} & =\left(\mathbf{H C M}^{\text {corr }}+\mathbf{H C M}^{\mathrm{ORM}}\right) \vec{\alpha}_{\mathrm{corr}} \\
& =\mathbf{H C M}_{24 \times N_{\text {corr }}} \vec{\alpha}_{\text {corr }} \tag{4}
\end{align*}
$$

The dimension of so called harmonic correction matrix HCM is determined by the number of segments and by the number of correctors being used. Equivalent to the equation for a one-segment-integral kick angle, the harmonic correction matrix can be split up into a corrector scheme part and a part depending on the ORM, strength and length of the quadrupoles ${ }^{4}$.

It is aimed to solve equation (4) for sinusoidally distributed integral kick angles. To achieve this a singular value decomposition is used, since in general $\mathbf{H C M}$ is not a square matrix. A formalism to solve such an equation system utilizing singular value decomposition (SVD) is given in [3]. The whole formalism was implemented into the accelerator control system.

Figure 4 is comparable to Figure 3. Indeed, the corrector kick angles do not fit desired curve, but the integral kick angles, by means of summarizing corrector and quadrupole kick angles, fit well to the curve. The errors of the fit parameters as well as $\chi^{2} /$ dof $=0.925$ show the advantage of the new correction scheme for this example.

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## Beam Dynamics and EM Fields


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    ${ }^{1}$ This includes for example an inverted source of polarized electrons, a fast ramping booster synchrotron as well as a fast ramping stretcher ring.

[^1]:    ${ }^{2}$ The frequency is set according to the number of whole precessions $a \gamma$.

[^2]:    ${ }^{3}$ Since the beam position monitors are mounted very close to the quadrupoles, the measured position nearly equals to the displacement inside the quadrupoles.
    ${ }^{4}$ If the $\mathbf{H C M}{ }^{\text {ORM }}$ part is not considered, the solution of equation (4) holds results which are equivalent to those using the previous correction scheme.

