LONGITUDINAL PHASE SPACE TOMOGRAPHY AT J-PARC RCS

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Abstract

The longitudinal phase space tomography, which was evolved to retrieve the two-dimensional beam distribution in the longitudinal phase space with computer tomography algorithms, is very useful diagnostic tool in the accelerator domain. The simple reconstruction tool was developed for the J-PARC RCS with the convolution back projection method for the beam storage mode. On the assumption that the longitudinal profiles should not be disturbed for one period of the synchrotron oscillation, such two-dimensional profiles can be reconstructed easily from one-dimensional bunch beam profiles, which are measured for every turn by the wall current monitor.

INTRODUCTION

In order to observe two-dimensional beam distributions in the longitudinal phase space, the simple reconstruction tool was developed with longitudinal tomography (LT) algorithm for the 3-GeV rapid cycling synchrotron (RCS) at Japan Proton Accelerator Research Complex (J-PARC) [1][2]. The LT algorithms were evolved with applying the X-ray computer tomography (CT) algorithms for the medical to the accelerator beam physics [3]. Recently various LT algorithms have been developed to retrieve the two-dimensional profile of the original object from a set of projected histograms of this object. We adopt the Convolution Back-Projection (CBP) method [4] for the LT algorithms in the beam storage mode of the RCS. In this paper, we demonstrate the reconstruction of the longitudinal distribution from the experimental data and present the comparison with the longitudinal beam tracking simulation. We discuss the technical issues and applicability of the longitudinal tomography.

RECONSTRUCTION TOOL

Convolution Back Projection Method

If the original distribution is described as $f(x, y)$, the projected histogram taken at angle θ can be expressed as

$$g(r,\theta)=\int_{-\infty}^{\infty} f(r\cos\theta - s\sin\theta, r\sin\theta + s\cos\theta)ds\ , \quad (1)$$

where a new coordinate system (r, s) is obtained by a rotation of the (x, y) axis. And using the formula for the inverse Fourier transform, the object function $f(x, y)$ can be written in modified polar coordinates as

$$f(x,y)=\frac{1}{4\pi}\int_0^{2\pi}\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega\cos\theta,\omega\sin\theta)e^{j\omega r}|\omega|d\omega\right\}d\theta\ . \quad (2)$$

Here a filter function $H(\omega)$ is defined as $H(\omega) = |\omega|$ and then a new function $q(r, \theta)$ is defined as following formula:

$$q(x,y)\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega\cos\theta,\omega\sin\theta)e^{j\omega r}H(\omega)d\omega\ . \quad (3)$$

By substituting the equation (3) for (2), the object function $f(x, y)$ can be written as

$$f(x,y)=\frac{1}{4\pi}\int_0^{2\pi}q(r,\theta)d\theta\ . \quad (4)$$

If we obtain a set of the function $q(r, \theta)$, the original distribution can be calculated by the equation (4). Thus the function $q(r, \theta)$ is called back-projection. The inverse Fourier transform in (3) can be written as a convolution:

$$q(r,\theta)\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}g(r',\theta)\cdot h(r-r')dr'\ , \quad (5)$$

where $h(r)$ is the inverse Fourier transform of the filter function $H(\omega)$ and it can be written as

$$h(r)=\frac{1}{2\pi}\int_{-\infty}^{\infty}H(\omega)e^{j\omega r}d\omega\ . \quad (6)$$

To summarise, the back projection $q(r, \theta)$ is calculated with the convolution of two functions $g(r, \theta)$ and $h(r)$ in (6), and the object function $f(x, y)$ is obtained by the equation (4).

Longitudinal Tomography with CBP Method

We developed the reconstruction tool with the CBP method and tested with the circulating beam bunch profiles, which are measured by the wall current monitor (WCM) at the RCS [5]. In the trial, the RCS was operated for the beam storage mode, thus the proton beam was stored in the ring and extracted at the injection energy of 181 MeV, and the longitudinal particles are captured in the stationary RF bucket and synchrotron frequency is constant in time. In the algorithm of the reconstruction tool with the CBP method, it is assumed that the

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longitudinal profiles should not be disturbed for one period of the synchrotron oscillation because the non-linearity of large amplitude synchrotron motion have scarcely effect on the motion during one synchrotron oscillation period. On this assumption, two-dimensional distributions can be reconstructed easily from the measured one-dimensional beam bunch profiles without the hybrid method, which incorporates the particle tracking [3].

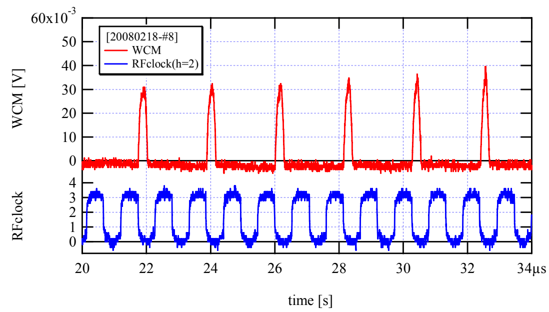

Figure 1: The typical beam bunch shape acquired by the WCM and the RF-clock generated by the LLRF.

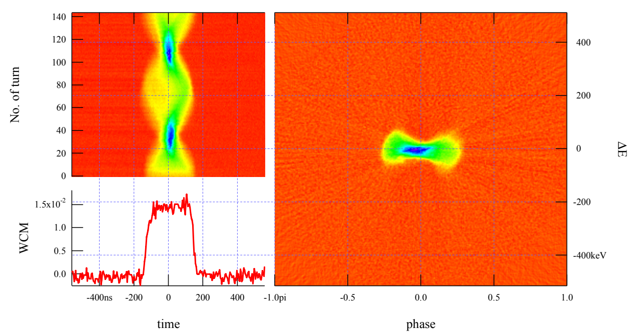

Figure 2: the reconstructed beam distribution in the longitudinal phase space with the CBP method.

Fig. 1 shows the typical measurement results of the circulating bunch signal by the WCM and the RF-clock signal generated by the low level RF (LLRF) system [5]. A set of projected histograms can be obtained by cutting out from the WCM signal at the every rise timing of the RF clock. Fig. 2 shows the results of the reconstruction after the multi-turn injection. Left figures show the mountain plot for one synchrotron oscillation period and the first projected histogram. Right figure shows the two-dimensional reconstructed distribution in the longitudinal phase space. In the beam studies condition, the 24 bunches were injected in the ring and the first injected bunch rotates in the phase space for 1/7 period of the synchrotron oscillation for the beam injection period, the shape of longitudinal distribution in the phase space is similar to a bow tie.

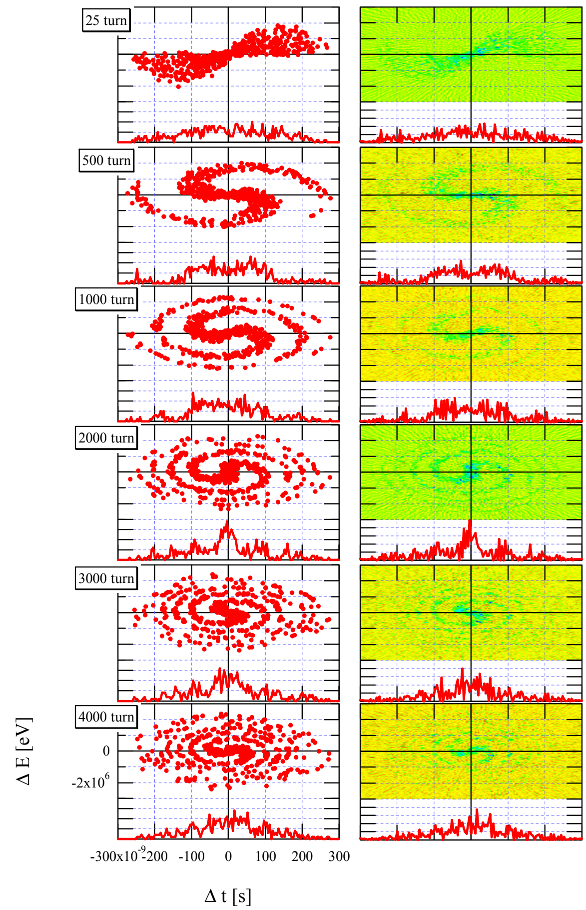

Figure 3: The comparison of the reconstructed beam distribution with the original simulated particle distribution. Left figures show the simulated particle behavior in the longitudinal phase space and right figures show the reconstructed beam distribution.

Comparison with the Tracking Simulation

In order to prove useful as an experimental tool for the RCS beam studies, we curried out the particle tracking simulation and reconstructed the longitudinal phase space distribution from the simulation results with the CBP method. The longitudinal particle behavior is simulated on condition that the number of bunch for multi-turn injection is 25turns, particle number of every bunch is 20, the bunch length is 560ns, and RF phase is 0 degree. The one-dimensional projected histogram is calculated from the simulated 2-dimansional particle distributions at every turn. And from a set of the projected histograms in one synchrotron oscillation period, the two-dimensional distributions are reconstructed with the CBP method. Fig. 3 shows the comparison of the reconstructed beam distributions with the original simulated particle distributions. These results of the comparison suggest that the reconstruction tool with the CBP method has a good reproducibility and reliability.

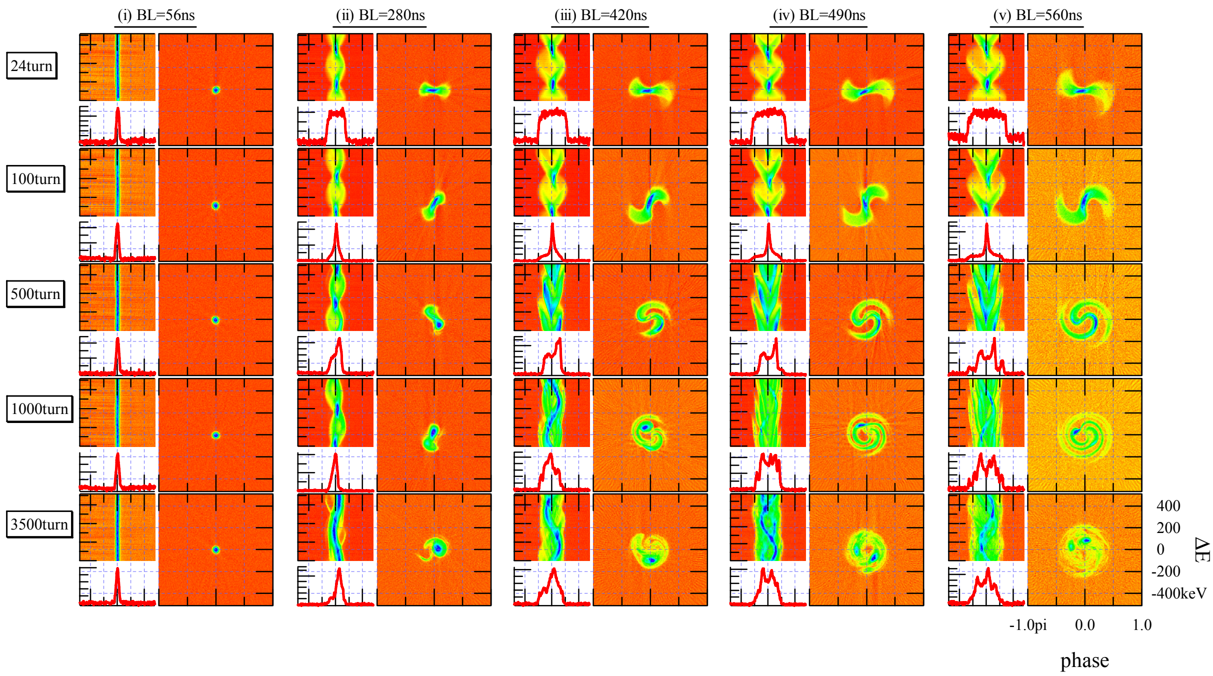

Figure 4: The experimental demonstration of the reconstructed distributions with various injection bunch length.

EXPERIMENTAL DEMONSTRATION

The WCM signals were acquired under the various conditions, in which the injection bunch length was changed from 56ns to 560ns, and the longitudinal distributions were reconstructed from the WCM signals as shown in fig. 4. The filamentation growth due to the non-linearity of the large amplitude synchrotron motion can be observed clearly, although the reconstruction tool takes no account of the non-linearity for one synchrotron oscillation period. The reconstruction tool is not affected by the non-linearity of the large amplitude synchrotron oscillation and we demonstrate that it is very useful and powerful tool to diagnose the beam dynamics.

OUTLOOK

In order to observe two-dimensional beam distributions in the longitudinal phase space, the simple reconstruction tool was developed using the LT algorithm with the CBP method for the J-PARC RCS. The comparison of the reconstructed beam distributions with the original simulated particle distribution suggested that the reconstruction tool with the CBP method has a good reproducibility and reliability. From the experimental demonstration, the filamentation growth due to the non-linearity of the large amplitude synchrotron motion can be observed clearly.

We demonstrated that the reconstruction tool with the CBP method is very useful and powerful to diagnose the longitudinal beam dynamics in the beam storage mode of the RCS. For the next step to reconstruct the longitudinal distribution in the acceleration mode, we need to improve the LT algorithm with the CBP method. The acceleration frequency sweeps from 0.938MHz to 1.67MHz in the acceleration mode of the RCS. Thus the step of the projection angle $\Delta\theta$, which appears on discretization of the equation (4), is not constant for one synchrotron oscillation period. The synchrotron frequency is calculated from the RF voltage, and the step of the projection angle $\Delta\theta$ is estimated. And we will demonstrate that the simple LT algorithm with the CBP method can be adopted in the various beam operation from both the simulation and the beam experiment, in the future plan.

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