HIGH POWER THZ FEL SOURCE BASED ON FFAG BETATRON

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Abstract

A novel source of high power sub-mm waves is proposed that combines two well-known technologies – a betatron induction FFAG accelerator and a free electron laser (FEL). The system is configured as an FEL oscillator: the electron beam circulates in bi-periodic FFAG lattice and the external optical resonator maintains beam-radiation overlap through multiple orbits. Initial analysis shows that FEL gain and very high extraction efficiency are possible with modest injected beam current. A simplified interaction model and preliminary analysis results are presented.

INTRODUCTION

FEL technology has been considered for years as a leading contender to enable very high average power directed energy applications. This interest was predominantly triggered by defense applications, such as Strategic Defense Initiative during the '80s, or the on-going Navy FEL project. However, these endeavors imply large machines, long development times and multi-billion dollars budgets. The complexity is partially due to the relatively poor FEL extraction efficiency at optical wavelength: to obtain 1 MW of average power one has to deploy ~ 100 of MW of high quality electron beam, which is challenging. In fact, the only feasible way to obtain such powers is through recycling the electron beam, either by direct recirculation or in the energy recovery linac (ERL). Direct recirculation is problematic for optical frequency FELs, since the electron beam quality degrades after a single path. The ERL approach, while more promising, is very costly, requiring superconducting radio-frequency (RF) technology to support the ultra-high average power operation.

Table 1: Rough Estimate of the beam parameters to support 10 kW average FEL output at NIR and at 1 mm (300 GHz) wavelengths.

High Power FEL	Optical	THz
Wavelength	1.5 μm	1000 μm
Beam Energy	60 MeV	6 MeV
Peak current	100 A	20 A
Duty cycle	0.07 %	10 %
Emittance	3 mm-mrad	40 mm-mrad

If one, on the other hand, considers FEL technology for microwave generation, the dynamics changes considerably, as the beam quality becomes less of an issue. In order for electrons to radiate coherently, the beam emittance must be small enough to achieve good phase space overlap with the photon beam, but at 1 mm wavelength this is a very loose constraint. Table 1 shows a crude estimate of electron beam parameters necessary to achieve 1 MW average output from an FEL at optical wavelength and at 300 GHz. The optical wavelength FEL parameters demand a state of the art photoinjector driven electron beam, with superconducting RF driver and energy recuperation design, which is very challenging and very expensive. While the average power, current and duty cycle requirements for the 300 GHz case are still challenging, the beam quality is very forgiving, and a direct beam recirculation approach becomes feasible.

TECHNICAL APPROACH

RadiaBeam Technologies is developing a novel industrial circular accelerator, the Radiatron (Fig. 1), which is a modern rethinking of the betatron [1]. The Radiatron incorporates two key improvements to the conventional betatron: the deployment of a Fixed Field Alternating Gradient (FFAG) focusing scheme; and the use of modern, low loss magnetic materials. In combination, they increase the average current possible in the betatron by orders of magnitude, allowing it to match the higher-power capability of RF accelerators at a fraction of the cost, and with the potential for improved reliability.

The Radiatron is a circular induction accelerator: the electron beam is injected at the inner radius and is accelerated by the ramping magnetic field in the core, which creates a 2.5 KV voltage across a ceramic accelerating gap in the vacuum chamber. The strong-focusing scaled FFAG lattice contains the beam to circular trajectories of increasing radius throughout the acceleration cycle, while new electrons are being injected at the lower orbit. The presently chosen parameters of Radiatron were designed to achieve a favorable performance comparison to a typical industrial linac, while unlike the linac, the Radiatron does not have high maintenance RF components, thus it could become a very reliable and turn-key system.

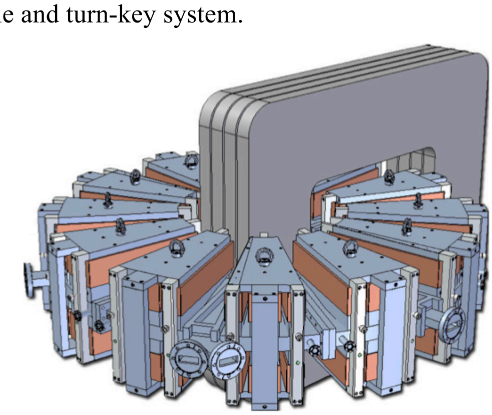

Fig. 1: CAD rendering for industrial Radiatron.

The emittance and extraction energy in Radiatron can easily match the parameters listed in the target parameters for the 300 GHz FEL listed in Table 1, except for the peak current. However, extraction is not necessary for this application. To bridge the 3 order of magnitude gap between 20 mA extracted current available in the present design of the Radiatron, and the requirements for a THz FEL operation, we have to consider the total recirculating current in Radiatron, which is on the order of 20 A given that each electron traverses the circular orbit about 2000 times before the extraction. It will be shown later in this manuscript, that the FEL process itself could provide a mechanism for the electron beam to slow down and accumulate around resonant orbit, resulting in pseudo-monoenergetic, and well localized beam of a very high current density. In order to enable FEL operation in the oscillator configuration, beam trajectory inside the Radiatron has to be modulated (Fig. 2a) and co-aligned to the optical resonator. The electron beam is accelerated towards the resonant orbit, where the resonator forces the radiated power back into the machine resulting in FEL start-up and amplification [2].

FEL EQUATIONS IN CIRCULAR GEOMETRY

Due to the alternating polarity magnets, the electron orbits in an FFAG accelerator are not simply circular, but modulated by periodic component of the magnetic field. Locally, these "scalloped" orbits are analogous to the ones in a planar wiggler, where the resonant condition is such that the radiation slippage per wiggler period is equal to the radiation wavelength,

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right). \quad (1)$$

By analogy to a conventional wiggler, bi-periodic racetrack FFAG geometry can be chosen (Fig. 2a), of which the magnetic field profile can be expressed in cylindrical coordinates as,

$$\vec{B}(R,\varphi) = -B_0\left(\frac{R}{R_0}\right)^k \left[1-\cos(4\varphi)+\kappa\cos(4n\varphi)\right]\hat{\mathbf{y}}, \quad (2)$$

where k is the FFAG scaling factor and n is the number of sector magnets within each leg of the racetrack. When the electron beam is orbiting in the described magnetic lattice it radiates and an optical resonator forces the beam interaction with self-induced spontaneous emission, which can generate the FEL field build up and amplification. To demonstrate bunching of the electron beam in Radiatron FEL of the geometry shown in Fig. 2(a), one has to analyze the dynamics of the energy exchange between an electron beam orbiting around the average radius R_0 and a co-aligned continuous radiation field in the resonator shown in Fig. 2(b). The optical symmetry axis approaches the beam at the distance $L=15/16\ R_0$, with respect to the betatron symmetry axis. Further discussion is limited to a smooth trajectory approximation ($k^2/n^2 \ll 1$), and only a relativistic radial region is considered ($\gamma \gg 1$).

(a)

(b)

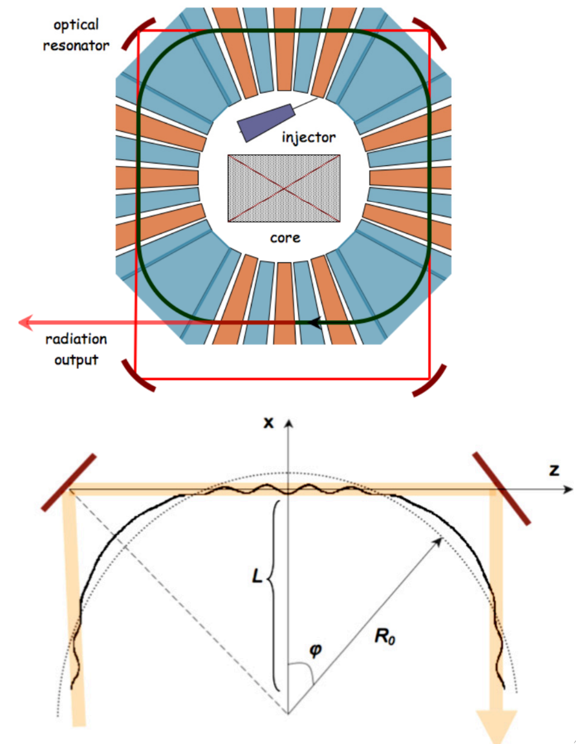

Fig. 2: Geometry of the proposed system (a); the photon beam overlaps with the electron beam when it accelerates to reach the resonant orbit (b).

In a steady state (small gain) approximation, assuming Gaussian transverse distribution of the radiated beam, one can express the radiated field in cylindrical coordinates as,

$$\vec{E}(\varphi,R) \simeq E_0\left\{ \begin{array}{c} e^{i(k_r R\sin\varphi-\omega_r t)} e^{-\frac{(R\cos\varphi-L)^2+y^2}{\sigma(\varphi)^2}} \times \\ \times\left(\hat{\mathbf{r}}\cos\varphi-\hat{\mathbf{j}}\sin\varphi\right) \end{array}\right\}, \quad (3)$$

Proceeding with the energy exchange equation derivation, similar to more conventional approach in planar FEL [3], one obtain a system of equations for slow (period-averaged) phase and energy evolution of the system,

$$\left\{ \begin{array}{l} \dfrac{d\bar{\theta}_n}{d\psi} \approx -\dfrac{n}{\varepsilon}\dfrac{\bar{\eta}}{k+1} \\ \dfrac{d\bar{\eta}}{d\psi} = \left(\dfrac{n+2}{n}\right)\dfrac{\kappa K_a[JJ]}{8\gamma_0\varepsilon} e^{-\frac{\bar{\eta}^2}{2\sigma_\eta^2}} \sin\bar{\theta}_n + \dfrac{K_v}{8\pi\gamma_0} \end{array} \right., \quad (4)$$

where $\varepsilon \equiv 1/2\gamma_0{}^2 + \kappa^2/32n^2$, $\psi \equiv 4\varphi$ is a sector-phase (2π phase advance per sector), $\alpha_n \equiv \kappa^2/64n^2\varepsilon$, $[JJ] \equiv J_0(\alpha_n) - J_1(\alpha_n)$, and $K_v \equiv eV/m_e c^2$.

Equations (4) are similar to pendulum equations for a planar wiggler [4]; however, the two major differences are

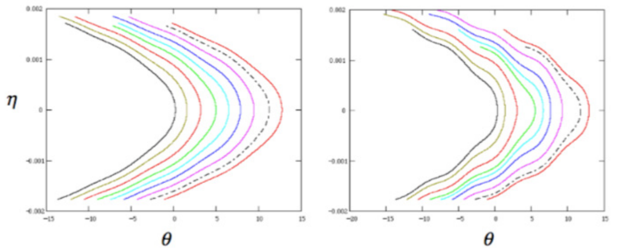

Fig. 3: Numerical solutions of equations (3) for different amplitudes of the radiation field. The stronger field generates visible bunching at the resonant wavelength.

large rate of phase variation (due to orbit elongation phase advance), and presence of the acceleration term K_V. The numeric solution to these equations, shown in Fig. 3, clearly demonstrates bunching formation at the resonant wavelength, which is a profound signature of the FEL interaction.

Microbunching in the electron beam $B(\psi)$ leads to a stronger interaction and affects the field amplitude, which leads to FEL instability. Once again, following the approach of the conventional FEL theory, one can introduce the beam of N particles, and write down a system of *2N+1* equations for slow phase, normalized energy of every particle, as well as the amplitude of the radiation field K_a, all as a function of sector-phase ψ,

$$\begin{cases} \dfrac{d\bar{\theta}_j}{d\psi} = -\dfrac{n}{\varepsilon}\dfrac{\bar{\eta}_j}{k+1} + Kr_{\beta,j}(\psi) \\ \dfrac{d\bar{\eta}_j}{d\psi} = -\left(\dfrac{n+2}{n}\right)\dfrac{\kappa[JJ]}{8\gamma_0\varepsilon} e^{-\frac{\bar{\eta}_j^2}{2\sigma_\eta^2}} \operatorname{Im}\left\{K_a(\psi)e^{-i\bar{\theta}_j}\right\} + \\ \qquad + \dfrac{K_v}{8\pi\gamma_0} - \Lambda \operatorname{Re}\left\{B(\psi)e^{-i\bar{\theta}_j}\right\} \\ \dfrac{dK_a(\psi)}{d\psi} = \dfrac{3K_a^{SE}}{8\pi} iB(\psi) - \delta K_a(\psi) \\ B(\psi) = \iint d\bar{\eta}\, d\bar{\theta}\, e^{-\frac{\bar{\eta}^2}{2\sigma_\eta^2}} e^{i\bar{\theta}} F(\bar{\theta},\bar{\eta},\psi) \end{cases} \quad . \quad (5)$$

The systems of equations (5) fully describes the system evolution, on a slow (period-averaged) timescale, including particle slippage due to betatron motion around equilibrium orbit, $r_{\beta,j}(\psi)$; although the space charge effects including tune perturbations has not been considered. As the electron beam accelerates in the betatron, the head of continuous electron beam accelerates to the resonant energy, and the field build-up in the resonator starts from noise. As the interaction intensifies, the microbunching of the electron beam results in further increase in field strength. This start-up from noise process has been studied numerically, using Eq. (5), and initially random phase space distribution at an energy significantly below the resonance. Preliminary simplified numerical simulations, shown in Fig. 4, demonstrated fast field growth for the system parameters listed in Table 1.

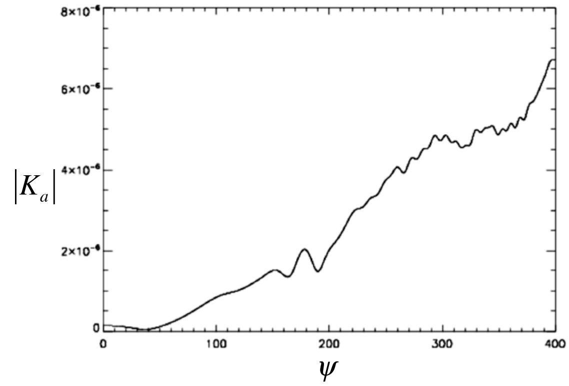

Fig. 4: Preliminary numerical simulation of the start-up from noise process shows strong growth in the radiation field intensity over the first 13 orbital turns.

Following the start-up some interesting dynamics may occur at the vicinity of the resonant orbit. As the part of the beam at resonance undergoes microbunching, and it radiates more energy and its acceleration slows down. At the same time, new particles continue to arrive at the constant rate, being accelerated from the inner orbits where FEL interaction is insignificant. As a result the density of the beam grows around the resonant orbit, further contributing to the radiated power increase. As the density increase leads to the interaction increase, at some point the bunched beam density reaches a level where the overall power loss by the beam to the radiation field is equivalent to the power gain by the same fraction of the beam due to inductive acceleration in betatron.

At this point equilibrium is reached where all of the recirculating current is condensed in the vicinity of the resonant orbit, and any energy transferred to the electron beam by the inductive core is being re-radiated. While dynamics of such equilibrium states is outside the scope of this paper, this qualitative argument suggests possibility of a highly energy-efficient radiation regime, where electrons stay in the vicinity of the betatron resonant orbit and like in transformer, directly transfer energy from the inductive core power supply, into the THz radiation beam.

CONCLUSION

It was shown that in the THz spectral range, the FEL beam quality requirements are much more relaxed and can be obtained in an inexpensive recirculating electron accelerator, such as betatron. Initial formalism and start up from noise in such system has been examined.

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