COHERENT ELECTRON COOLING*

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Abstract

Cooling intense high-energy hadron beams remains a major challenge in modern accelerator physics. Synchrotron radiation is still too feeble, while the efficiency of two other cooling methods, stochastic and electron, falls rapidly either at high bunch intensities (i.e. stochastic of protons) or at high energies (e-cooling). In this talk a specific scheme of a unique cooling technique, Coherent Electron Cooling, will be discussed. The idea of coherent electron cooling using electron beam instabilities was suggested by Derbenev in the early 1980s, but the scheme presented in this talk, with cooling times under an hour for 7 TeV protons in the LHC, would be possible only with present-day accelerator technology. This talk will discuss the principles and the main limitations of the Coherent Electron Cooling process. The talk will describe the main system components, based on a high-gain free electron laser driven by an energy recovery linac, and will present some numerical examples for ions and protons in RHIC and the LHC and for electron-hadron options for these colliders. BNL plans a demonstration of the idea in the near future.

INTRODUCTION

Cooling intense high-energy hadron beams poses a major challenge for modern accelerator physics. The synchrotron radiation emitted from such beams is feeble; even in the Large Hadron Collider (LHC) operating with 7 TeV protons, the longitudinal damping time is about thirteen hours. None of the traditional cooling methods seems able to cool LHC-class protons beams. In this paper, a novel method of coherent electron cooling based on a high-gain free-electron laser (FEL) is presented. This technique could be critical for reaching high luminosities in hadron and electron-hadron colliders.

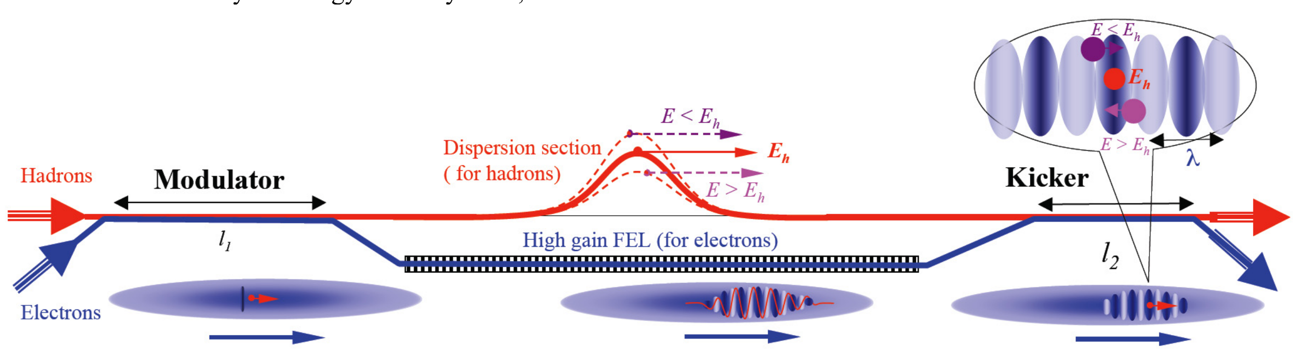

Figure 1: A general schematic of the Coherent Electron Cooler (CEC) comprising three sections: A modulator; an FEL plus a dispersion section; and, a kicker. The FEL wavelength, λ, in the figure is grossly exaggerated for visibility.

Table 1: Estimates of cooling times (in hours)

Collider	Species $^{A}El^{Z}$	Energy GeV/n	Synch. radiation	Electron cooling	CeC, 3D	FEL λ, μm
RHIC	$^{197}Au^{79}$	130	∞	~ 1	0.02	3
RHIC	$^{1}p^{1}$	325**	∞	~ 30	0.1	0.5
LHC	$^{207}Pb^{82}$	2,750	10	~ 4 104	0.2	0.07
LHC	$^{1}p^{1}$	7,000	13	∞	1	0.01

The electron-beam parameters of the energy recovery linac designed at BNL (Ne=3.2 1010 per bunch, peak current 100 A, $\varepsilon_{\perp n}$ =3mm mrad and σγ=0.33) were used for estimating e-cooling.

* *This calculation done for eRHIC having 30% higer energy of ptotons, which would be possible with upgraded DX magnets*

Hadron beams in storage rings (colliders) do not have a strong natural cooling mechanism, such as synchrotron radiation of lepton beams, to reduce their energy spreads and emittances. However, cooling hadron beams transversely and longitudinally at the energy of collision might significantly increase the luminosity of high-energy hadron colliders (LHC, Tevatron) and future electron-hadron colliders (eRHIC, ELIC and LHeC). Such improvement may be critical for discovering new physics beyond the standard model, and for attaining a better understanding of nuclear matter.

Presently, two efficient traditional cooling techniques are used for hadron beams; electron cooling [1], and stochastic cooling [2]. Unfortunately, the efficiency of electron cooling rapidly falls with increases in the beam's energy, and while the efficacy of stochastic cooling is independent of the particles' energy, it quickly declines with their number and their longitudinal density [2]. Accordingly, both methods cannot cool TeV-range proton beams with typical linear density ~ 10^{11}-10^{12} protons per nanosecond. This paper describes FEL-based mechanism that holds promise to cool high intensity proton beams at 250 GeV (RHIC) in under 10 minutes and proton beams at 7 TeV (LHC) in under an hour [3], [4].

Since the early 1980s, various possibilities have been proposed for using the electron-beam's instabilities to enhance electron cooling [5]. In this paper we present, and fully evaluate, a specific scheme to accomplish this. Our proposed coherent electron cooling (CeC) scheme is based on the electrostatic interaction between electrons and hadrons that is amplified by a high-gain FEL [3]. This CeC mechanism bears some similarities to stochastic cooling but incorporates the enormous bandwidth of the FEL-amplifier. In this paper focus is on the fundamental physics principles underlying coherent electron cooling; lengthy detailed considerations and in-depth analysis of various effects will appear elsewhere [6]. Fig. 1 is a schematic of a coherent electron cooler comprised of a modulator, a FEL-amplifier, and a kicker. The figure also depicts some aspects of coherent electron cooling.

PRINCIPLES OF CEC OPERATION

In CeC, the electron- and hadron-beams have the same velocity, v:

$$\gamma_o = \frac{E_e}{m_e c^2} = \frac{E_h}{m_h c^2} = 1/\sqrt{1-\frac{\mathrm{v}^2}{\mathrm{c}^2}} >> 1, \quad (1)$$

and co-propagate in vacuum along a straight line in the modulator and the kicker. The CeC works as follows: *In the modulator*, each hadron (with charge *Ze* and atomic number *A*) induces a density modulation in electron beam that is amplified *in the high-gain FEL*; *in the kicker,* the hadrons interact with the electric field of the electron beam that they have induced, and receive energy kicks toward their central energy. The process reduces the hadrons' energy spread, i.e., cools the hadron beam.

The details of this process are as follows: The co-moving frame (c.m.) of reference, wherein the electron- and hadron- beams are at rest, is the most natural one for describing the processes *in the modulator*. In the c.m. frame, the motion of the electrons and hadrons is non-relativistic, so that the process can be described from first principles. Let's note that the velocity spreads of the electrons and hadrons are highly anisotropic with $\sigma_{v_{x,y}} >> \sigma_{v_z}$, where *z* is direction of beams' propagation [7]. In the modulator, the positively charged hadron attracts electrons, thereby creating a cloud of them.

When a hadron moves with constant non-zero velocity $\vec{\mathrm{v}}_\mathrm{h} = \hat{x}\mathrm{v}_x + \hat{y}\mathrm{v}_y + \hat{z}\cdot\mathrm{v}$ [8], the electron cloud follows it with some lag $\Delta\zeta \propto \mathrm{v}_z/\omega_p$. The typical dimensions of this disk-shaped electron cloud (a pancake) are given by the dynamics Debye radii:

$$\mathrm{R}_{D_\alpha} \propto \left(|\mathrm{v}_\alpha| + \sigma_{v_\alpha}\right)/\omega_p; \quad \alpha = x,y,z;$$

where $\omega_p = \sqrt{4\pi n_e e^2/\gamma_o m_e}$ is the plasma frequency of electron beam in the c.m. frame, n_e is the lab-frame electron density, and $-e$ and m_e , respectively, are the charge and the mass of the electron. It can be show analytically (for an infinite plasma [9, 10]) that the total charge induced by the hadron in electron plasma is given by the simple formula:

$$q(\varphi_1) = -Ze\cdot(1-\cos\varphi_1), \quad (2)$$

where $\varphi_1 = \omega_p t$ is the phase-advance of plasma oscillation in the modulator. Eq. (2) holds for a simple case of a cold plasma [11], and a warm anisotropic plasma [12] for both resting and moving hadrons. Direct computer simulations support this result [4,13]. For a given length of a modulator, l_m, the phase advance $\varphi_1 = (\omega_p l_m)/(\gamma_o \mathrm{v})$ is inversely proportional to the beam's energy; in a very high-energy collider (like LHC) an additional buncher may be required to complete the cloud's formation (see [3]).

The electron beam emerging from the *modulator* carries information about individual hadrons imprinted in pancake-type density distortions with a total induced charge of about that of the hadron (per pancake). In the lab-frame, the beam's transverse dimensions remain the same as in the c.m. frame, while its longitudinal size contracts by the Lorentz factor, $\gamma_o >> 1$, making the pancakes very thin.

Following the modulator, the electrons pass through a wiggler – *a high-gain FEL* – wherein the induced density modulation is amplified so that it becomes a packet of alternating high- and low-density "pancakes". The period of this modulation is that of the FEL wavelength:

$$\lambda = \lambda_w\left(1 + \langle a_w^2 \rangle\right)/2\gamma_o^{\ 2}, \quad (3)$$

where λ_w is the wiggler's period, and $\vec{a}_w = e\vec{A}_w/mc^2$ is the dimensionless vector potential of the wiggler. If the pancakes are significantly shorter than the FEL wavelength (i.e., $\mathrm{R}_{D_z}/\gamma_o << \lambda$), they will be amplified similar to the shot noise (δ-functions in z-direction), viz., the case well known in the basic theory of FELs [14]. We are exploring the exact FEL response (i.e., its Green function) using 3D FEL codes [4]. Here focus is on the most important longitudinal part of the FEL response, a wave-packet modulated with the wavelength λ:

$$G(\zeta) = G_o \mathrm{Re}\left(K(\zeta)\cdot e^{ik\zeta}\right); \zeta = \mathrm{z} - \mathrm{v}t; \mathrm{k} = \frac{2\pi}{\lambda}, \quad (4)$$

where G_o is the maximum FEL gain, and $K(\zeta)$ is a complex normalized envelope of the gain function (4) with $|K|_{\max} = 1$. Let's denote location of the $|K(\zeta)|$ maximum as ζ_{peak}. Fig. 2 shows an example of the gain envelope. The origin, ζ=0, corresponds to the location of the initial distortion. Each hadron induces an FEL-amplified wave-packet of longitudinal density modulation in the electron beam:

$$\rho(\zeta) \cong \mathrm{k}q(\varphi_p) \cdot G(\zeta) \equiv \mathrm{k}G_o q \cdot \mathrm{Re}\left(K(\zeta) \cdot e^{ik\zeta}\right).$$

The modulation results in the longitudinal electric field

$$\mathbf{E}(\zeta) \cong X \cdot \mathbf{E}_o \cdot \mathrm{Im}\left(K(\zeta) \cdot e^{ik\zeta}\right); \mathbf{E}_o = 4\pi G_o e / S, \quad (5)$$

where S is the beam's transverse area, and $X = q/e \cong Z(1-\cos\varphi_1) \sim Z$. For a round electron beam, which is of interest here, S can be expressed via the optical β-function of the lattice and, $\varepsilon_{\perp n}$, the normalized transverse emittance of the electron beam: $S = 2\pi\beta\varepsilon_{\perp n}/\gamma_o$, yielding $\mathbf{E}_o = 2G_o e\gamma_o/\beta\varepsilon_{\perp n}$.

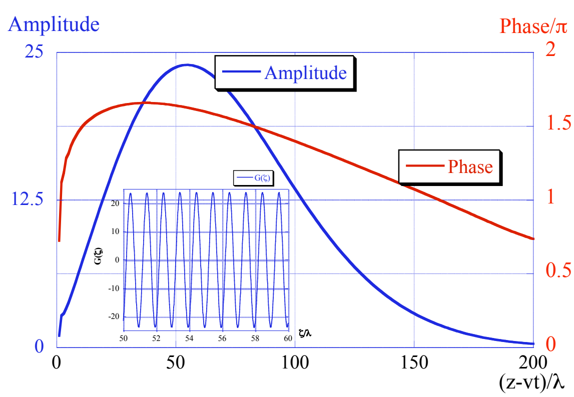

Figure 2: The amplitude (blue line) and the phase (red line, in the units of π) of the FEL gain envelope after 7.5 gain-lengths (300 period). Total slippage in the FEL is 300λ, λ=0.5 μm. A clip shows the central part of the full gain function for the range of ζ={50λ, 60λ}.

One should note that the shot noise of electron beam also is amplified in the FEL and each electron will generate wave-packet given by eq.(5) with *X=1*. For a FEL operating in a linear regime, which can be achieved by properly selecting the length and gain of the FEL [18], the density modulation in the electron beam and the resulting electric field is a direct linear superposition of the wave-packets induced by all hadrons and electrons:

$$\mathbf{E}_{total}(\zeta) = \mathrm{E_o} \cdot \mathrm{Im}\, X \cdot \left(\sum_{i,hadrons} K(\zeta - \zeta_i) e^{ik(\zeta-\zeta_i)} - \sum_{j,electrons} K(\zeta - \zeta_j) e^{ik(\zeta-\zeta_j)} \right).$$

In the kicker, both beams co-propagate again, and the longitudinal electric field inside the electron beam affects the hadrons' energy. The kick of the hadron's energy is [4]

$$\Delta E_{h,i} = l_k \cdot Ze \cdot \mathbf{E}_{total}(\zeta_i + \Delta\zeta_i) \cdot f(\varphi_2), \quad (6)$$

where l_k is the length of the kicker, $f(\varphi) = \sin\varphi/\varphi \sim 1$, $\varphi_2 = (\omega_p l_k)/(\gamma_o \mathrm{v}) \leq 1$ is the phase-advance of plasma oscillation in the kicker, and $\Delta\zeta_i$ is the delay of the hadron with respect to the corresponding distortion induced by it in the modulator (see details below).

To take into account the "cross-talking" of particles, we follow the method developed in a traditional stochastic cooling [2]. We also use a well-established fact for storage ring FELs [15], i.e., that any correlations between hadrons at the scale of the FEL wavelength are washed away after one turn in the collider $\left\langle K(\zeta_m - \zeta_i + \Delta\zeta_m) \cdot e^{ik(\zeta_m - \zeta_i + \Delta\zeta_m)} \right\rangle = 0$ for any $m \neq i$. It means that only a self-induced field ($m = i$, see eq. below) can accumulate on average into damping (or antidumping), while fields generated by other particles create only random diffusion. The resulting evolution equation (as a function of the turn number, n, with $' \equiv d/dn$) of the RMS hadron beam's energy spread ($\delta = (E - E_h)/E_h$) is:

$$\begin{gathered} \left\langle \delta^2 \right\rangle' = -2\xi \left\langle \delta^2 \right\rangle + D; \\ \xi = -g\left\langle \delta_i \, \mathrm{Im}\left(K(\Delta\zeta_i) e^{ik\Delta\zeta_i}\right)\right\rangle / \left\langle \delta^2 \right\rangle;\ D = g^2 N_{eff}/2; \\ g = G_o \frac{Z^2}{A} \frac{r_p}{\varepsilon_{\perp n}} \left\{ 2 f(\varphi_2)(1-\cos\varphi_1) \frac{l_2}{\beta} \cdot \right\}, \end{gathered} \quad (7)$$

where $r_p = m_p c^2 / e^2$ is the classical proton radius, and N_{eff} is the effective number of hadrons and electrons within one FEL correlation length $\Lambda_k = \iint |K(\mathrm{z}-\zeta)|^2 d\zeta$:

$$N_{eff} \cong N_h \frac{\Lambda_k}{\sqrt{4\pi}\sigma_{z,h}} + \frac{N_e}{X^2} \frac{\Lambda_k}{\sqrt{4\pi}\sigma_{z,e}}. \quad (8)$$

with $\sigma_{z,e}$ and $\sigma_{z,h}$, respectively, being the RMS lengths of electron- and hadron-bunches. This "cross-talking" of the particles imposes a well-known limit [2] $\xi_{\max} \propto N^{-1}_{eff}$. Fortunately, the bandwidth of FELs under consideration in this paper (Δf ~ 10^{13}-10^{15} Hz) is so large that this limitation does not play any practical role.

DETAILS OF CEC OPERATION

One of most important terms in eq. (6) is the hadron's delay:

$$\Delta\zeta_i = \Delta\zeta_{\mathrm{mod}}(\vec{\mathrm{v}}_{\mathrm{hi}}) - \Delta\zeta_{FEL} + \mathrm{v} \cdot \mathrm{T_{hi}} - \mathrm{L}_e, \quad (9)$$

where $\Delta\zeta_{\text{mod}}$ is the distance (in the lab frame) between the electron cloud and the hadron at the modulator's exit, $\Delta\zeta_{FEL} = N_w(\lambda - (1 - \text{v/c})\lambda_w)$ is the slippage of electrons caused by the curved trajectory in the FEL wiggler, L_e is the total length of electron-beam's trajectory, and T_{hi} is the travel time of the *i*-th hadron between the modulator and the kicker. For an ideal hadron traveling along design trajectory with the ideal energy ($\delta = 0$) $\Delta\zeta_m$=0,the $\Delta\zeta_o$ delay can be adjusted simply by changing the length of the trajectory in the dispersion section. Let's choose $\Delta\zeta_o \cong \zeta_{peak}$ for the hadron to arrive at the kicker section near the peak of the corresponding electron-beam modulation $K(\Delta\zeta_i)$; furthermore, (adjusting within $\{\Delta\zeta_o - \zeta_{peak}\} \in \{\lambda/2, \lambda/2\}$) we will select $Arg\left(K(\Delta\zeta_o)\cdot e^{ik\Delta\zeta_o}\right) = 0$, i.e., the hadron arrives at the crest of the corresponding wave-packet. The smooth dependence of $|K|$ allows both criteria to be satisfied: $K(\Delta\zeta_o)e^{ik\Delta\zeta_o} \cong 1$.

The delay (9) can be expanded for non-ideal hadron as follows:

$$\Delta\zeta_i - \Delta\zeta_o = D_l\cdot\delta - R_{26}x_o + R_{16}x'_0 - R_{46}y_o + R_{36}y'_o + O(\alpha^2),$$

where $D_l = R_{56} + \Delta\zeta_{\text{mod}}/\delta$ is the longitudinal dispersion of the hadron transport, including the lag in the modulator, and R_{ik}; $X(s) = R(s_o|s)\cdot X(s_o)$ are elements of the symplectic transport matrix from the modulator to the kicker for 6-vector $X^T = \{x, x', y, y', -ct, \delta\}$ (the term $O(\alpha^2)$ stands for higher order terms in the time-of-flight dependence). Let's note such dependence on transverse coordinates and angles is the direct consequence of the symplecticity of the transport matrix, and, for an achromatic lattice (i.e., $R_{i6} = 0,\ i = 1,2,3,4$) these linear terms vanish. Then, one has the following expression for the CeC decrement in the beam

$$\zeta \approx -g\langle\delta\cdot\sin kD_l\delta\rangle/\langle\delta^2\rangle, \qquad (10)$$

as well as the damping term for an individual hadron:

$$\delta'_{CeC} \approx -g\cdot\sin kD_l\delta, \qquad (11)$$

that clearly indicates the importance of the sign and the value of the longitudinal dispersion (let's use natural D_l>0 further in the paper). A hadron with higher energy, $0 < \delta < \pi/kD_l$, arrives at the kicker ahead of its respective clump of high density in the electron beam, and is pulled back (decelerated) by the beam's coherent field. Similarly, a hadron with lower energy, $-\pi/kD < \delta <_l 0$, falls behind and is dragged forward (accelerated) by its respective clump of high electron density.

The energy deviations are undergoing synchrotron oscillations $\delta = a\cdot\sin(\Omega_s n + \psi_s)$ and averaging eq. (11) yields

$$a' \approx -g\cdot J_1(kD_l a)$$

where J_1 is the first-order Bessel function of first kind. For small amplitudes, $a << 1/kD_l$, this entails exponential damping of the longitudinal oscillations [16]:

$$a = a_o\exp[-n\cdot gkD_l/2].$$

For larger amplitudes, damping and anti-damping ranges are separated by the roots of J_1: $kD_l a_1 \cong 3.8317$, $kD_l a_2 \cong 7.0156$, and so on. Thus, the natural value of longitudinal dispersion $kD_l \sim 1/\sigma_\delta$ provides a central energy-cooling range of ± 3.83 σ_δ. In practice, electron bunches usually are much shorter that hadron bunches and "the bunch average" CeC decrement has an additional multiplier of σ_{ze}/σ_{zh} [17]. Assuming that $kD_l\sigma_\delta = 1$, $\beta \cong f(\varphi_2)\cdot l_2$ and $\varphi_1 = \pi$, one obtains an important estimate for the decrement of coherent-electron cooling:

$$\xi_{CeC\ \ bunch} \leq 2\frac{G_o\sigma_{ze}}{\sigma_\delta\sigma_{zh}}\frac{Z^2}{A}\frac{r_p}{\varepsilon_{\perp n}}. \qquad (12)$$

with the FEL gain limited to $G_{o\max} \propto 10^3$ for protons, and to $G_{o\max} \propto 10^2$ for heavy ions to avoid saturation of the FEL [18]. It is particularly striking that the CeC decrement does not directly depend upon the hadron beam's energy. The presence of the hadron's longitudinal emittance, $\sigma_\delta\sigma_{zh}$, in the denominator of eq.(12) means that cooling enhances further this cooling decrement.

Similar to redistributing the decrements of synchrotron-radiation damping [16], one can redistribute the longitudinal damping of the CeC process to transverse directions:

$$\xi_l + \xi_{t1} + \xi_{t2} = \xi_{CeC},$$

where $_{t1}$ and $_{t2}$ stand for two transverse modes of betatron motion (for uncoupled motion, it is simply *x* and *y*). The easiest way to couple to the transverse motion (for example, *x*) is to install a chromatic chicane for the electron beam after the FEL, to tilt the slices of density modulation (Fig.3), and to make the electric field dependent on *x*:

$$\Delta E_{h,i} = \Delta E_o\left(k\left(D_l\delta + R^e{}_{26}x\right)\right).$$

In combination with non-zero transverse dispersion ($\eta_x \neq 0$) in the location of the kicker, this scheme couples the longitudinal cooling with the transverse motion:

$$\xi_\perp = \xi_{CeC\,o}\frac{\eta_x}{D_l}R^e{}_{26}.$$

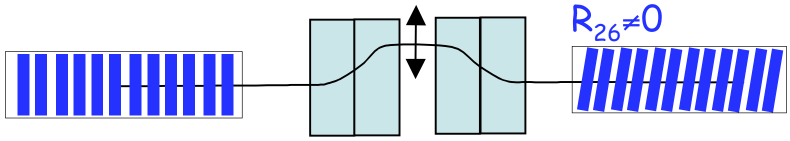

Figure 3: A chromatic chicane tilts the wave-fronts of charge- density modulation in an electron beam, making the electric field dependent on the horizontal position.

Proper coupling between the horizontal and vertical motions, which one can controlled by SQ-quadrupoles, ensures further redistribution of between the two betatron modes. Thus, the CeC can cool all three degrees of freedom of hadron- beam's phase space with about one third of its total decrement per degree of freedom.

DISCUSSION

Naturally, there is multitude of second-order effects and dependence on the hadron- and electron-beams' parameters, which may reduce effectiveness of CeC process. We considered a variety of the effects and parameters [3,4,6]; none negated the CeC concept. A sub-set of similar effects was considered by authors of another exciting cooling concept, namely Optical Stochastic Cooling (OSC) [19.20]. It is unfortunate that the short length of this article does not allow us to compare effectiveness of the CeC and OSC. Here we would like to say only that in contrast with OSC, where protons interacts with TEM field via radiation in a wiggler, the CeC utilizes electrostatic field of the electron beam. Therefore, the CeC does not suffer from weakness for the radiation from a proton, i.e. a well known fact that the radiation of a charged particle in a wiggler is inversely proportional to the square of its mass. Detailed comparison of CeC with OSC and other cooling method will be published elsewhere [6].

We conclude that that our proposed FEL-based coherent electron cooling has the potential to become a revolutionary cooling technique for high-energy hadron colliders, and consequently, will increase their productivity.

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