# COMBINING MULTITURN AND CLOSED-ORBIT METHODS FOR MODEL-INDEPENDENT AND FAST DETERMINATION OF OPTICAL FUNCTIONS IN STORAGE RINGS 

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## Abstract

Multiturn / turn-by-turn data acquisition is a new source for Twiss parameter determination in storage rings, while closed-orbit measurements are a long-known tool for diagnostics with conventional low-frequency beam position monitor (BPM) systems, being available at almost every storage ring. The presented method aims to join the advantages of multiturn and closed-orbit measurement methods.

For uncoupled optics, there are only two correctors per oscillation plane and two multiturn BPMs needed in one drift space of a storage ring for model-independent measurement of $\beta$ and betatron phase functions at all BPMs along the ring, including conventional ones. This is a costeffective alternative to exclusive multiturn BPM usage in storage rings, resulting in the same amount of information. This method can be extended to include betatron coupling [1].

In addition, a possible experimental setup needed for multiturn data acquisition using a bunch-by-bunch feedback system is described. By applying an uncritical coherent excitation to coupled bunch modes, the accuracy of the multiturn data acquisition may be significantly improved, enabling the use of smaller drift spaces.

## CONCEPT

Closed-orbit perturbations originating from two dipole correctors in each betatron oscillation plane can be directly linked to Twiss parameters, if the optical functions at the two dipole corrector positions are known. If the correctors are inside a drift space with multiturn BPMs at its ends, this information can be obtained experimentally by measuring beam centroid motion in transverse phase space.

Thus the method presented here consists of two steps:

## Step 1) Determination of Optical Functions within a Drift Space via Multiturn BPMs

If there is just a drift space between two multiturn BPM positions, it is possible to determine the absolute $\beta$ function at all multiturn BPM positions in the ring (see [3] where this is shown in detail). Although this technique is demonstrated in the $x-s$ or ring plane with uncoupled betatron motion for simplicity, it is possible to include vertical motion and betatron coupling [3].

The transverse position $x_{n j}$ and angle $x_{n j}^{\prime}$ of the undamped beam centroid at BPM $j$ after a short transverse

[^0]

Figure 1: The multiturn phasors $X_{1}, X_{1}^{\prime}$ span up an area an the complex plane that is equal to the Courant-SnyderInvariant $\Upsilon_{x}$
kick and $n$ turns around the ring can be written (see Ref. [4]) as

$$
\begin{equation*}
x_{n j}=\sqrt{\Upsilon_{x} \beta_{j}} \cos \left[\phi_{j}+\mu_{x} n\right] \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
x_{n j}^{\prime}=-\sqrt{\frac{\Upsilon_{x}}{\beta_{j}}}\left[\alpha_{j} \cos \left(\phi_{j}+\mu_{x} n\right)+\sin \left(\phi_{j}+\mu_{x} n\right)\right] \tag{2}
\end{equation*}
$$

with the horizontal Courant-Snyder invariant $\Upsilon_{x}$, betatron phase $\phi$, horizontal betatron phase advance per turn $\mu_{x}=$ $2 \pi Q_{x}$ and optical parameters $\alpha, \beta$. Since these sequences have sinusoidal form, each sequence can be described using only one phasor

$$
\begin{equation*}
X_{j}=\quad \sqrt{\Upsilon_{x} \beta_{j}} e^{i \phi_{j}}, \quad X_{j}^{\prime}=-\sqrt{\frac{\Upsilon_{x}}{\beta_{j}}}\left[\alpha_{j}+i\right] e^{i \phi_{j}} \tag{3}
\end{equation*}
$$

In a drift space of length $l$ with BPMs $j=1,2$ at its ends, the phasors relate to each other, given by conditions of the intercept theorem

$$
\begin{equation*}
X_{1}^{\prime}=X_{2}^{\prime}=\frac{X_{2}-X_{1}}{l} \tag{4}
\end{equation*}
$$

Thus, one can determine $X_{1 / 2}^{\prime}$ from experimental data in a drift space. It is then possible to determine the ring-global invariant $\Upsilon_{x}$ because it equals the area spanned up by the phasors $X_{1}, X_{1}^{\prime}$ in the complex plane (see Fig. 1). Then, $\beta_{j}, \phi_{j}$ can be determined using the absolute value and complex angle of $X_{j}$ at all multiturn BPMs. Also, the Twiss parameter $\alpha$ that is proportional to the slope of $\beta(s)$ is obtainable (only inside the drift section).

Using further calculations [1], one can determine the $\beta$ and $\phi$ functions continuously inside the drift space because


Figure 2: Exemplary plots of orbit perturbations observed using two dipole correctors $k=1,2$ of the used drift section at DELTA. The waveform applied to the correctors was a sinusoid with a frequency below 1 Hz , each applied current corresponds to a different color. The black curve was computed using FFT on the measured waveforms
they depend on very few parameters that are determined by the measurement, e.g. $\beta(s)$ is a parabola inside the drift. This method is robust against decoherence phenomena because of its calculations in frequency domain [1].

## Step 2) Applying Small Closed-orbit Perturbations using Correctors inside the Drift Section

The description of a closed-orbit perturbation $x$ at the position of a BPM $j$ by an additional kick $\theta_{k}$ of a dipole corrector $k$ can be written as [2]

$$
\begin{equation*}
x_{j k}=C_{Q} \sqrt{\beta_{j} \beta_{\tilde{k}}} \cos \left(\phi_{j}-\phi_{\tilde{k}}\right) \theta_{k} \quad \text { with } \phi_{j}>\phi_{\tilde{k}} \tag{5}
\end{equation*}
$$

If the perturbations originate from two dipole correctors $k=1,2$ inside the drift section used in step $1, \beta_{\tilde{k}}, \phi_{\tilde{k}}$ are already known. Thus the closed-orbit perturbations only depend on the optical functions $\beta_{j}$ and $\phi_{j}$ at the position of the BPMs $j$ where the perturbation is measured, and an additional factor $C_{Q}$ (depending on betatron tune) assumed constant during the measurement. One gets a system of two equations (5) for each BPM $j$ with the two unknowns $\beta_{j}$ and $\phi_{j}$. This system is solvable with two exceptions that can be avoided in practice. These are linear dependency of both corrector perturbations, equivalent to a difference in betatron phase $\Delta \phi_{12}=n \pi$ for $n \in \mathbb{Z}$ between both correctors. The second exception is the same condition for the multiturn BPM positions. A new variable

$$
\begin{align*}
u_{j}=\arctan \left(\frac{1}{\tan \Delta \phi_{1 \tilde{1}}}-\right. & \left.\frac{1}{\sin \Delta \phi_{1 \tilde{2}}} \sqrt{\frac{\beta_{\tilde{2}}}{\beta_{\tilde{1}}}} \frac{x_{j 1}}{x_{j 2}}\right)  \tag{6}\\
& + \begin{cases}\pi & \text { for } x_{j 2}<0 \\
0 & \text { for } x_{j 2}>0\end{cases}
\end{align*}
$$



Figure 3: Map of the DELTA synchrotron radiation facility
is introduced. Using this variable, one obtains

$$
\begin{gather*}
\beta_{j}=\frac{x_{j 2}^{2}}{\beta_{\tilde{2}} C_{Q}^{2} \theta^{2}}\left(1+\tan ^{2} u_{j}\right)  \tag{7}\\
\phi_{j}= \begin{cases}u_{j}+\phi_{\tilde{2}}+\mu_{x} / 2 & \text { for } \phi_{j}>\phi_{\tilde{2}}, \\
u_{j}+\phi_{\tilde{2}}-\mu_{x} / 2 & \text { for } \phi_{j}<\phi_{\tilde{2}} .\end{cases} \tag{8}
\end{gather*}
$$

as solution of the equation system. Since conventional lowfrequency BPM systems can measure closed-orbit perturbations, $\beta$ and $\phi$ can be determined at the position of all BPMs in the ring, including conventional analog BPMs. If the applied dipole kick $\theta_{k}$ of the corrector electromagnets is not known, it is possible to scale the perturbation values using the multiturn drift measurement results.

## EXPERIMENTAL SETUP AT DELTA

The synchrotron radiation facility DELTA (see Fig. 3), located at TU Dortmund University, consists of a linac, a full-energy synchrotron and a 1.5 GeV electron storage ring with a circumference of 115.2 m . The storage ring exhibits three insertion devices and 54 double-view BPMs. 44 of them work with analog BPM electronics [5] ( 10 Hz sampling rate, black arrows), while the remaining 10 BPMs incorporate multiturn-capable readout electronics [6] (2.6 MHz sampling rate, red arrows). Up to now, the multiturn capability of BPM 39-41 is not yet set up.

For the measurements presented here, the drift space ( $l=5.137 \mathrm{~m}$ ) between BPM 14 and BPM 15 is used. The electromagnetic undulator between these BPMs has been turned off and exhibits small correction coils that remained unused until now. Via the accelerator control system [8] [7], a sinusoidal current with a frequency $<0.2 \mathrm{~Hz}$ and an amplitude $I_{\text {corr }}=3 \mathrm{~A}$ is applied subsequently on the first and the last correction coil of the undulator. For the multiturn measurements, a diagonal "slotted-pipe" type kicker is used [9].

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Figure 4: Measurements of the horizontal $\beta$-function and phase for two different beam currents $I_{\text {beam }}$. Shown are the theoretical $\beta$-function (red) and phase (green) from a lattice model, the corresponding values from 7 multiturn BPMs (black crosses) and the results from the combined method (blue circles). In the drift space at $s \approx 30 m$, $\beta_{x}$ and $\phi_{x}$ can be obtained continuously (indigo line).

## RESULTS AND COMPARISON

Figure 4 shows an exemplary result of the presented method. This method is a combination of closed-orbit and multiturn techniques and comprises their advantages and disadvantages. Its advantages are

- fast measurement procedure - slower than fullmultiturn, but faster than closed-orbit-only methods.
- less synchronization issues than full-multiturn - only two multiturn BPMs have to be synchronized.
- model-independence - the method is independent of existing ring models and their accuracy.
- compatibility - results of the method can be converted to full-multiturn-equivalent data, and then used by procedures using this kind of data.

Possible error sources are given by

- imperfect beam position measurements, e.g. by pincushion distortion - this is a common error of all BPM-based methods
- timing difference errors of the two used multiturn BPMs - this is a problem common to multiturn measurements.


## Planned New Experimental Setup

Figure 5 shows the layout of the bunch-by-bunch feedback system to be installed in the DELTA storage ring [10]. It enables to drive transverse modes in a stable way by an excitation signal generated by the iGp12 module. The signal frequency is then tuned to the frequency of the transverse mode.
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Figure 5: Layout of the planned bunch-by-bunch feedback system

This system may be used to obtain multiturn data, thus replacing the short pulses of the installed kicker system. Since the oscillation is driven, no decoherence damping should occur, which would make it possible to get oscillation information from $\approx 6 \cdot 10^{4}$ turns [6]. This would increase resolution of the combined method by a factor $\approx 30$.

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