## NSLS-II FAST ORBIT FEEDBACK WITH INDIVIDUAL EIGENMODE COMPENSATION\*

Yuke Tian<sup>#</sup>, Li Hua Yu, BNL, Upton, NY 11973, U.S.A.

## Abstract

This paper presents the NSLS-II fast orbit system with individual eigenmode feedback compensation. The fast orbit feedback system is a typical multiple-input and multiple-output (MIMO) system. Traditional singular value decomposition (SVD) based fast orbit feedback systems treat each eigenmode the same and the same compensation algorithm is applied to all the eigenmodes. In reality, a MIMO system will have different frequency responses for different eigenmodes and thus it is desirable to design different compensation for each eigenmode. The difficulty with this approach comes from the large amount of computation that needs to be done within the time budget of the orbit feedback system. We designed and implemented the NSLS-II fast orbit feedback (FOFB) system with individual eigenmode compensation by taking advantage of the parallel computation capability of field programmable gate array (FPGA) chips.

## **INTRODUCTION**

NSLS-II is a third generation 3GeV storage ring with ultra-low emittance [1]. The low emittance requires a very stable electron beam orbit. Applying the common rule (beam stability < 10% of beam size), one discovers that the NSLS-II needs to hold submicron beam orbit stability. The stringent orbit stability requires the orbit feedback system to be able to suppress various noises from low frequency ground motion to high frequency mechanical vibration [2].

In an orbit feedback system, there are many beam position monitors (BPMs) to monitor the orbit and many correctors to correct the orbit. It is a typical multiple-input and multiple-output (MIMO) system and singular value decomposition (SVD) is commonly used for the feedback calculation. To correct the ill-conditioned response matrix, truncated SVD (TSVD) and Tikhonov regulation is often applied to improve the orbit feedback performance [3].

One common feature for the traditional SVD based orbit feedback algorithm is that it applies the same controller dynamics (such as PID loop) to all eigenmodes. It will be desirable to control each eigenmode with a different controller and thus each eigenmode has different compensation in the frequency domain [4]. This approach provides many benefits for both system analysis and orbit correction. The challenge for such individual eigenmode compensation is that the feedback system needs to finish much more calculations within the time budget of the fast orbit feedback system. In this paper, we present a FPGAbased fast orbit feedback algorithm solution for such an

\*Work performed under the auspices of the U.S. Department of Energy  $^{\#}$ ytian@bnl.gov

approach. To simplify the discussions, only one corrector plane is considered in this paper. The algorithm for the other corrector plane is the same and can be treated independently.

## **ALGORITHM**

## FOFB Without Orbit Eigenmode Decoupling

Using the common FOFB and SVD notations, we assume there are M BPMs and N fast correctors in the orbit feedback system. Orbit displacements **d** (M component vector) and corrector kick angles  $\vec{\theta}$  (N-component vector) have the following relationship:

$$\mathbf{R}\boldsymbol{\theta} = \mathbf{d} \tag{1}$$

When  $M \ge N$ , the SVD of matrix R can be represented as

$$\mathbf{R}_{M\mathbf{x}\mathbf{N}} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}$$
(2)

Where U is a MxN matrix with orthonormal column vector set.  $V^T$  is the transpose of the NxN matrix V which also has orthonormal column vector set.  $\Sigma$  is an NxN diagonal matrix with non-negative elements and we refer to its eigenvector as the orbit eigenmodes discussed in this paper.

Using SVD, we can get the inverse response matrix,

$$\mathbf{R}^{-1} = \mathbf{V} \boldsymbol{\Sigma}^{-1} \mathbf{U}^{\mathrm{T}}$$
(3)

And the required corrector strength is found to be,

$$\theta = R^{-1}d \tag{4}$$

In practise, the actual corrector kick angles are calculated with a PID or PI controller.

$$\theta_{act} = Q(z)\theta \tag{5}$$

Where Q(z) is used to present any compensator controller such as PID.

The above describes a simple SVD-based FOFB algorithm. From equation (4), we can see that the calculation needed for one corrector strength is M multiplications and an accumulation (MAC). The remaining controller (PID) needs a few (assuming k) MAC for each of the corrector strengths. Then the total calculation needed for one corrector strength is,

$$L_{1-correctorr} = M + k$$
(6)

The total calculation for all correctors' strength is,

$$L_{\text{N-corrector}} = N(M+k)$$
(7)

# FOFB With Orbit Eigenmode Decoupling and Individual Eigenmode Compensation

The best way to describe the FOFB with decoupling and individual eignemode compensation is to use the diagram in Figure 1.



Figure 1: FOFB with eigenmode compensation.

In Figure 1,  $\vec{e}$  is the noise in the system and  $\vec{c}$  is the eigenspace projection of each input.  $d_{gold}$  is the golden orbit and we set it to zero for a simpler discussion.

$$\mathbf{c} = \mathbf{U}^{\mathrm{T}} \left( \mathbf{d} + \mathbf{e} \right) \tag{8}$$

We use c(n) to represent the eigenspace projection at feedback cycle n for input signal.

$$c(n) = U^{T}(d(n) + e(n))$$
 (9)

For each of the components of c(n), an individual digital controller can be designed to carry on various standard compensations. We use  $Q_i(z)$  to represent the digital controller applied only on the *ith* eigencomponent of c(n). Combining all the single mode compensation, we can write the total compensation in eigenspace as,

$$Q(z) = \begin{bmatrix} Q_1(z) & 0 & 0 & 0 \\ 0 & Q_2(z) & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & Q_N(z) \end{bmatrix}$$
(10)

Equation (10) decouples the MIMO problem into many single input single output (SISO) problems, for which control theory has many standard treatments. Let's see the effect of Q(z) by calculating the eigencomponent of c in the (n+1) feedback cycle.

In cycle n, the corrector strength is,

$$\theta(n) = V\Sigma^{-1}Q(z)c(n)$$
(11)

And the feedback signal is,

$$d(n + 1) = U\Sigma V^{T} (V\Sigma^{-1}Q(z)c(n)) + e(n + 1) (12)$$

Since 
$$VV^T = V^TV = I$$
 and  $U^TU = I$ ,  
 $d(n+1) = UQ(z)c(n) + e(n+1)$  (13)

In eigenspace, the new projection vector is,

$$c(n + 1) = U^{T}d(n + 1) = Q(z)c(n) + U^{T}e(n + 1)$$
  
Or,  
$$\begin{bmatrix} c_{1}(n+1) \\ c_{2}(n+1) \end{bmatrix} = \begin{bmatrix} Q_{1}(z) & 0 & 0 & 0 \\ 0 & Q_{2}(z) & 0 & 0 \end{bmatrix} \begin{bmatrix} c_{1}(n) \\ c_{2}(n) \end{bmatrix}_{+U^{T}e(n + 1)}$$

$$\begin{bmatrix} \dots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{c}_{N}(n+1) \end{bmatrix} \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & Q_{N}(z) \end{bmatrix} \begin{bmatrix} \dots \\ \mathbf{c}_{N}(n) \end{bmatrix}$$

(14)

Equation (14) shows that the digital controller has a pure effect only on the *ith* eigencomponent of c(n) and

#### **Instrumentation and Controls**

#### Tech 05: Beam Feedback Systems (hardware)

doesn't affect other eigencomponents. The second term of equation (14) represents the noise projection on the eigenspace, where it can be individually suppressed. This is the exact advantage of doing digital compensation in eigenspace.

Because we decouple the input vector in eigenspace and design compensators for each eigenmode, the calculation will be much larger than the FOFB without decoupling, as described in equation (5). Let's calculate the total calculation needed for one corrector strength.

From equation(9), the decoupling calculation amount is,

$$L_{decoupling} = NxM$$
(15)

WEODN4

Assume we have digital compensations for all the eigenmodes (in reality we might only treat a few significant components), then the total calculations in eigenspace are,

$$L_{eigenspace} = Nxk$$
 (16)

Here we use k to represent the average digital compensator calculation amount. Since  $\Sigma^{-1}$  is a diagonal matrix for proportional gain control, it can be included in the digital compensator  $Q_i(z)$ .

Once we get the compensated vector, we need to calculate correctors' strengths. One corrector strength is a summary for all compensated eigenmodes,

$$L_{ouptut} = NxN \tag{17}$$

Adding equation (15), (16) and (17) gives the total calculation needed for one corrector strength is,

$$L_{1-correctorr} = N(M + k + N)$$
(18)

The total calculation to get all corrector strengths is,

$$L_{N-corrector} = N^{2} (M + k + N)$$
(19)

reative Commons Attribution Comparing equation (6), (7), (18) and (19), we can see that the calculation amount is much larger for the decoupling algorithm. For NSLS-II, there are 240 BPMs and 90 fast correctors; we assume a simple PID compensator (k=3); equation (18) gives us the total calculation (MAC) needed for one corrector strength,

$$L_{1-correctorr_NSLS_{II}} = N(M + k + N) = 29970$$
 (20)

And the total calculation for all the correctors is,

$$L_{N-corrector} = N^{2}(M + k + N) = 2697300$$
 (21)

About 2.7million MAC need to be finished within the orbit feedback calculation time budget. For NSLS-II's 10KHz fast orbit feedback, the total time budget is 100us. Besides the orbit feedback calculation, much time should be reserved for data communication: distributing all 240 BPM data around the 800 meter ring and sending setpoints to 90 fast correctors around the ring. If we assume that half of the time budget (50us) is spent on orbit feedback calculation, then the equation (18) and (19) are 600million MAC per second and 5400million MAC second respectively. As a reference, the high end DSP chips on the market give about 200million floating point (MFLOP) calculation capability.

It is clear that the individual eigenmode compensation provides us with flexible control for the MIMO system. It also challenges us to solve the large computation problem. Fortunately, the parallel processing nature of FPGA and its powerful DSP function blocks allows us to achieve the calculation requirement. Moreover, the two-tier data communication structure of the NSLS-II control system provides an efficient and reliable mechanism for both BPM data and corrector data transfer.

**IMPLEMENTATION** 

Architecture



Figure 2: NSLS-II FOFB architecture.

Figure 2 shows the block diagram of the NSLS-II fast orbit feedback system. There are 30 cells in the NSLS-II storage ring and this diagram shows the main FOFB components in one cell: cell controller, BPM, corrector power supplies, and EPICS IOC. A two-tier communication is designed to distribute the data among the ring. At the lower tier, local BPM data is delivered to the cell controller through a 2.5Gbps and redundant serial device interface link (SDI). The setpoints for the corrector are also delivered to power supply controller (PSC) through a 100Mbps serial interface link. At the higher tier, the similar 2.5Gbps SDI link between cell controllers is used to deliver all BPM data around the ring. The cell controller is the central piece for the orbit feedback system: it has all the BPM data, it carries all the orbit feedback calculation, and it controls the local power supplies. The cell controller has a Gigabit Ethernet port to communicate with EPICS IOC.

Due to the distributed character of the system, we don't need to deal with the large calculation described in equation (22). Each cell controller need only calculate the 3 local corrector strengths as shown in equation (21).

## Calculation in FPGA

Figure 3 show the FPGA calculation structure for the 3 local correctors. One can notice that the decoupling calculation (equation 15) is done in parallel for all the eigenmodes (90 for NSLS-II). So the calculation time for decupling (equation 15) is reduced to 240MAC. The compensation for each eigenmode (equation 16) is also done in parallel since they are independent of each other. For the output calculation (equation 17), the FPGA only needs to calculate its local corrector strengthen, which needs N (90 for NSLS-II) MAC for each corrector. And the three local correctors strength are calculated in parallel.



Figure 3: FPGA calculation structure.

The total FPGA calculation time is about 330 MAC ((240+90). During this time, many parallel calculations are carried out.

We have implemented the detailed FOFB calculation in FPGA. Since FPGA calculation is based on fixed point algorithm, we compared it with the floating point calculation to show that the loss of accuracy for FPGA fixed point calculation is minimal. The detailed implementation and discussions will be discussed in other papers.

### SUMMARY

This paper discussed the NSLS-II FOFB algorithm with eigenmode compensation. It shows that this algorithm provides better controllability for the orbit feedback system by decoupling the feedback signal into eigenspace and compensating for each eigenmode. It shows the calculation challenges for this approach. It also shows how the NSLS-II FOFB architecture is able to solve this challenge.

#### ACKNOWLEDGEMENTS

Orbit feedback system consists of many critical subsystems to make it work. We like to thank Bob Dalesio for architecting the two-tier communication structure. We like to thank Joseph Delong for implementing the SDI link. We like to thank to the Om Singh, Kurt Vetter, Joseph Mead, Kiman Ha, Alfred Dellapenna and Yong Hu for their efforts of BPM development. We also want to thank George Ganetis, Wing Louie and John Ricciardelli for the power control system development. Also special thanks to Sam Krinsky, Igor Pinayev, Guobao Shen and Professor William Levine for the useful orbit feedback algorithm discussions.

### REFERENCES

- S. Ozaki, et.al, "Philosophy for NSLS-II Design With Sub-nanometer Horizontal Emittance", Proceedings PAC07, p. 77(2007).
- [2] L. Yu, "Performance Calculation On Orbit Feedback For NSLS-II", Proceedings PAC05, p.1036(2005)
- [3] C.H.Kuo et,al, "Fast Orbit Feedback System Commissioning Of The Taiwan Light Source", Proceeding PAC09, p.3965(2009)
- [4] M.G.Abbott et.al, "Performance And Future Development Of The Diamond Fast Orbit Feednback System", Proceedings of EPAC08, p.3257