HELICAL CHANNELS WITH VARIABLE SLIP FACTOR FOR NEUTRINO FACTORIES AND MUON COLLIDERS*

C. Yoshikawa#, C. Ankenbrandt, Muons, Inc., Batavia, IL 60510, U.S.A
D. Neuffer, K. Yonehara, Fermilab, Batavia, IL 60510, U.S.A.

Abstract

In order to realize a muon collider or a neutrino factory based on a muon storage ring, the muons must be captured and cooled efficiently. For a muon collider, the resulting train of bunches should be coalesced into a single bunch. Design concepts for a system to capture, cool, and coalesce a muon beam are described here. In particular, variants of a helical channel are used, taking advantage of the ability to vary the slip factor and other parameters of such a channel. The cooling application has been described before [1,2]; this paper reports recent studies of a system that includes two novel concepts to accomplish capture and coalescing [3] via a slip-controlled helical channel.

INTRODUCTION

The helical cooling channel (HCC) [1] was invented to achieve efficient ionization cooling in all three degrees of freedom, i.e. in 6-D phase space. However, there is considerable flexibility in the design of helical channels, so other applications are also possible. In particular, the magnetic parameters, RF parameters, and contents of the volume (e.g. vacuum or gas, slab absorbers or wedge absorbers) can be varied, allowing the design of helical channels for other purposes: to capture muons upstream of the cooling section, to allow extreme cooling (EPIC [4]), and to coalesce multiple bunches into a single bunch downstream of the cooling section. This paper focuses on the upstream capture subsystem, called a quasi-isochronous helical channel (QIHC), and the downstream bunch merger subsystem, called the bunch-coalescing helical channel (BCHC [3]).

Providing multiple functions in the same type of magnetic channel greatly simplifies the transitions between subsystems, since the parameters of the channel can be varied adiabatically to accomplish the matching. The resultant front end that provides multiple functions in a single helical channel is likely to be simpler and less expensive than the baseline muon collider front end [5] and its associated bunch merger [6,7]. Besides the QIHC and BCHC, other structures in the channel provide the initial capture of muons into RF bunches as well as cooling in the HCC and EPIC before the bunch merging. These other subsystems are included as necessary parts of the whole system, but this paper focuses on the QIHC and BCHC.

CONCEPTS UNDERLYING THE QIHC

The QIHC captures a train of muon bunches in RF buckets by varying the channel parameters in order to cause the RF bucket area to increase monotonically as the beam propagates downstream. The bucket area is given by:

$$A_{bucket} \cong \frac{16}{w_{rf}} \sqrt{\frac{eV'_{\max} \lambda_{RF} m_{\mu} c^2}{2\pi|\eta_H|}} \left[\frac{1-\sin(\varphi_s)}{1+\sin(\phi_s)}\right] \quad (1)$$

where

- the term in brackets is an approximation for the moving-bucket factor
- w_{rf} is the RF frequency in radians/second
- V'_{max} is the maximum E-field voltage gradient
- λ_{rf} is the RF wavelength
- m_μ is the mass of the muon
- φ_s is the synchronous particle RF phase, and

η_H is the slip factor, derived in [1] for an HCC as:

$$\eta_H = \frac{\sqrt{1+\kappa^2}}{\gamma\beta^3}\left(\frac{\kappa^2}{1+\kappa^2}\hat{D} - \frac{1}{\gamma^2}\right) \quad (2)$$

where $1/\gamma_T^2 = \left(\kappa^2/(1+\kappa^2)\right)\hat{D}$ and the dispersion factor $\hat{D}$ relates to apparatus quantities and design momentum via:

$$\hat{D}^{-1} = \frac{a}{p}\frac{dp}{da} = \frac{\kappa^2+(1-\kappa^2)[(B\sqrt{1+\kappa^2}/pk)-1]}{1+\kappa^2} - \frac{(1+\kappa^2)^{3/2}}{pk^2}\frac{\partial b_\phi}{\partial \rho} \quad (3)$$

in which

- p is reference momentum; a is reference radius
- $\kappa = p_{transverse}/p_z$ = helix pitch
- B is the solenoid B_z
- $k = 2\pi/\lambda$; λ is helix period, and
- $\dfrac{\partial b_\phi}{\partial \rho}$ is the quadrupole component.

Thus, the RF bucket area in the QIHC can be adjusted by varying the gradient of the dipole field ($\partial b_\phi/\partial\rho$), the reference momentum (p), the accelerating phase (φ_s), the transition energy γ_t, and the maximum gradient (V'_{max}).

DESIGN & SIMULATION LAYOUT

The QIHC and BCHC are integral parts of a proposed front end shown in Figure 1 that exploits the flexibility of the helical channel. The simulations described here involved 100k 8 GeV protons on a Hg target in a MERIT-like configuration [8] followed by a tapered capture solenoid with a $B_z(z)$ profile similar to that of the baseline design [5], but modified to end at 4 T instead of 2 T. Subsequent to the tapered solenoid is a first straight RF buncher in vacuum for 20 m to capture lower energy pions/muons in the useful range, followed by a second straight RF buncher in high-pressure hydrogen gas to allow higher electric field gradients together with variable amounts of Be foils. This material provides transverse precooling of the muons and also allows the otherwise useless higher energy pions and protons to interact, thereby possibly creating additional useful muons. Following the second straight is the QIHC, which not only serves to enhance capture via enlarged RF buckets, but also provides the matching from a straight

*Work supported in part by DOE SBIR grant DE-SC00002739
#cary.yoshikawa@muonsinc.com

solenoid channel into the HCC, which has a pitch angle of 45°. The HCC provides 6-D cooling of multiple muon bunches and is followed by EPIC for even further cooling. The BCHC takes the train of cooled muon bunches and coalesces them into a single bunch. If space charge precludes bunch merging at this low energy, the bunches will be accelerated before coalescing.

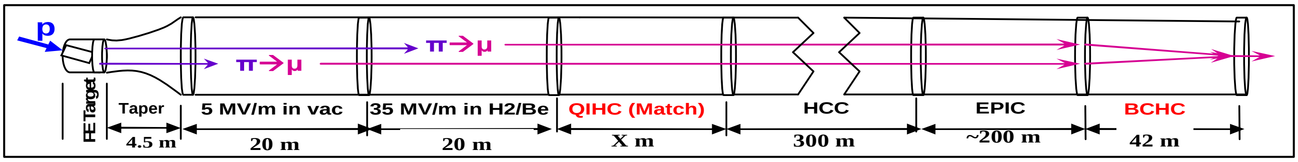

Figure 1: Layout showing a tapered solenoid for initial π/μ capture, 20 m of buncher with 5 MV/m in vacuum, 20 m of buncher with 35 MV/m in H_2 gas plus π degrader via Be windows, the QIHC which also provides matching of trajectories from straight solenoid into the HCC, the HCC that cools multiple muon bunches, the EPIC for extreme cooling, and the BCHC to merge the cooled bunches into a single muon bunch.. Acceleration may be needed before entering the BCHC if space charge precludes bunch merging at low energies.

QIHC DESIGN & SIMULATION

Figure 2 shows the longitudinal dynamics for μ^-'s at the end of a previous QIHC design [9]. That design phased the RF cavities to keep a constant momentum for the reference particle as it traversed a matching section based on gradually increasing coil displacements [2].

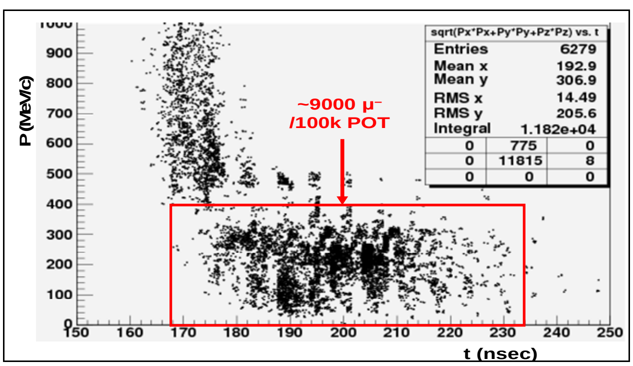

Figure 2: Longitudinal dynamics of μ-'s at end of a previously designed QIHC.

The current iteration of the QIHC design was motivated by the fact that the output emittance of the muon beams exiting the second straight solenoid (ε_T=11 mm-rad, $\varepsilon_{||}$= 378 mm-rad) did not match the acceptance of the HCC (ε_T = 20 mm-rad, ε_L = 40 mm-rad). Hence, the new QIHC must also provide longitudinal cooling. In the new system, the parameters of the first 51m of the front end are adjusted to provide a monotonically increasing bucket area to optimize capture. Those parameters include γ_t, the accelerating phase φ_s, and the reference momentum, as illustrated in Figure 3.

Figure 4 shows a relatively high yield of pions and muons (~0.1 per proton on target for 150 MeV/c $\leq$ p $\leq$ 450 MeV/c) exiting the second straight and heading for the QIHC. Since the QIHC will be receiving muons from a straight channel, it must start below transition, so a desirable transition momentum corresponds to ~450 MeV/c. Reception of longitudinally hot muons from a straight channel also suggests that the QIHC operate at a small pitch, κ, to provide longitudinal cooling.

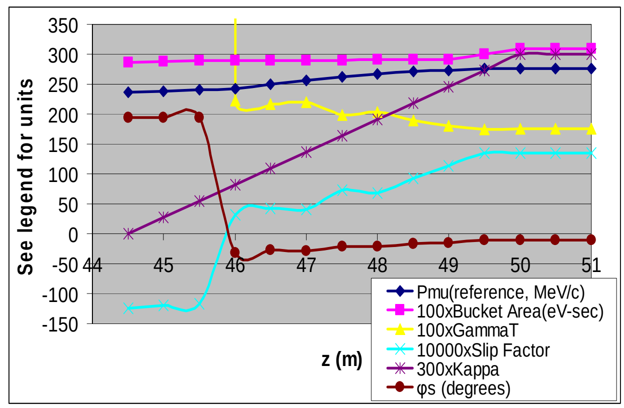

Figure 3: Parameters for monotonically increasing bucket area in the first part of a QIHC matching section.

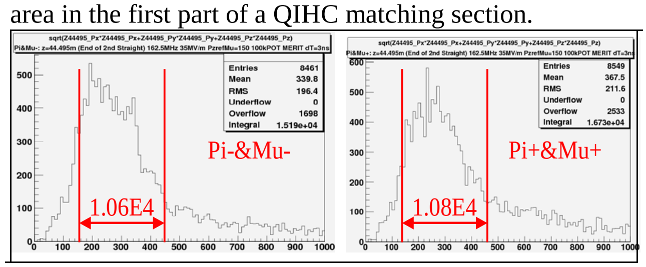

Figure 4: Momenta (MeV/c) of π's and μ's at the end of the second straight that enters the QIHC from 100k POT.

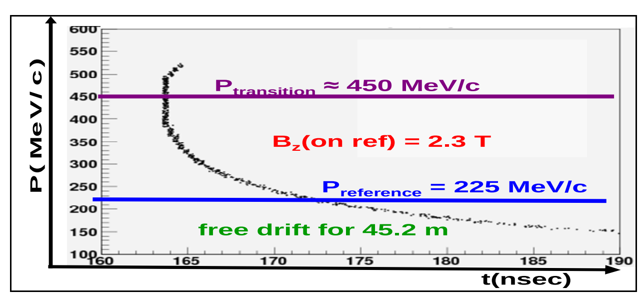

Figure 5: Longitudinal dynamics of a QIHC with κ=0.25.

A design for the magnetic fields of a QIHC that achieves the above criteria can be realized; its longitudinal dynamics are shown in Figure 5. To enhance longitudinal cooling, maximal use of a cylindrically symmetric Be wedge (1.48 mm thick on reference every 10 cm) along with 60 atm of H_2 gas at 293K (at the knee of the Paschen breakdown curve [10]) provided the largest ε_L acceptance compared to other cases with less use of wedges. For example, it provides a 19% enhancement of the ε_L acceptance compared to a case that has no wedges but the same total energy loss provided by only the gaseous H_2(200atm) as shown in Table 1. However, the equilibrium ε_L at ~30 m is lowest for the case without any wedge. So, a configuration to consider is to incorporate the maximal amount of Be wedges for the first ~20m of the κ=0.25 QIHC for largest longitudinal acceptance, followed by a region with no wedges to achieve the smallest equilibrium ε_L.

Table 1: Effect of varying amounts of Be wedge on ε_L. Maximal use of wedge has 60 atm of H_2. Equal use of wedge (E_{loss}(wedge)=E_{loss}(H_2)) has 100 atm of H_2. No use of wedge has 200 atm of H_2. All cases have same total energy loss in materials (E_{loss}(wedge)+E_{loss}(H_2)).

	H2:200 atm	H2:100 atm	H2: 60 atm
ε_L(m-rad) at z=0m	0.13300	0.14810	**0.15820**
ε_L(m-rad) at z=30m	**0.09619**	0.09843	0.09901

CONCEPTS UNDERLYING THE BCHC

The BCHC has a large slip factor to facilitate coalescing. The RF gymnastics that occur in the BCHC consist of three steps, as follows:

1. A frequency incommensurate with the bunch spacing is applied to the muon bunch train for two purposes:
   a. To apply a phase rotation within individual bunches to reduce the energy spread and increase the time spread of each bunch.
   b. To apply energy offsets between bunch centers that put early arriving bunches at higher energies and late arrivals at lower energies. Multiple frequencies may be used if needed to linearize the rotations.
2. A drift (with no RF) of the bunches in a helical channel to align the bunches in time. The slip factor is positive, so higher energy bunches take a longer time travelling downstream than lower energy bunches. The drift distance needed to align the bunches, which can be shortened by a large slip factor, is given by:

$$\delta(c\tau) = \eta_H \frac{\delta E}{m_\mu c^2} z \quad (4)$$

3. When the bunches are time aligned, they encounter RF voltage to capture them into a single bunch.

BCHC DESIGN & SIMULATION

To illustrate the concepts underlying the BCHC, a design to merge 11 bunches with 5 nsec spacing (200 MHz) into a single bunch over a longitudinal distance of ~42m was simulated. A purely longitudinal simulation provides promising results as shown in Figure 6. An RF frequency of 204.08 MHz is applied to the 11 bunches that are initially separated with 5 nsec (200 MHz) spacing in order to induce phase rotation within each bunch as well as cause energy offsets between bunch centers as shown from Figure 6(a) to Figure 6(b). At the end of a 32.5 m-long drift in a region with η=0.43, the bunches are aligned. RF is then applied to capture ~95% of the muons in a single bunch. This successful longitudinal simulation motivates a full 3-D one using G4beamline [11].

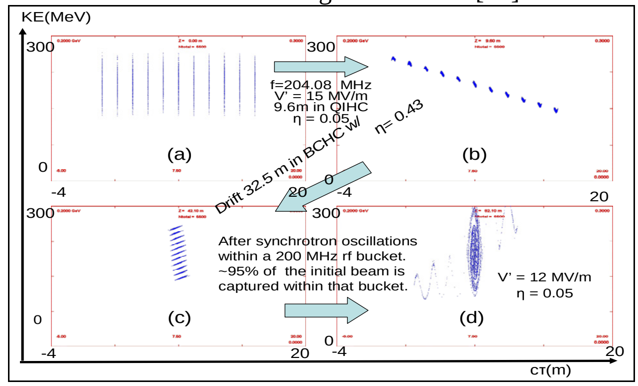

Figure 6: A longitudinal simulation of the BCHC. The 11 initial bunches with 5 nsec (200 MHz) bunch spacing are shown in (a). The energy displaced bunch centers and phase rotated muons within bunches after application of off-resonant frequency are shown in (b). At end of a 32.5 m long drift, the bunches are aligned in (c). RF is applied to capture ~95% of the muons in a single bunch in (d).

SUMMARY & FUTURE PLANS

The flexibility of the helical channel to capture, cool, and coalesce bunches of muons has been illustrated via this preliminary design of a front end for a NF/MC that promises to be simpler and more cost-effective than the baseline design. As the design is preliminary, it is too early to compare its performance with the baseline. Regarding capture, further studies including longitudinal cooling will enhance its effectiveness. For bunch coalescing, the 1D simulations provide promising results, which is the source of excitement for anticipated results from 3D simulations that are soon to come.

REFERENCES

[1] "Six-dimensional muon beam cooling...in a helical dipole channel", Y. Derbenev and R. P. Johnson, Phys. Rev. STAB 8, 041002 (2005).
[2] K. Yonehara et al., "A Helical Cooling Channel System for Muon Colliders," IPAC10, MOPD076
[3] D. Neuffer, "Use of Helical Transport Channels for Bunch Recombination," MuCool Note 548
[4] V. Morozov et al., "Epicyclic Twin-Helix…Ionization Cooling," IPAC10, MOPEA042
[5] Muon Accelerator Program (MAP) Proposal, http: //map.fnal.gov/proposals/pdfs/MAPproposal-R6d.pdf
[6] R. Fernow, "Estimate of Front-End Magnetic Requirements," NFMCC Tech. Note 529 (2008)
[7] J. Gallardo et al., "Bunch Merging," 2007 MCDW, http://www.cap.bnl.gov/mumu/conf/collider-071203/talks/JGallardo1-071203.pdf
[8] H. Kirk, "The MERIT High-Power Target Experiment at the CERN PS," EPAC08, WEPP169
[9] C. Yoshikawa, "Quasi-Isochronous Muon Collection Channels," IPAC10, WEPE073
[10] P. Hanlet et al., "High Pressure RF Cavities in Magnetic Fields," EPAC06, TUPCH147
[11] G4beamline, T. Roberts, http://www.muonsinc.com