END-TO-END SIMULATION OF AN INVERSE CYCLOTRON FOR MUON COOLING*

K. Paul†, E. Cormier-Michel, Tech-X Corporation, Boulder, CO 80303, USA
T. Hart, D.J. Summers, University of Mississippi-Oxford, University, MS 38677, USA

Abstract

Neutrino factories and muon colliders require significant cooling of the muon beam. Most muon cooling channels are long and expensive single-pass structures, due to the difficulty of injecting very large emittance beams into a circular device. Inverse cyclotrons can potentially solve the injection problems associated with other circular cooling channels, and they can potentially provide substantial initial cooling of the beam. We present the first end-to-end (injection to extraction) simulations of an inverse cyclotron for muon cooling, performed with the particle-in-cell code VORPAL. We study the cooling capability of the device as well as potential limitations due to space charge effects and material interactions with the beam.

INTRODUCTION

Intense muon beams are needed for both neutrino factories and muon collider designs, presenting a novel and challenging problem to the accelerator community: how can one produce, capture and cool such an intense muon beam as would be needed for a neutrino factory or muon collider? The only known method for producing muons in the numbers required is through the strong interactions of high-energy protons in a high-density target. This produces an intense pion shower, with 10% of the protons leading to a captured pion in a very strong solenoid magnet. The captured pions are allowed to expand into lower-field strength solenoids for transport while they decay into muons. The resulting muon beam has a very large emittance. To achieve the small beam size necessary to accelerate muons to store in a neutrino factory racetrack and to achieve high luminosity at a muon collider, significant cooling of the beam must be done. [1, 2, 3, 4]

Conventionally explored muon cooling channels are long and expensive, owing mainly to the fact that these channels require strong radio-frequency (RF) gradients and strong transverse focusing to achieve rapid cooling. [5, 6, 7, 8] Multi-pass synchrotron ring-like channels are much less expensive than conventional linear designs, but injection into these devices is challenging because of the large emittance of the initial muon beam. Large admittance inverse cyclotrons, however, can potentially provide beam cooling with the advantages of a multi-pass device without the difficulties associated with injection into a standard synchrotron ring. Inverse cyclotrons have been used to produce modest, low energy anti-proton and muon beams for many years. These devices have the potential for providing beam cooling for high-intensity muon beams, but their capabilities in this regard are still mostly unexplored.

THE INVERSE CYCLOTRON

The first inverse cyclotron (or anti-cyclotron or cyclotron trap) was constructed as a way of stopping anti-protons for LEAR at CERN. This device was later transported to PSI for use in stopping μ^- beams. [9, 10, 11, 12, 13, 14, 15] Particles start at the outer-most radius of the inverse cyclotron at injection. After injection, the particles lose energy in a low-density gas that fills the interior of the cyclotron magnet. The particles accumulate in the core of the cyclotron and are ejected through the "top" by electrostatic or pulsed electric fields. A new inverse cyclotron trap is being constructed for FRIB by researchers at Michigan State University.

For the inverse cyclotron to be of use for neutrino factories or muon colliders, the inverse cyclotron must accept a large-emittance, high-intensity muon beam. For ionization cooling, it must also strongly focus the beam. Strong-focusing in modern cyclotrons is normally provided with spiraling sectored magnets. The effective density of material in the inverse cyclotron must be much higher than ion-stopping cyclotrons to accommodate both the short lifetime of the muon and the initial high energy of the beam. These constitute high demands on the design of the inverse cyclotron for intense muon beams.

If gas is used in the cyclotron interior, high-intensity muon beams will result in large ionization rates that can be problematic for operation with electrostatic ejection. Studies have shown most of the ionization in the inverse cyclotron occurs away from the central core where the ejection fields are localized [16, 17]. However, the very high densities required to stop high-energy muons in a fraction of their lifetime—typically $\sim$10 bar—make this statement untrue. Densities of this magnitude can suppress breakdown in the gas, but also prevent low-energy muons from being ejected at all.

Space-charge is another potential problem with high-intensity muon beams when they stop in the core. To counter the potential Coulomb explosion of the stopped beam, a Penning-style trap can be placed in the core. In a previous study [18], the capabilities of a Penning-style trap to hold the 2×10^{12} muons desired for a muon collider bunch were explored. The study showed that the required

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† kpaul@txcorp.com

fields are strong but potentially capable of containing the bunch. Alternately, reducing the charge in a bunch and going to multiple bunches may be a reasonable compromise.

If high-density gas proves to be too great of a difficulty, it may be possible to consider replacing the high-density gas with thin wedges of low-Z solid, such as lithium hydride. The wedges can be placed at the focal points of the cyclotron beam lattice to maximize the cooling effect. If lithium hydride is a problem, then other materials can be used, such as carbon.

VORPAL SIMULATIONS

VORPAL [19, 20] is a commercial electromagnetic finite-difference, time-domain (FDTD) simulation framework, originally developed for simulating laser-plasma interactions. Since its inception, VORPAL has been developed to include the electrostatic solvers and material interactions needed in accurate simulation of the inverse cyclotron, among many other features. In this study, we use VORPAL to simulate a simple six-sector, non-spiraling cyclotron filled with a uniform, high-pressure hydrogen gas.

The fields describing the cyclotron are taken from an expansion out of the central plane, where we have assumed a nominal field in the mid-plane of the cyclotron of

$$B_0(r,\theta) = Cr^k \left(1 + f\cos(N\theta)\right)\mathcal{F}(r) \,, \qquad (1)$$

where $\mathcal{F}(\nabla)$ denotes the shape of the fringe-fields in the mid-plane, describing the fall-off of the fields as r decreases due to the fact that the cyclotron magnetic do not extend into the cyclotron core. For this study, we assume the fringe fields in the mid-plane can be approximated by the function

$$\mathcal{F}(r) = \frac{1}{2}\left(1 + \tanh\left(\frac{r - r_1}{w}\right)\right) \,. \qquad (2)$$

From the mid-plane fields, an expansion is done out of the mid-plane to fourth order in z, as described by [21]. The values used in this study for the parameters in the equations above are given in Table 1.

Table 1: Mid-plane Magnetic Field Parameters

C	k	f	N	r_1	w
1.77 T/m^k	0.6	$\sqrt{2}$	6	0.2 m	0.1 m

The core of the cyclotron contains a simplified Penning-style trap. The magnetic field of the trap is provided by a pair of Helmholtz coils with a central peak field of 2.4 T, the fields for which are approximated by a gaussian in the mid-plane and expanded out of the mid-plane in the same way as the cyclotron fields. The electrostatic field in the core is approximated by a uniform spherical ball of charge of radius 2.5 cm—opposite in sign and equal in magnitude to the injected muon beam.

With the field specifications described above, the reference orbit for the cyclotron was found by tuning the starting radius of a $p = 180$ MeV/c μ^+ near the theoretically predicted value given by $p = erB(r)$. For $p = 180$ MeV/c, $r = 0.464$ m, and the particle is placed in the simulation in the center of one of the cyclotron magnets. With a $\rho = 2.5$ kg/m^3 (30 bar) hydrogen gas in the interior of the cyclotron, the reference muon spirals into the core in approximately 300 ns. The reference trajectory is depicted in Figure 1.

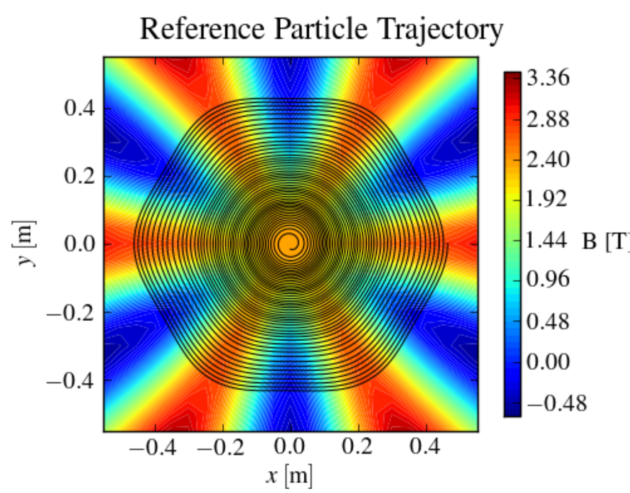

Figure 1: Trajectory of the reference particle with initial momentum of $p = 180$ MeV/c and initial radius of $r = 0.464$ m. The color contour behind the trajectory depicts the mid-plane strength of the magnetic field in Tesla.

The acceptance of this design was explored with various beam widths and momentum spreads, all with space-charge effects and stochastic material effects disabled. The largest beam size with the least loss was used for the simulations, which has a radius of 5 cm and a momentum spread (in all components) of 15 MeV/c. The beam is assumed to be 100 ns long, and we assume only one-tenth of the maximum beam charge needed for a muon collider (2×10^{11} muons). With this much charge in the beam, the electrostatic fields at the locations of the presumed electrodes (top and bottom of the cyclotron, $z = \pm 0.2$ m) are less than 7.2 kV.

After 400 ns, the beam has accumulated in the core. At this point, the electrostatic trap is ramped down over 100 ns while an ejection field of $E = 500$ MV/m, directed in the z direction, is ramped up in the same time. This ejection field is unrealistic, but it is required in this simulation the friction force produced by the high-density gas. Future designs will avoid this problem by using lithium hydride wedges and low-density helium gas.

With space-charge and stochastic material effects enabled, 97% of the beam accumulates in the core, with distributions show in Figure 2. When the trap fields are ramped down, very little Coulomb expansion occurs because of the strong axial magnetic field. As the extraction fields are ramped up, the beam is accelerated toward the top of the cyclotron.

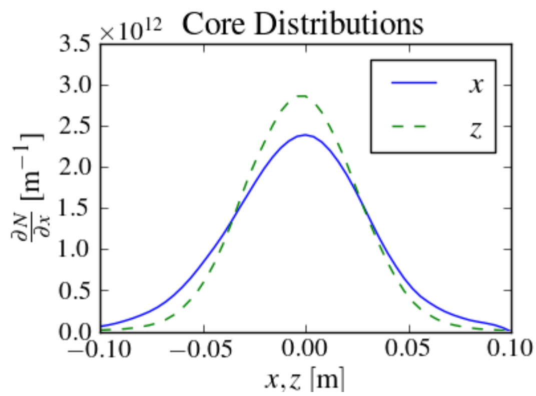

Figure 2: Distributions of the muons in the core of the cyclotron just before extraction.

At the ejection surface ($z = 0.2$ m), we do not expect the energy of the beam to be meaningful because the extraction fields must be so large to compensate for the high-density gas. The time spread of the beam at ejection greatly depends on the length of the extraction field ramp time and the strength of the fields themselves, so the time spread is also ignored. The transverse distribution, however, is meaningful, and it changes very little from when it is in the core due to the strong axial magnetic field and the electrostatic trap. Hence, we can assume the normalized transverse emittance in this simulation will be similar in more realistic ejection scenarios. We measure the final transverse emittance to be $\epsilon_{T,N} = 3.08$ mm-rad.

Based on our initial beam, we measure the initial normalized transverse emittance to be $\epsilon_{T,N} = 2.06$ mm-rad, and the initial normalized longitudinal emittance is $\epsilon_{L,N} = 1.08$ m-rad. Ejection results in transverse and longitudinal distributions to flip because ejection is normal to the cyclotron plane. The extracted normalized transverse emittance is more than $300\times$ smaller than the initial normalized longitudinal emittance. More realistic simulations must yet be performed to assess the effect on the final longitudinal emittance, however.

CONCLUSIONS & FUTURE WORK

The inverse cyclotron shows promise as means of cooling the intense muon beam, though many difficulties with the design have yet to be overcome.

The high-density gas will need to be replaced in the core to remove the need for excessively large extraction fields. Currently, designs are being investigated that use lithium hydride wedges placed in the low-β regions between the cyclotron magnets. A thin horizontally oriented foil can be used in the core in place of higher-density gases. This appears to be a promising approach, but a full study of this design is ongoing.

Realistic designs for the electrostatic trap must be incorporated into the simulation. Studies with realistic designs have been performed, but the fields have not been incorporated into the end-to-end simulations because of their complexity, typically requiring very high resolution. Such simulations will be performed in the future.

Finally, realistic injection in to the cyclotron requires a degrader, normally a series of wedges, through which the beam is sent to cause the beam to lose enough energy to be captured by in the cyclotron. This has been studied in preliminary simulations, but was not simulated in this study. This kind of *energy loss injection* will need to added to future simulations, as well. Similarly, capture and transport in a radio-frequency quadrupole must be simulated following extraction.

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