CSR FIELDS FROM USING A DIRECT NUMERICAL SOLUTION OF MAXWELL'S EQUATIONS*

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Abstract

We discuss the properties of the coherent electromagnetic fields of a very short, ultra-relativistic bunch in a rectangular vacuum chamber inside a bending magnet. The analysis is based on the results of a direct numerical solution of Maxwell's equations together with Newton's equations. We use a new dispersion-free time-domain algorithm which employs a more efficient use of finite element mesh techniques and hence produces self-consistent and stable solutions for very short bunches. We investigate the fine structure of the CSR fields including coherent edge radiation. This approach should be useful in the study of existing and future concepts of particle accelerators and ultrafast coherent light sources.

INTRODUCTION

The coherent synchrotron radiation (CSR) fields have a strong action on the beam dynamics of very short bunches, which are moving in the bends of all kinds of magnetic elements. They are responsible for additional energy loss and energy spread; micro bunching and beam emittance growth. These fields may bound the efficiency of damping rings, electron-positron colliders and ultrafast coherent light sources, where high peak currents and very short bunches are envisioned. This is relevant to most high-brightness beam applications. On the other hand these fields together with transition radiation fields can be used for beam diagnostics or even as a powerful resource of THz radiation. A history of the study of CSR and a good collection of references can be found in [1]. Electromagnetic theory suggests several methods on how to calculate CSR fields. The most popular method is to use Lienard-Wiechert potentials. Other approach is to solve numerically the approximate equations, which are a Schrodinger type equation. These numerical methods are described in [2]. We suggest that a direct solution of Maxwell's equations together with Newton's equations can describe the detailed structure of the CSR fields [3].

METHOD

The main strategy of the method:

- implicit scheme
- Fourier expansion
- traveling mesh
- ensemble of particles

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Modeling ultrafast phenomena requires a special algorithm for solving the electromagnetic equations. This algorithm must be free of frequency dispersion which means that all propagating waves must have their natural phase velocity, completely independent of the simulation parameters like a mesh size or a time step. We suggest an implicit algorithm which does not have on stability issues and employs a more efficient use of finite element mesh techniques. This method can produce self-consistent stable solutions for very short bunches. We have already used this same approach for wake field calculations [4]. To employ the implicit scheme we transform Maxwell's equations to the second order equations. Formulas for the numerical approximation of an equation of second order are given in [3]. In this publication we also gave an analysis and comparison of the explicit and implicit schemes with application to CSR field calculations.

CSR FIELD DYNAMICS

We will start with pictures of the electric field force lines. We will try to understand how a bunch field remakes itself when a bunch is rotated in a magnetic field. We have calculated the electromagnetic field of a Gaussian bunch initially moves along the vacuum chamber very close to the speed of light. At some point the bunch enters a vertical magnetic field of a bend. Fig. 1 shows snapshots of the electric field lines at different time moments. Before entering a bend the bunch has only a transverse field, which can be seen as a set of vertical lines. A new field that is generated in a bend is a set of ovals, which increase in size with a time. We can outline two time periods of the field formation. The first is when a bunch is moving inside the region of it's initial transverse field. The first two plots in Fig. 1 are related to this first period. The second period

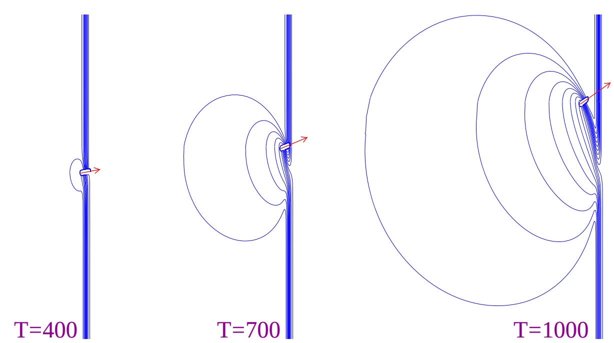

Figure 1: Snapshots of electric field lines of a bunch, which is moving in a magnetic field. White boxes show bunch contours. Red arrow show directions of a bunch velocity

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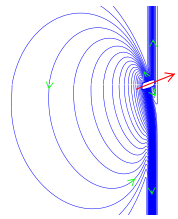

Figure 2: Detailed structure of the field pattern. Red arrow shows the direction of the bunch velocity. Green arrows show the field line direction.

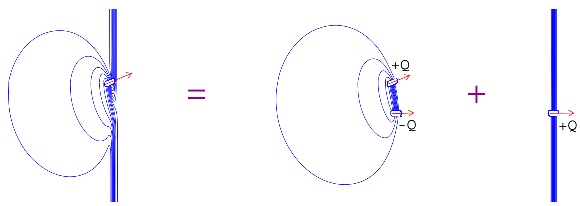

Figure 3: Decomposition of the field of a bunch moving in a magnetic field into two fields: a field of a dipole (middle plot) and a field of a bunch moving straight in initial direction (right plot). Red arrows show directions of a bunch velocity.

starts when the bunch is delayed so much that it is out of the region of the initial transverse field. The last plot in Fig. 1 shows this situation. The transverse field continues to move straight with the speed of light, so we may consider it to be the field of the edge radiation in a bend. A more detailed picture of the field lines is shown in Fig. 2. One can be see that the upper field lines take the position of the lower lines and a part of the lower field lines take the position of the upper lines. However at the far ends the transverse field lines continue traveling in the same initial direction. It is easy to explain such behavior if we present this field as a sum of two fields $\vec{E} = \vec{E}_{dp} + \vec{E}_{in}$ First field $\vec{E}_{dp}$ is the field of a dipole, which consists of two oppositely charged bunches. One bunch is the "real" one with a positive charge. This bunch is rotated in the magnetic field while the other bunch is a "virtual" one, which has an opposite charge and travels straight in the initial direction. Second field $\vec{E}_{in}$ is the field of another "virtual", but positively charged bunch, which travels straight along the initial direction. Naturally the virtual bunches together sum to zero. This decomposition is shown in Fig. 3. We note that this decomposition can also help to improve the accuracy of the numerical calculation of the force acting on the bunch particles because we remove the strong initial bunch field. A dense set of field lines in Fig. 2 also reveals a fine structure of the field in front of a bunch. This γ-type region is common in a classical synchrotron radiation. The characteristic wavelength of the synchrotron radiation or an equivalent value of the bunch length is $\sigma = R/\gamma^3$. Ref. [6] has a picture of the field lines for the case of a relativistic factor $\gamma = 6$. An equivalent value of the bunch length is very close to our bunch length. Fig. 4 shows this finite structure together with a plot from a Ref.[6]. We can state that the γ-region before a bunch is very close for both cases. As was mentioned above we can distinguish two stages of the field formation. Now we can separate the interaction with the field $\vec{E}_{in}, \vec{B}_{in}$ and the field $\vec{E}_{dp}, \vec{B}_{dp}$. The interaction with the field $\vec{E}_{in}, \vec{B}_{in}$ continues only for the time when a bunch is inside the region of this transverse field. We can estimate the distance D^+ (or equivalent time) when a bunch leaves this region. This means that the bunch delay δ must be more than the bunch length 2σ $\delta = \rho\varphi - \rho\sin\varphi \approx \rho\frac{\varphi^3}{3} \geq 2\sigma$ from this relation we have $D^+ = \rho\varphi \geq (6\rho^2\sigma)^{1/3}$ The distance D^+ is 37% less than a characteristic overtaking length $L_0 = 2(3\sigma\rho^2)^{1/3}$ according to [5]. The transverse field is located only near the bunch, in the region, which can be approximated by the bunch size σ_x or σ_y or the bunch length σ. A bunch leaves the transverse field much earlier when his transverse displacement Δx exceeds the field region $\Delta x = \frac{1}{2}\rho\varphi^2 \geq \sigma$ so the distance D^+ is $D^+ = \rho\varphi \geq \sqrt{2\rho\sigma}$ The interaction of the bunch with the field $\vec{E}_{dp}$ continues for a long time. Fig. 5 shows the absolute value of the electric field on the horizontal plane in the vertical center of the vacuum chamber in consecutive time steps. The white oval shows the real bunch contour(90% of particles). When a dipole is created an electric field appears between a real bunch and a virtual bunch. This field increases in value and reaches a maximum value when the bunches are completely separated and then it goes down as the bunches more apart leaving fields only around the bunches. The bunch acquires an energy loss while interacting with the field $\vec{E}_{dp}$. To study the fields acting on the particles inside the bunch we calculated the distribution of a collinear force $F_{\parallel} = \vec{J}_b \cdot \vec{E}$ and a transverse force $F_{\perp} = \left(\vec{J}_b \times \vec{E}\right)_x$ We have found some very exciting fine structure of the force acting on the particle in the bunch. Fig. 6 show a distribution of forces on the horizontal plane in the vertical center of the vacuum chamber at three time moments. The transverse force is the well known space-charge force, which probably is compensated by a magnetic force in the ultra-relativistic case. The collinear force is responsible for an energy gain or an energy loss. The red color corresponds to acceleration and

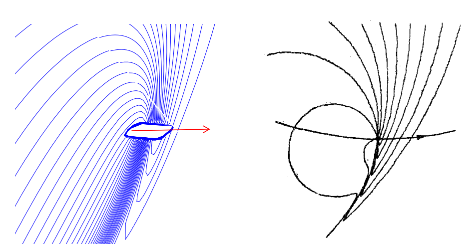

Figure 4: Fine structure of the field pattern in front of a bunch. The left plot shows field lines near a bunch. The right plot presents a picture from the Ref. [6] for $\gamma = 6$. The plots are rotated and scaled in order to have the same direction for the velocities and the same bending radius.

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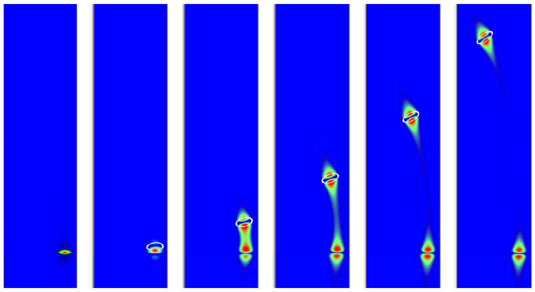

Figure 5: Absolute value of the electric field $\vec{E}_{dp}$ on the horizontal plane in the vertical center of the vacuum chamber in consecutive time steps. The red color corresponds to the maximum; the blue - to the minimum value of the field. The white oval shows the real bunch contour.

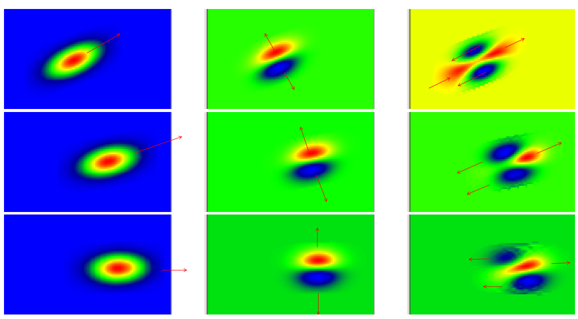

Figure 6: The left three vertical plots show a bunch charge distribution. The middle plots show a transverse force. The right plots show the collinear force, which is responsible for an energy gain or energy loss. The red arrows show the direction of the bunch velocity and forces. The starting plot is at the bottom.

energy gain ad the blue color corresponds to deceleration and energy loss. We did not include these forces in the beam dynamics simulation in order to make the physical picture clear. We see here that the forces on the bunch are very complicated. The particles, which are in the center of the bunch, in front of the bunch and at the end are accelerating, whereas the particles at the boundaries are decelerating. This means that a bunch gets an additional energy spread in the transverse direction. The total effect is deceleration and the bunch loses energy. This complicated structure of the collinear field is very important. A bunch will get an additional transverse energy spread, which can not be compensated. This energy spread in the magnetic field immediately generates emittance growth. This effect can limit the efficiency of the magnetic bunch compressors and as a result the efficiency of FELs. As we mention above an ultra-relativistic bunch and CSR fields are moving together and interact for a long time. However one can see a field, which propagates straight ahead from the initial beam X-position . This field can be seen very well when the bunch gets a large horizontal displacement. Fig. 7 shows the distribution of the magnetic field. Fig. 8 shows images of the coherent radiation in the form of transverse magnetic field

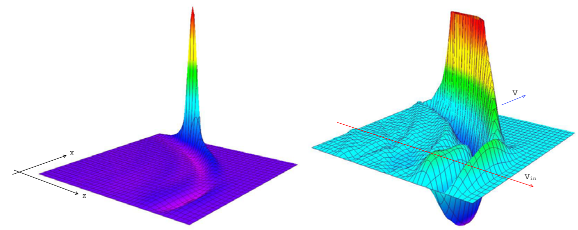

Figure 7: Coherent edge radiation. Distribution of the magnetic field on the horizontal plane in the vertical middle of the vacuum chamber. The large peak corresponds to the bunch field. The right picture is a magnified image of the left picture. Note the scales in the X and Z directions are different A red arrow shows the initial bunch X-position and the direction of the bunch velocity. A blue arrow shows the direction of the bunch velocity at this time.

distributions on the vertical planes of the vacuum chamber. Left and right set of vertical plots correspond to different longitudinal positions. Each plot in a set shows a distribution at a different time. At first we see an image of edge radiation, then the image of synchrotron radiation and finally a bunch field image. The calculated images of the coher-

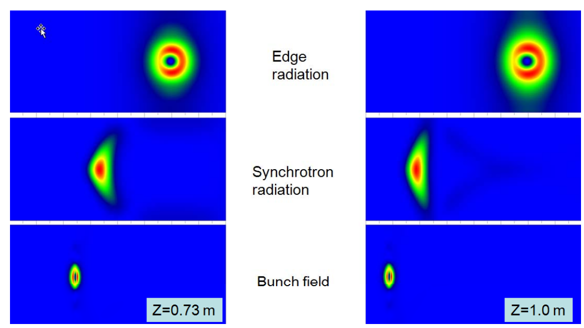

Figure 8: Images of radiation in the form transverse magnetic field distributions. The left and right set of vertical plots correspond to different longitudinal positions. Each plot in a set shows a distribution at a different time. First comes an image of edge radiation, then the image of synchrotron radiation and finally a bunch field image.

ent edge radiation look very similar to the images, which we have seen on the YAG screen after the dump magnets, which bend down the beam at LCLS.

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