# ACTION AND PHASE JUMP ANALYSIS FOR LHC ORBITS* 

O.R. Blanco ${ }^{\dagger}$, J.F. Cardona ${ }^{\ddagger}$<br>Physics Department - Universidad Nacional de Colombia, Bogota, Colombia


#### Abstract

Action and phase orbit analysis is implemented to detect magnetic errors in LHC orbits of late 2009 run. The last achievements in the theory of action and phase jump analysis have been included to reduce action and phase plots noise and to increase precision on the calculation of linear errors. The validation of the implementation is performed by MAD-X simulations of the LHC lattice V6.5, where dipole and quadrupole errors are included and recovered within $1.53 \%$. Then, the implementation is applied to experimental orbits, taken from the 2009 run during November and December, where several interaction regions are analyzed.


## THE ACTION AND PHASE METHOD

## The Method Briefly

When a magnetic error $\theta_{z}$ is present at some particular point $s=s_{\theta}$, it was shown in [1] that the trajectory of the particle can be described independently before and after the error:

- Before the error $\left(s<s_{\theta}\right)$

$$
\begin{equation*}
z(s)=\sqrt{2 J_{0} \beta_{z}(s)} \sin \left(\psi_{z}(s)-\delta_{0}\right) \tag{1}
\end{equation*}
$$

- After the error $\left(s>s_{\theta}\right)$

$$
\begin{equation*}
z(s)=\sqrt{2 J_{1} \beta_{z}(s)} \sin \left(\psi_{z}(s)-\delta_{1}\right) \tag{2}
\end{equation*}
$$

where $J_{0}$ and $\delta_{0}$ are the action and phase before de error while $J_{1}$ and $\delta_{1}$ correspond to the action and phase after the error. Here $z$ represents either the $x$, or the $y$ plane. With these equations, using the Courant-Snyder parameters to propagate the particle trajectory through the error and after some algebra and trigonometric identities we obtain the magnetic error as:

$$
\begin{equation*}
\left|\theta_{z}\right|=\sqrt{\frac{2 J_{1}+2 J_{0}-4 \sqrt{J_{1} J_{0}} \cos \left(\delta_{1}-\delta_{0}\right)}{\beta\left(s_{\theta}\right)}} \tag{3}
\end{equation*}
$$

[^0]The magnetic error $\theta_{z}$ could be generated by any the multiple components in the magnetic field multipole expansion.

$$
\begin{align*}
\theta_{x}= & B_{0}-B_{1} x\left(s_{\theta}\right)+A_{1} y\left(s_{\theta}\right)+2 A_{2} x\left(s_{\theta}\right) y\left(s_{\theta}\right) \\
& +B_{2}\left[-x^{2}\left(s_{\theta}\right)+y^{2}\left(s_{\theta}\right)\right]+\cdots  \tag{4}\\
\theta_{y}= & A_{0}+A_{1} x\left(s_{\theta}\right)+B_{1} y\left(s_{\theta}\right)+2 B_{2} x\left(s_{\theta}\right) y\left(s_{\theta}\right) \\
& +A_{2}\left[x^{2}\left(s_{\theta}\right)-y^{2}\left(s_{\theta}\right)\right]+\cdots \tag{5}
\end{align*}
$$

with $A_{n}=B^{\prime} l a_{n} / B \rho$ and $B_{n}=B^{\prime} l b_{n} / B \rho$
When a magnetic error is present, there is a noticeable change in the local action and phase of the orbit specially at the interaction regions where usually there are significant changes in the beta functions (see Fig. 2). Magnetic error strength and sign is recovered directly from the calculation and magnetic errors are calculated by linear o power regressions according to [2], [3], [4].

For dipolar errors, no dependence with any of the transverse coordinates is expected.

$$
\begin{equation*}
B_{0}=\theta_{z} \tag{6}
\end{equation*}
$$

For quadrupolar errors, linear dependency with the transverse coordinates is expected.

$$
\begin{align*}
A_{1} & =\frac{\theta_{x} y\left(s_{\theta}\right)+\theta_{y} x\left(s_{\theta}\right)}{x^{2}\left(s_{\theta}\right)+y^{2}\left(s_{\theta}\right)}  \tag{7}\\
B_{1} & =\frac{\theta_{y} y\left(s_{\theta}\right)-\theta_{x} x\left(s_{\theta}\right)}{x^{2}\left(s_{\theta}\right)+y^{2}\left(s_{\theta}\right)} \tag{8}
\end{align*}
$$

For sextupolar errors, quadratic dependency with the transverse coordinates is expected and multiple orbits might be needed.

## Requirements

The action and phase method is performed along the coordinate $s$ of the accelerator to obtain $\left(J_{i}, \delta_{i}\right)$, segment by segment, on each $i^{t h}$ segment. It means, it requires:

- BPMs readings: Two before the error (to calculate $\left(J_{0}, \delta_{0}\right)$ ), two after the error (to calculate $\left(J_{1}, \delta_{1}\right)$ )
- The lattice model: To obtain the Courant-Snyder parameters of the accelerator
- Multiturn trajectories: To increase the precision in the polynomial fitting (quadrupolar and sextupolar errors mainly), it is better if the multiturn trajectory is made with high amplitude oscillations


Figure 1: Lattice. (top) The entire lattice, (middle) Zoom in over an interaction region, (bottom) Zoom in over an arc. The 1's represent dipoles and 2's represent quadrupoles.


Figure 2: $\beta$ functions. The higher values are localized over the interaction regions.

## THE LHC IN SIMULATIONS

The LHC lattice model version 6.5 for MAD-X 4.01 was used during the simulations. Figure 1 shows schematically the appearance of the entire lattice, an interaction region and an arc. Lattice functions and beam trajectory position at each BPM are read from output files of the MAD-X simulations

## Error Simulations

Dipolar and quadrupolar errors were included in the accelerator.

Orbits with the errors were obtained by MAD-X simulations with the following parameters: Experiments off, period lhcb1, Lattice model V6.5, Energy $450[\mathrm{GeV}]$ (injection), Particle PROTON

A orbit corrector was used to generate orbits with different amplitudes for different simulations.

Calculations were made along the entire $s$ coordinate, BPM by BPM, using custom scripts written in different languages: C, C++, BASH, PERL, Python V2.5.

Figure 3 shows the action and phase jump at the error $s_{\theta}$ calculated from the simulated orbit. In figure 4 , a quadrupolar magnetic error of $\mathrm{K} 1=1 \times 10^{-3}$ was included and recovered at $s_{\theta}$, showing that the magnetic error precision decreases from $\pm 0.01 \%$ to $\pm 1.53 \%$ when random


Figure 3: A magnetic error $\theta_{z}$ is located in the lattice and a orbit corrector is used to produce orbits with different amplitudes.


Figure 4: Plot used to calculate the quadratic magnetic error. Precision is decreased when random noise is included in the orbits.
noise between $1 \sim 10 \%$, similar to the experimental data is included in the simulations. Simulations were done with sextupoles off but we know they introduced additional noise in the third significant figure which can be neglected when compared with the experimental noise.

## THE LHC EXPERIMENTAL ORBITS

## The Data

Data taken from the 2009 LHC run during November and December was processed. It consisted of beam transverse positions of one bunch in multiple turns in sdds format. After applying the method, high noise appears in the action and phase plot.

Several techniques were used to reduce the influence of the noise, but best results were obtained with phase range selection. From a large number of turns corresponding to one orbit file, only turns in phase ${ }^{1}$ are used to calculate one principal orbit, adding turns with same phase and subtracting turns with phase $\bar{\psi}+\pi$.

The magnetic error must be calculated at a $s_{\theta}$ position where the $\beta$ functions are high. Figure 5 shows that $s_{\theta}$

[^1]

Figure 5: Selection of the $s_{\theta}$ where the error is calculated.
is different for the $x$ and $y$, coordinates. If both planes are supposed to be analyzed at the same time, for example, looking for skew or coupling error, then $s_{\theta}$ should be a point where none of them is a minimum.

## IR3

As an example, figure 6 shows the phase difference before and after the IR3. The result of performing the analysis in this region is shown in figure 7 where a main quadrupole error is detectable, but more data will be needed to determine it more precisely.


Figure 6: Interaction region 3. A phase jump is observed from data.

## CONCLUSIONS

Even though simulations show that possible LHC magnetic errors can be recovered within $1.53 \%$ uncertainty, the data shows significant noise in the action and phase plots. In this paper, we have shown a way to reduce this problem through careful selection of turns (phase range selection) and building difference orbits from these selected turns.

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Figure 7: Interaction region 3. A phase jump is observed from data.

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    $\dagger$ orblancog@bt.unal.edu.co
    $\ddagger$ jfcardona@unal.edu.co

[^1]:    ${ }^{1}$ Each turn has a different averaged phase $\bar{\psi}$, but in this case we choose only those that are similar within a range that should be chosen by looking the turns along the fraction of the accelerator that is under analysis.

