WEDGE ABSORBER DESIGN AND SIMULATION FOR MICE STEP IV*

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Abstract

In the Muon Ionization Cooling Experiment (MICE), muons are cooled by passing through material, then through RF cavities to compensate for the energy loss; which reduces the transverse emittance. It is planned to demonstrate longitudinal emittance reduction via emittance exchange in MICE by using a solid wedge absorber in Step IV. Based on the outcome of previous studies, the shape and material of the wedge were chosen. We address here further simulation efforts for the absorber of choice as well as engineering considerations in connection with the absorber support design.

EMITTANCE EXCHANGE IN THE MUON IONIZATION COOLING EXPERIMENT

The Muon Ionization Cooling Experiment (MICE) [1] is an international experiment based at Rutherford Appleton Laboratory in U.K. Ionization cooling is achieved in MICE baseline by the placement of absorbing material in the beamline. The absorbing material reduces beam momentum, which is replaced only in the longitudinal direction by RF cavities, resulting in a net reduction of emittance. Overall, transverse emittance is reduced while longitudinal emittance stays the same or increases slightly due to stochastic processes in the energy loss.

We plan to demonstrate emittance exchange with MICE. In emittance exchange a dispersive beam is passed through a wedge-shaped absorber. Muons with higher energy pass through more material and experience greater momentum loss. In this way longitudinal emittance of the beam can be reduced either in addition to, or even instead of transverse emittance reduction. Emittance exchange is vital for the cooling section of a Muon Collider and has been considered as an upgrade option to the Neutrino Factory.

The measurement of longitudinal emittance reduction in MICE will test the accuracy of the absorber physics models in a different geometry, demonstrate that the physics of emittance exchange is well understood, and demonstrate emittance exchange in a real magnetic lattice.

A first simulation study of wedges in MICE was made in [2], where it was shown that even a large emittance dispersive beam could be passed through MICE Step IV with acceptably small non-linear effects given care in the way the beam is selected.

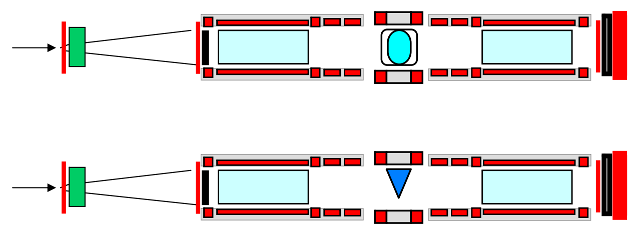

Figure 1: The geometry of MICE, side view. Top: the liquid hydrogen absorber module in MICE Step IV; bottom: the liquid hydrogen absorber is replaced with the wedge absorber.

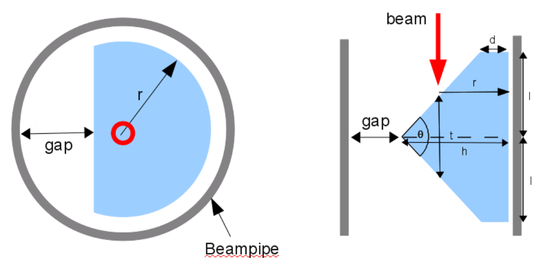

Figure 2: Schematic of the wedge geometry, which is parameterized by the on-axis thickness t, opening angle θ and radius r.

SIMULATION GEOMETRY

In this study a wedge-shaped absorber is simulated in a straight solenoid channel. The geometry is shown in Fig. 1. The case considered here is MICE Step IV, where MICE is operated in flip mode without RF cavities. The focussing system has symmetry in the transverse planes x and y, and the absorber is at an optical waist with no beam kinetic angular momentum. The dispersion function is assumed to be at a waist and the dispersion direction aligned with the wedge.

The choice was made to use a lithium hydride (LiH) absorber, and time permitting, a polyethylene (C_2H_4) absorber. LiH is a solid with low average Z and low Z/A resulting in less multiple scattering and energy straggling than polyethylene for a given energy loss and hence a generally better cooling performance. LiH is a restricted material due to the nature of its production, making it expen-

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sive and difficult to procure. There may also be some handling and safety issues associated with LiH. Polyethylene is readily available and widely used for many industrial applications, so it is easy to procure and there are no handling issues specific to this material.

The wedge is modeled by the intersection of a triangular prism with a cylinder, as shown in Fig. 2. The wedge absorber is parameterized by the thickness on-axis, which determines the energy lost by a reference particle, and the opening angle of the wedge, which governs the emittance exchange.

Available Beams

The MICE beam line has been shown in simulation to generate matched beams with emittances in the range 3 to 10 mm and momenta in the range 140 to 240 MeV/c. This gives us a good range of parameters with which to populate phase space for beam selection.

Table 1: Parameters of the simulated beam at the wedge center. D_i are the dispersions.

Parameter	Value
Reference p [MeV/c][1]	200
Transverse emittance [mm][2]	6
Transverse β [mm]	420
Transverse α	0
Longitudinal emittance [mm]	90
Longitudinal β [ns]	10
Longitudinal α	0
RMS energy spread [MeV]	25.1
D_x [mm]	200
D_y [mm]	0

Control of dispersion has not been planned for the MICE beam line, and is expected to be challenging. In this study, it is assumed that dispersion will be introduced using a beam selection algorithm similar to the one described in [3]. The parameters of the beam used in simulation, corresponding to a beam matched to the canonical MICE lattice and with typical emittances, are listed in Table 1.

Wedge Choice

The main criterion for the wedge absorber choice is that a strong cooling be observable. The cooling performance for the wedge of choice with the beam described above is shown in Fig. 3 and compared to two other options: 60° and 30°. LiH wedges were simulated with 75.4 mm on-axis thickness, corresponding to about 12 MeV energy loss at 200 MeV, and various opening angles.

[1] At the lattice start.

[2] The transverse distribution was generated ignoring the effects of dispersion, such that the calculated emittance is different from the nominal emittance listed here.

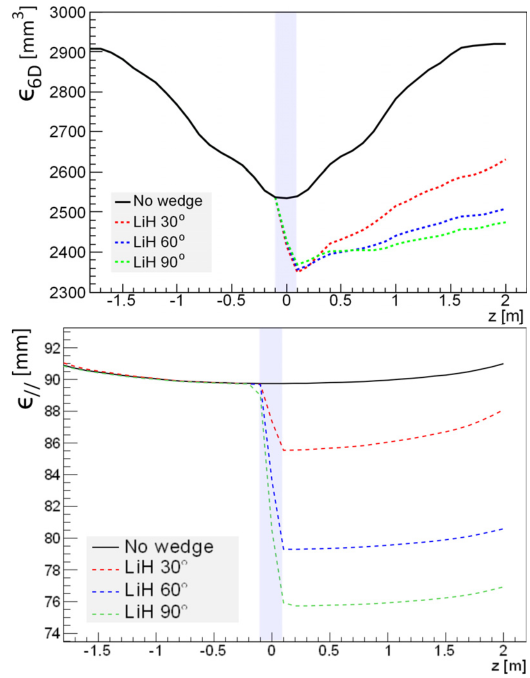

Figure 3: Simulated emittance along the beam line for canonical beam parameters and a dispersion of 200 mm.

For larger wedge angles, $\partial/\partial x(dE/dz)$ is more pronounced so that the longitudinal partition function is larger, resulting in more longitudinal cooling. In most cases the wedges heat in transverse phase space, with more heating for larger opening angles. The key part of this experiment is to demonstrate longitudinal emittance reduction. In light of this, the smaller opening angle wedges are disfavored as the longitudinal cooling signal is too weak. On the other hand, the decision to limit ourselves to a maximum opening angle of 90° is motivated by the fact that the gap between the beam pipe and the absorber apex (see Fig. 2) becomes very large, and too much beam misses the wedge for larger opening angles.

SIX-DIMENSIONAL EMITTANCE PRESERVATION

Some concerns were raised about the fact that the six-dimensional emittance shown in Fig. 3 changes when there is no material in the channel, violating Liouville's theorem. To show that the phase volume does not change, but rather that the approximation that is used to estimate the corresponding emittance is not precise for strongly nonlinear systems, the following technique was employed.

Liouville's theorem states that for Hamiltonian systems the phase space density is constant. Consider the transfer map characterizing the effect of the lattice on the phase space between any two given points z_0 and z_1:

$$\vec{X}_1 = \mathcal{M}(\vec{X}_0),$$

where $\vec{X}_0$ is the vector of phase space variables at position

z_0, $\vec{X}_1$ is the vector of phase space variables at z_1, and $\mathcal{M}$ is the nonlinear transfer map defined as the flow of the underlying system of differential equations. The fact that the phase space volume is conserved follows from the condition that holds for all Hamiltonian systems:

$$\det(\text{Jac}((M))) = 1. \quad (1)$$

Thus, to test that the phase space volume is conserved, we need to check that the condition of Eq. 1 holds everywhere in the area of phase space where the beam is.

The MICE Step IV magnet configuration was implemented in the COSY INFINITY [4] code capable of calculating high-order approximations of nonlinear transfer maps via Taylor expansions of the flow of the system of differential equations describing the effect of the lattice on the particles. For consistency, the magnetic field calculation results were compared to another beam physics code, g4beamline [5]. The results are consistent, as shown in Fig. 4. A ninth-order transverse map from the symmetry point ($z = 0$) to a point outside the last magnet ($z = 3.3$ m) was calculated in COSY. A ninth-order Taylor expansion is sufficient to accurately represent the nonlinearities of the system in question. Once the tansfer map was obtained, its partial derivatives provided all the information required to calculate the determinant of the Jacobi matrix at any point of interest in phase space.

It was found that the determinant is equal to 1 everywhere in the area larger (over 8σ) than the beam size under consideration. Figure 5 shows the deviation of the determinant from 1 as a function of x and y (as an example – the same holds true for all other phase space coordinates). The deviation of $O(10^{-11})$ results from the fact that the ninth-order Taylor polynomial approximation is not sufficient for larger amplitudes.

The fact that the determinant is 1 implies that the phase space volume is constant. However, the six-dimensional emittance approximation produced by using the second-moment matrix does not reflect that. In general, such an approximation works well for linear and weakly nonlinear systems, or when the beam is paraxial. None of these conditions hold for the MICE magnets and beam. Thus, there is a need for a better estimate of the six-dimensional emittance, and the corresponding study is underway. One of the proposals is to calculate the phase space volume based on the Voronoi algorithms. The issue with that approach is that it is rather computationally demanding, especially in six dimensions, to be used for routine emittance calculations.

The single-particle nature of the experiment allows another approach to improve precision. We can reconstruct the trajectory of each individual particle from the point where it is last measured in the first tracker ($z = -2$ m) to the upstream edge of the wedge absorber, and again from the point of measurement in the second tracker ($z = 2$ m) backwards to the downstream edge of the absorber, and compare the two emittances.

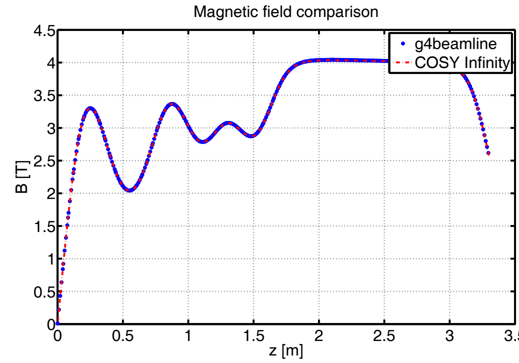

Figure 4: COSY Infinity and g4beamline field approximation comparison.

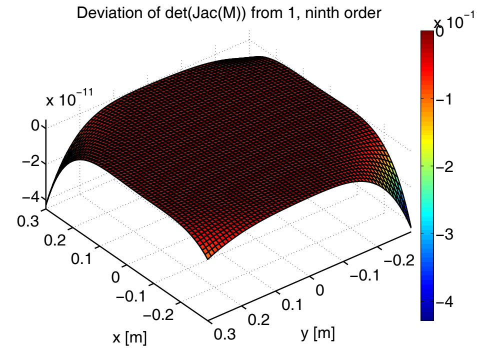

Figure 5: Ninth-order transfer map Jacobian deviation from 1.

WEDGE ABSORBER ENGINEERING AND TESTING

The LiH absorber is being fabricated based on the schematic shown in Fig. 2. For engineering and budgetary purposes the absorber will be comprised of two identical half-wedges with 45° opening angle each. A flat solid absorber for transverse emittance tests has been completed recently, and the expected completion date for the wedge is May 2011. Engineers at Fermilab are preparing a test stand for thermal tests to be held prior to shipping the absorber to RAL. The wedge support structure is being designed at RAL and Imperial College.

REFERENCES

[1] An International Muon Ionization Cooling Experiment (MICE), Proposal to the Rutherford Appleton Laboratory, Tech. rep. MICE-NOTE-GEN-0021, 2003.

[2] C. T. Rogers, P. Snopok, Wedge Absorber Simulations for the Muon Ionization Cooling Experiment, Proc. COOL 2009.

[3] C. T. Rogers, Statistical Weighting of the MICE Beam, Proc. EPAC 2008.

[4] K. Makino, M. Berz, COSY INFINITY version 9, NIM A558 (2005) 346-350.

[5] T. Roberts, www.muonsinc.com.