# ANALYSIS OF SINGLE SPIKE RADIATION PRODUCTION AT SPARC

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#### Abstract

The single spike operation regime has been analysed in the case of the SPARC injector and free-electron-laser. Various beams with charge from 100 to 500 pC and energy from 100 to 150 MeV are studied, for different undulator matching, with different production conditions and performances.

## **INTRODUCTION**

In the FEL emission two different regimes occur depending on the length  $L_b$  of the beam.

If the  $L_b$  is larger than  $2\pi$  times the cooperation length L<sub>c</sub>, the radiation presents a longitudinal structure constituted by several chaotic peaks, while, if the length of the beam is shorter than  $2\pi L_c$ , the emission produces a radiation pulse shaped in one single spike [1]. This regime occurs because the radiation emitted by the electrons, travelling from the tail towards the head of the beam, covers all the distance inside the bunch in a time shorter than few gain times, correlating all the particles. The properties of this regime are well-known in 1d: however, the study of single-spike ultra-short radiation in the X rays range [2], as well as in the visible light [3], by means of start-to-end simulations from the photocathode to the end of the undulator, has shown that transverse and non-homogeneity effects due to radiation diffraction and to non-ideal characteristics of the electron beam such as emittance and energy spread change considerably the properties of the emission process. A fundamental problem is also how to produce a suitable beam. We present, therefore, some numerical start-to-end FEL simulations made for realistic set of parameters in the case of the SPARC FEL, for some different beam with charge from 100 to 500 pC and energy from 100 to 150 MeV, and with different matching conditions. The performance of the various bunches are compared and the most interesting of them are discussed.

## SCALING LAW

The single spike operation requires that the beam length  $L_b$  satisfies the following requirement:

 $\begin{array}{c} L_{b}\leq 2\pi L_{c} \qquad (1)\\ \text{with } L_{c}=L_{c1d}\left(1+\eta\right)\\ \text{where: } L_{c1d}=\lambda/(\sqrt{3}\ 4\pi\rho) \text{ and }\eta \text{ is defined as in [4]:} \end{array}$ 

$$\eta = 0.45 \eta_{d}^{0.057} + 0.55 \eta_{\epsilon}^{1.6} + 3 \eta_{\gamma}^{2} + 0.35 \eta_{\epsilon}^{2.9} \eta_{\gamma}^{2.4} + 51 \eta_{d}^{0.95} \eta_{\gamma}^{3} + 5.4 \eta_{d}^{0.7} \eta_{\epsilon}^{1.9} + 1140 \eta_{d}^{2.2} \eta_{\epsilon}^{2.9} \eta_{\gamma}^{3.2},$$
(2)

with  $\eta_{d} = L_{gld} \lambda / (4\pi \sigma_{x}^{2})$  term that accounts for radiation diffraction,  $\eta_{\varepsilon} = \frac{4\pi L_{gld} \varepsilon_{n,x}^{2}}{\sigma_{z}^{2} \gamma^{2} \lambda}$  for the emittance and

 $\eta_{\gamma}=4\pi\frac{L_{gld}}{\lambda_{u}}\frac{\delta\gamma}{\gamma}~~$  for the energy spread effects. In these last

expressions  $L_{g1d} = \lambda_v / (\sqrt{3} 4\pi\rho)$  is the 1d gain length,  $\epsilon_{n,x}$  the normalized transverse emittance,  $\delta\gamma/\gamma$  the energy spread and  $\lambda$  is the radiation wavelength given by the resonance condition  $\lambda = \frac{\lambda_u (1 + a_w^2)}{2\gamma^2}$ .

The FEL parameter  $\rho$ , in terms of the beam average current I, of the radial r.m.s dimension  $\sigma_x$  of the beam, of the undulator parameter  $K_0 = \sqrt{2} a_w$  and period number  $k_u = 2\pi/\lambda_u$ , of the Lorentz factor of the beam  $\gamma$  can be written as:

$$\rho = \left[\frac{1}{16} \frac{I}{I_A} \frac{K_o^2 [JJ]^2}{\gamma^3 \sigma_x^2 k_u^2}\right]^{1/3}$$
(3)

where  $I_A=17$  KA is the Alfven current and JJ =(J\_0(\xi)-J\_1(\xi)), J's are Bessel function of argument  $\xi = \frac{a_w^2}{2(1+a_w^2)}$ .

In (3) the current I is defined as I=cQ/L<sub>b</sub> with L<sub>b</sub> the whole beam length if the beam current is flat top, or  $L_b=\sqrt{2\pi}\sigma_z$  with  $\sigma_z$  the rms length, if the longitudinal beam profile is Gaussian.

The single spike condition is:

and the Q vs L<sub>b</sub> scaling law becomes

$$L_{b}=2\pi L_{c1d} (1+\eta) \qquad (4)$$

$$Q = \left(\frac{\pi^{2} I_{A}}{3\sqrt{3}c}\right) \left(\frac{\lambda_{u}(1+a_{w}^{2})^{3}}{K_{0}^{2} [JJ]^{2}}\right) \left(\frac{\sigma_{x}^{2}}{L_{b}^{2} \gamma^{3}}\right) (1+\eta)^{3}$$
(5)

where in the factor  $\eta$  a further irrational dependence on I is contained.

The number of spikes in the radiation pulse is: 
$$N_s = L_b/(2\pi L_{c1d}(1+\eta)).$$
 (6)

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In the SPARC operation, the electron bunch is compressed by means of the technique of the velocity bunching. As the electron beam enters in the sections of the linac not in crest, the final energy of the beam is in general lower than the nominal value of 150 MeV. Furthermore, the control of the emittance is easier for beams with charge smaller than the nominal value of 1 nC. We have therefore considered beams with energies lower than 150 MeV, in the range between 100 and 140 MeV, with charge from 100 to 500 pC. We have adjusted the undulator parameter aw in order to obtain wavelengths close to 500 nm. The compression factor C has been limited to 3-3.5 for a FWHM length of the beam of 110-170 µm. For these parameters the projected emittance is 1.5-3 mm mrad and the total energy spread is around 1%.

With all these quantities fixed, one can obtain the transition from the multiple to single spike regime by varying the matching of the beam to the undulator, changing the value  $\sigma_x$  along the propagation in the wiggler. In this way the gain and cooperation lengths increase and the number of spikes gets lower.



Figure 1: N<sub>s</sub> vs  $\sigma_x$  for Q=500pC,  $\varepsilon_x$ =2 mm mrad,  $\Delta \gamma / \gamma$ =0.38%.

In Fig 1, the number of spikes N<sub>s</sub> is shown as function of  $\sigma_x$  for Q=500pC,  $\varepsilon_x=2$  mm mrad,  $\Delta\gamma/\gamma=0.38\%$ ,  $\lambda=500$  nm. As can be seen the single spike condition can be achieved with a value of  $\sigma_x$  of the order of 400 µm.

The transverse dimension of the beam during the radiation phase can be controlled by changing the value of the focussing quadrupoles in the drifts of the undulator.

# **OPERATION AT 100-500 pC, 100-130 MeV**

Several different beams have been simulated in the case of the beam line and undulator of SPARC [5] by scaling the parameters from the 1 nC working point by means of the scaling law at the cathode  $\sigma_{xyz} \sim Q^{1/3}$  [6], and using a laser pulse length of  $\sigma_t=1$  psec, illuminating a region of R=0.4 mm. The compression of the beam has been done

by means of the technique of the velocity bunching, recently tested at SPARC [7]. All the simulations have been made by means of the code PARMELA [8]. In these series of simulations we have changed the charge of the beam from 100 to 500 pC and the energy from 100 to 140 MeV.

In Table 1, the main parameters for some examples of the beams studied are reported.

Table 1: Parameters of Some of the Beam Studied

Beam	Q pC	Energy MeV	$\epsilon_{x,n} \ \mu m$	Δγ/γ %	С	$\sigma_{z,}$ $\mu m$	I <sub>peak</sub> kA
1	100	121.6	0.8	0.7	3	133	90
2	300	121.6	1.7	1.	3.5	162	300
3	500	121.6	2.7	1.2	3.4	126	500

These beams have been injected in the undulator of SPARC to produce radiation at 500 nm. The radiation simulations have been made by uploading the electron phase spaces in GENESIS 1.3 [9].

Single spike pulses can be obtained in all conditions, by changing the current in the focussing quadrupoles placed in the drifts between the sections of the undulator for reaching the right transverse dimension of the electron beam. The main characteristics of the radiation are presented in Table 2.

As can be seen, the total energy of the radiation at the end of the undulator remains very low for the cases with small charge.

In the case with 500 pC, however, the energy yield achieves  $2 \mu J$ .

Table 2: Radiation Properties: N<sub>s</sub>: Number of Radiation Spikes, Pmax: Maximum Value of the Power Emitted, E: Total Energy of the Pulse,  $\sigma^{z_{rad}}$ :Radiation Length, bw: Normalized Bandwidth

Beam	$\sigma_{x}$	Ns	P <sub>max</sub>	Е	$\sigma^{z,}_{rad}$	bw
	μm		MW	μJ	μm	%
1	70	1-2	1.3	0.29	53	0.9
2	100	2	3.2	0.84	58	1.5
3	119	1	11	2	44	1

The phase space of this last case, together with the current profile I (A) vs t (sec) are presented in Fig 2.

In Fig. 3 the slice emittance  $\varepsilon_x$  and  $\varepsilon_y$  (left axis) and the slice energy spread  $\Delta \gamma / \gamma$  are shown along the beam coordinate t (s).

We have obtained the single spike condition by varying the transverse dimension inside the undulator by means of the quadrupoles in the drifts. In Fig. 4 two typical cases are reported. The red curve represents the transverse dimension  $\sigma_x$  vs z with the field of the quadrupoles set at dB/dz=5T/m, for the blue one, instead, dB/dz=1.7. The average value of  $R=(\sigma_x^2+\sigma_y^2)^{1/2}$  along the line is 195 µm in the first case, while R=258 µm for the second.

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Figure 2: Left axis:  $\gamma$  vs t. Right axis: Current I (A) vs t for Q=500 pC and E=121.6 MeV.



Figure 3: Left axis: Slice transverse emittance  $\varepsilon_x$  (black curve) and  $\varepsilon_y$  (blue curve) (in mm mrad) along the beam vs t (sec). Right axis: Normalized energy spread  $\Delta\gamma/\gamma$  (red curve) for 500 pC and 121.6 MeV.



Figure 4: Q=500, E=121.6 MeV. Red curve: $\sigma_x$  vs z for dB/dz=5T/m, Blue curve: dB/dz=1.7T/m.



Figure 5: blue: pulse shape P (W) vs s( $\mu$ m) at z= 14 m. Red curve: spectrum vs  $\lambda$ (nm) for Q= 500 pC, E=121.6MeV, dB/dz=1.7 T/m.

The number of spikes obtained in these cases is reported in Fig 1 by blue stars. In Fig 5 the radiation pulse and spectrum at the end of the undulator are shown for the case with dB/dz=1.7 T/m.

The characteristics of the spectrum can be detected with the spectrometer set at SPARC at the end of the fifth section, while the temporal shape of the pulse requires the development of a FROG device based on Transient Gradient non linear interaction [9].

# **CONCLUSIONS**

The production of single spike, clean pulse of radiation in the SPARC device at 500 nm has been analysed for realistic parameters. The velocity bunching method of compression allows to obtain bunches with very much peaked profiles. In this way, only the part of the charge in the higher current slices contributes to the radiation emission. The radiation pulse is short and single spiked, the energy yield ranging between fractions of  $\mu$ J with charge of 100 pC to tens  $\mu$ J at 500 pC.

#### REFERENCES

- [1] R. Bonifacio et al. PRL 73 (1994) 70
- [2] J. Rosenzweig et al. NIMA 593, 45 (2008)
- [3] M.Boscolo et al, NIMA 593 137 (2008)
- [4] M. Xie in Proc. of the 1995 PAC, IEEE, Piscataway, NJ,1995, p.183
- [5] M. Ferrario et al. PRL 99 (2007) 234801
- [6] M. Boscolo et al.: SPARC BD-note BD-07-006
- [7] M. Ferrario et al., "Velocity bunching at SPARC" these proceedings
- [8] L.M. Young, LANL Report No.LA-UR-96-1835
- [9] S. Reiche: NIM A 429 (1999) 243
- [10] G. Marcus et al.: FEL Conference 2008. TUPPH018

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