# SIMULATIONS OF EMITTANCE MEASUREMENT AT CLIC \*

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#### Abstract

A proposal for a CLIC emittance measurement line using laser-wire beam profile monitors is presented. Results of simulations and optimizations are given. Estimates of the impact of beam size as well as statistical and machinerelated errors on the measurement accuracy are discussed.

#### INTRODUCTION

To meet challenging performance specifications of the future CLIC collider a precise monitoring of the beam transverse space and a precise measurement of beam characteristics will be necessary. In particular, an accurate determination of the transverse beam sizes at various points of the machine upstream of the final focus is essential for a control of emittance preservation and for ensuring full luminosity.

In the present paper we describe a proposal of a section for emittance measurements for CLIC. The section is located at the end of Ring-to-Main-Linac Line (RTML), just before entrance to the main linac. Taking into account that the beam to be measured is of micron sizes and that the measurement method must be non-invasive, it is proposed to use laser wire (LW) beam profile monitors based on a scattering laser photons with the electrons or positrons of the beam. The idea was first proposed in [1], its further development for the future linear colliders and a schematic setup for a LW beam profile monitors are discussed in Ref. [2]. Results of first successful measurements of electron beams of micron size at the ATF [3] and PETRA-III [4] facilities have been reported. A detailed analysis of a LW diagnostic section of the International Linear Collider (ILC) is given in Ref. [5]. We discuss a procedure of the reconstruction of the beam emittances and present results of simulations.

## RTML EMITTANCE MEASUREMENT SECTION

The transverse beam characteristics are described by the standard  $4 \times 4$  symmetric beam envelope matrix. If the vertical and horizontal beam oscillations are decoupled the matrix is of block-diagonal form

$$\sigma = \begin{pmatrix} \Sigma_{xx} & 0\\ 0 & \Sigma_{yy} \end{pmatrix},\tag{1}$$

where  $\Sigma_{xx}$  and  $\Sigma_{yy}$  are  $2 \times 2$  symmetric matrices. In this case the emittances can be determined by measuring the

beam profiles only in horizontal and vertical planes (2D measurements). In the general case the motion is fully coupled and the full beam matrix reconstruction, including coupling terms, is needed (see [5], [6]).

In what follows a 2D emittance measurement scheme is considered. In this case the projected (2D) and intrinsic emittances coincide (see definitions for example in [5]) and are given by

$$\varepsilon_x = \sqrt{\det \Sigma_{0,xx}},$$
 (2)

$$\varepsilon_y = \sqrt{\det \Sigma_{0,yy}},$$
 (3)

where  $\Sigma_{0,xx}$  and  $\Sigma_{0,yy}$  are the matrices at the entrance to the measurement section.

Let us consider an emittance measurement section equipped with N LW scanners located at points  $s_i$ . The beam matrix at these points is related to the one at the entrance  $s_0$  by

$$\sigma_i = R_i \sigma_0 R_i^T, \quad (i = 1, 2, \dots N), \tag{4}$$

where  $R_i$  is the transport matrix from  $s_0$  to  $s_i$ . Beam profile measurements with LW scanners allow to determine the matrix elements  $(\sigma_i)_{11} = \langle x^2 \rangle$  and  $(\sigma_i)_{33} = \langle y^2 \rangle$ . Assuming that the transport matrices  $R_i$  of the diagnostic line are uncoupled and using Eq. (4) one obtains the following systems of linear equations

$$\begin{aligned} (\sigma_i)_{11} &= (R_i)_{11}^2 (\sigma_0)_{11} - 2(R_i)_{11} (R_i)_{12} (\sigma_0)_{12} \\ &+ (R_i)_{12}^2 (\sigma_0)_{22}, \end{aligned}$$
(5)  
 
$$(\sigma_i)_{33} &= (R_i)_{33}^2 (\sigma_0)_{33} - 2(R_i)_{33} (R_i)_{34} (\sigma_0)_{34} \\ &+ (R_i)_{34}^2 (\sigma_0)_{44} \end{aligned}$$
(6)

for the horizontal and vertical planes, respectively. For N > 3 the systems are overdetermined and the values of  $(\sigma_0)_{kl}$  are found by a least-squares fit. Once the parameters of the beam matrix  $\sigma_0$  at the entrance to the diagnostics section are found the emittances are calculated using Eqns. (2), (3). The problem that arises in the matrix reconstruction is that due to measurement errors this procedure may lead to non-positive beam matrix, namely to negative values of  $\varepsilon_x^2$  and  $\varepsilon_y^2$  [5], [7], [8].

We would like to note that the 2D emittance measurement scheme has certain advantages in comparison with the 4D one, namely each monitor measures only x- and ybeam sizes and no scan of the beam profile along a rotated axis is needed. Also, the beam matrix reconstruction in the 2D case generates far less non-physical solution. As it is claimed in Ref. [8], the results of simulations show that the reconstruction of the full  $4 \times 4$  matrix in the presence of errors can be misleading. A disadvantage of a 2D section **Colliders** 

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is that the entrance beam must be uncoupled so that a skew correction section (SCS) must be added [6]-[8].

We will consider a 2D emittance measurement section consisting of four equal FODO cells matched to the incoming beam. In order to optimize the spot-size resolution the LW scanners are located just after the horizontally defocusing quadrupoles, where the vertical  $\beta$ -function is maximal. As it is shown in paper [5] the number of non-physical solutions of systems (5), (6) is minimal if the phase advance per cell is equal to

$$\mu = 180^{\circ}/N.$$
 (7)

For N = 4 the phase advance per cell in both planes is  $\mu = 45^{\circ}$ . In this case the beam phase space coverage is optimal and the beam at the measurement points has constant size in each plane (see Refs. [6]-[8] for similar proposals). According to the CLIC preliminary design electrons at the end of the RTML line will have the energy  $E_0 = 9$  GeV and horizontal and vertical normalized emittances  $\varepsilon_{N,x} \leq 600 \text{ nm} \cdot \text{rad}$  and  $\varepsilon_{N,y} \leq 10 \text{ nm} \cdot \text{rad}$ , respectively [9]. For a good control of the collider performance the emittances have to be measured with a precision better than 10 %. The aspect ratio at the scanner locations is

$$a \equiv \frac{\sigma_x}{\sigma_y} = \sqrt{\frac{\varepsilon_x}{\varepsilon_y}} \frac{1 - \sin \mu/2}{1 + \sin \mu/2} \approx 0.67 \sqrt{\frac{\varepsilon_x}{\varepsilon_y}} \approx 5.2 \quad (8)$$

Lower limits on the length  $L_{EMS}$  of the emittance measurement section are determined by the following characteristics:

(a) Minimal beam size  $\sigma_{min}$  that can be measured by the LW monitors. It is easy to show that

$$L_{EMS} \ge \frac{\sigma_{min}^2 \gamma}{\varepsilon_{N,y}} \frac{\sin \mu}{1 + \sin \mu/2} \approx 0.9 \left(\frac{\sigma_{min}}{1 \ \mu \text{m}}\right)^2 \text{ m} \quad (9)$$

(b) Strength  $k_Q$  of the quadrupoles to avoid having too strong chromaticity effects:

$$L_{EMS} \ge \frac{16\sin\mu/2}{k_Q l_Q} = 81.6 \left(\frac{0.075 \text{m}^{-1}}{k_Q l_Q}\right) \text{ m},$$
 (10)

where  $l_Q$  is the quadrupole length.

Assuming  $\sigma_{min} < 4.8 \mu m$  and realistic values of the integrated strength, say  $k_Q = 0.25 \text{ m}^{-2}$  and  $l_Q = 0.3 \text{ m}$ , limit (10) is more restrictive and gives  $L_{EMS} \ge 81.6$  m. This is the case we are going to consider. The optical layout of the section is shown in Fig. 1. Having fixed the lattice parameters as above and using the MAD-X code a suitable solution for the optics was obtained. The matched minimal and maximal values of the  $\beta$ -functions are  $\beta_{min} = 17.8$  m,  $\beta_{max} = 39.8 \text{ m}.$ 

## **BEAM MEASUREMENT SIMULATIONS**

The beam with Gaussian distribution in both planes was generated with the Python code, the particle tracking was **Colliders** 





Figure 1: The optics layout of the 2D emittance measurement section.

done with the PTC module of MAD-X and the beam parameters at each LW scanner were recovered with the Python again. Beam parameters were obtained from the Gaussian fit to a histogram of  $10^4$  electrons with the energy E = 9 GeV.

The beam size obtained from the LW scan after deconvolution of laser effects,  $\sigma_{scan}$ , (see [5] for details) is given by

$$\sigma_{scan}^2 = \sigma_e^2 + \sigma_{jit}^2, \tag{11}$$

where  $\sigma_e$  is the value of the beam size extracted from the measurement and  $\sigma_{jit}$  is the jitter of the location of the bunches within the bunch train at the entrance to the measurement section. The total relative error of the beam size is equal to

$$\left(\frac{\delta\sigma_e}{\sigma_e}\right)^2 = E_{scan}^2 + E_{jit}^2,\tag{12}$$

where  $E_{scan}$  is the contribution to the error from the LW scanner and  $E_{jit}$  is the error contribution from the jitter. Here we assume that the dispersion is absent and the corresponding errors are negligible.

First we simulated the relative error in the determination of the emittances due to the measurement error  $\delta \sigma_e / \sigma_e$ . The result for the vertical plane is shown in Fig. 2, for the horizontal plane the plot is similar. Here for simplicity the measurement errors at all LW scanner were supposed to be the same. Notice that for  $\delta\sigma/\sigma < 0.4$  the emittance error displays an approximately linear behavior. The dependence of the vertical emittance relative error on the magnitude of the beam jitter defined as a fraction of the initial beam size  $\sigma_{jit,y}/\sigma_y$  is shown in Fig. 3.

Similar to the result obtained in Refs. [5], [7], [8] we found that with the increase of the error of the beam size measurement the number of unphysical cases, i.e. cases with negative det  $\Sigma_{0,xx}$  and det  $\Sigma_{0,yy}$ , also increases. The plot in Fig. 4 shows the fraction of non-physical cases of the vertical plane matrix as a function of the beam measurement error. Comparing this plot with a similar one in



Figure 2: Relative error of the vertical emittance measurement as a function of the relative beam size measurement error.



Figure 3: Relative error of the vertical emittance measurements as a function of the beam jitter magnitude.

[5] one can see that the 2D measurement scheme is less sensitive to beam size measurement errors than the 4D one.

### CONCLUSIONS

A 2D emittance measurement section proposed for the RTML line of the CLIC collider consists of four FODO sections of the total length 81.6 m and is equipped with four LW beam profile monitors placed just after the horizontally defocusing quadrupoles. At the entrance to this section the horizontal and vertical beam oscillations must be decoupled, therefore it must be preceded by a skew correction section. In the ILC proposal the length of such section is about 120 m (see details in [7, 8]). From the plot in Fig. 2 one can see that in order to reconstruct the emittance with an error better than 10 % the relative beam size measurement error must be  $\delta \sigma_y / \sigma_y \approx 0.1$  that means that the accuracy of vertical beam size measurements must be better than  $\delta \sigma_y \approx 0.5 \ \mu m$ .

To complete the proposal of the CLIC emittance mea-



Figure 4: Fraction of simulations giving a non-physical beam matrix as a function of the beam measurement error.

surement section a few more issues must be addressed. In particular, it is important to analyze and simulate the LW Compton photons detection, signal extraction, and beam profile measurement, as well as to take into consideration the corresponding errors. This study would allow to get an estimate of the emittance error in case of commercial lasers and existing LW beam profile monitors. Alternatively, on the basis of required emittance measurement precision one could derive requirements on the LW system.

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