Single-knob beam line for transverse emittance partitioning

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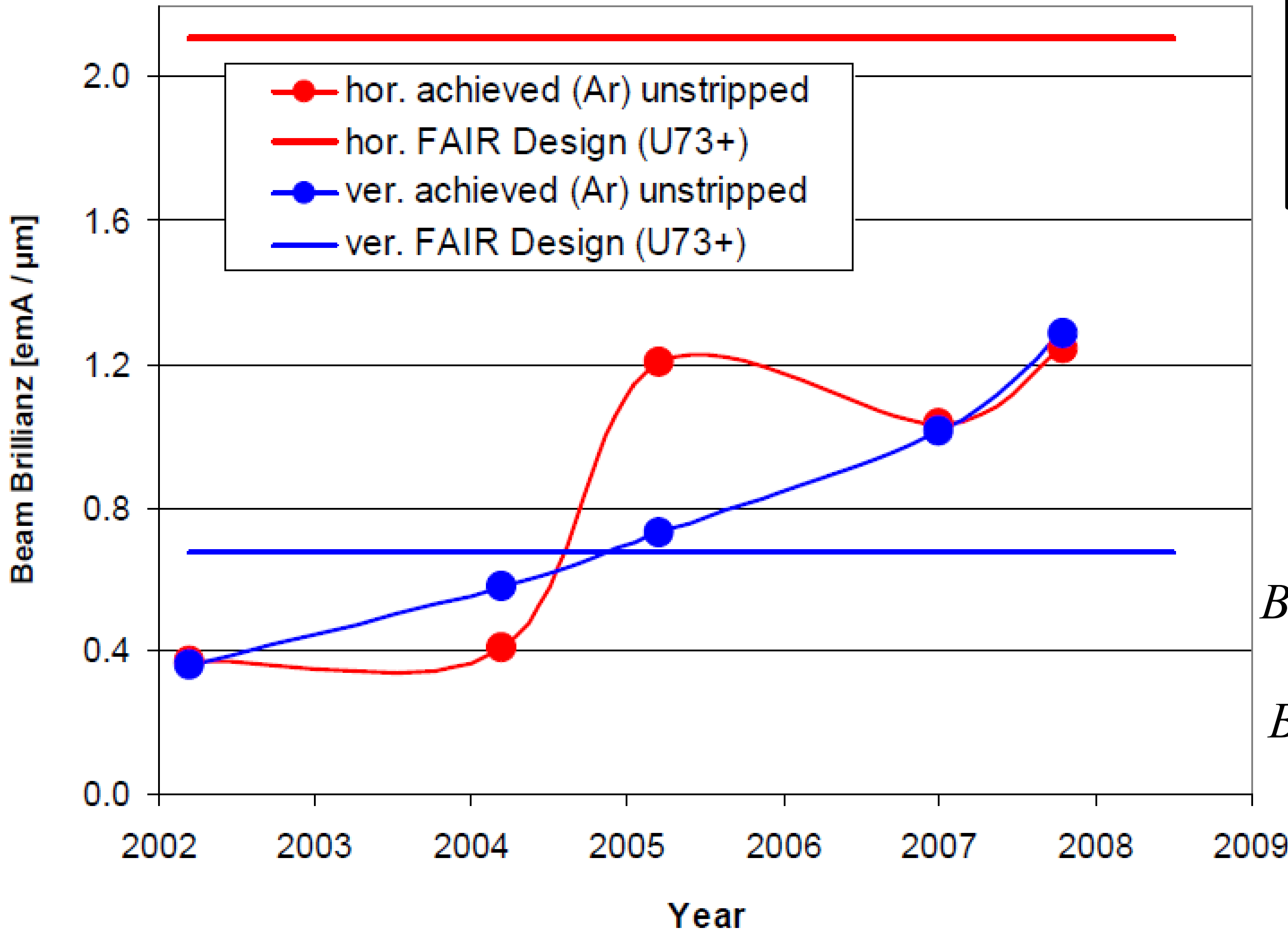
Outline

- Motivation
- Basic terms
- Emittance transfer experiment (Emtex)
- Decoupling and matching capability

FAIR Injector Linac Design Requirements



$$B_{x,y} = \frac{\frac{q}{A} \times I_{beam}}{\varepsilon_{x,y}}$$

B_x is not enough

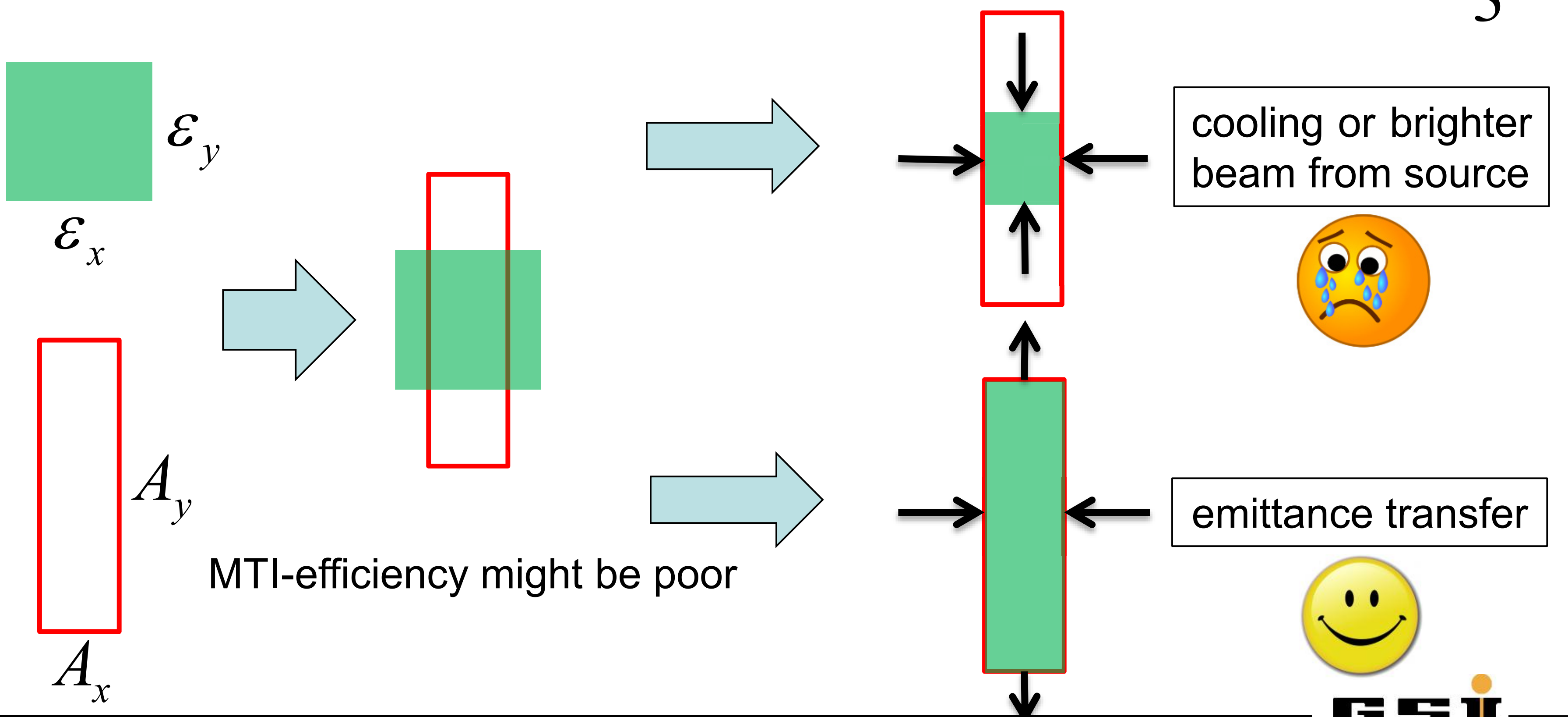
B_y is enough

$$\varepsilon_x \approx \varepsilon_y$$

Injection into Synchrotron: Acceptance versus Emittance

- Round beam is provided by the injector $\varepsilon_x \approx \varepsilon_y$

- Flat beam is required for synchrotron $A_x \approx \frac{1}{3} A_y$



Second Beam Moments & Emittancs

$$C = \begin{bmatrix} \langle xx \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle x'x \rangle & \langle x'x' \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle yx \rangle & \langle yx' \rangle & \langle yy \rangle & \langle yy' \rangle \\ \langle y'x \rangle & \langle y'x' \rangle & \langle y'y \rangle & \langle y'y' \rangle \end{bmatrix}.$$

If at least one of the elements of the off-diagonal sub-matrix is non-zero, the beam is x-y coupled.

$$\varepsilon_1 = \frac{1}{2}\sqrt{-tr[(CJ)^2] - \sqrt{tr^2[(CJ)^2] - 16|C|}},$$
$$\varepsilon_2 = \frac{1}{2}\sqrt{-tr[(CJ)^2] + \sqrt{tr^2[(CJ)]^2 - 16|C|}}.$$

Transverse eigen-emittances are equal to the transverse rms emittances when inter-plane correlations are zero.

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad M^T JM = J.$$

Eigen-emittances are invariant under symplectic transformations.

Beam through thin Solenoid with Foil in its Center

$$\begin{bmatrix} \varepsilon\beta & 0 & 0 & 0 \\ 0 & \frac{\varepsilon}{\beta} & 0 & 0 \\ 0 & 0 & \varepsilon\beta & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\beta} \end{bmatrix}$$

$$R_{in} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k_{in} & 0 \\ 0 & 0 & 1 & 0 \\ -k_{in} & 0 & 0 & 1 \end{bmatrix} \qquad R_{out} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -k_{out} & 0 \\ 0 & 0 & 1 & 0 \\ k_{out} & 0 & 0 & 1 \end{bmatrix}$$

$$k = k_{in} \neq k_{out}$$

$$k_{in} = k, k_{out} = \delta q k$$

$$\delta q = \frac{(B\rho)_{in}}{(B\rho)_{out}}, a = \delta q - 1$$

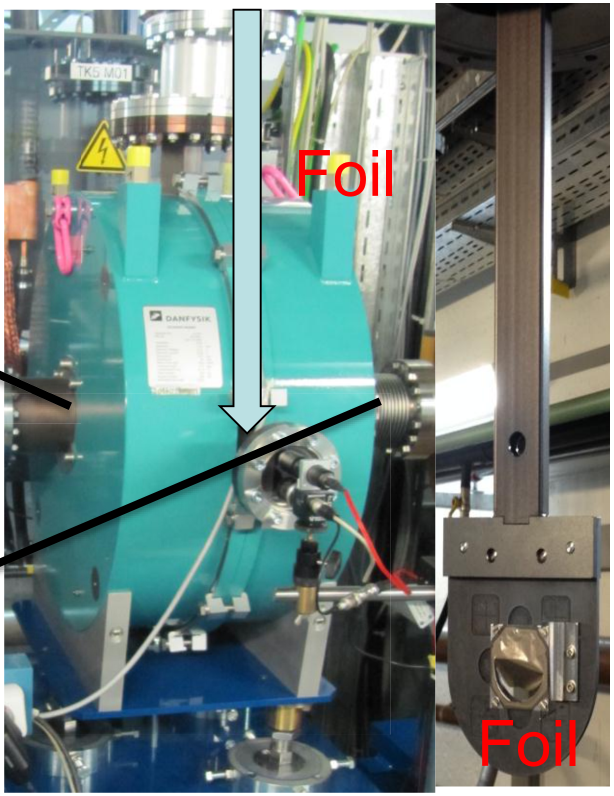
fringe field matrices

$$\begin{bmatrix} \varepsilon\beta & 0 & 0 & -k\varepsilon\beta \\ 0 & \frac{\varepsilon}{\beta} + k^2\varepsilon\beta & k\varepsilon\beta & 0 \\ 0 & k\varepsilon\beta & \varepsilon\beta & 0 \\ -k\varepsilon\beta & 0 & 0 & \frac{\varepsilon}{\beta} + k^2\varepsilon\beta \end{bmatrix}$$

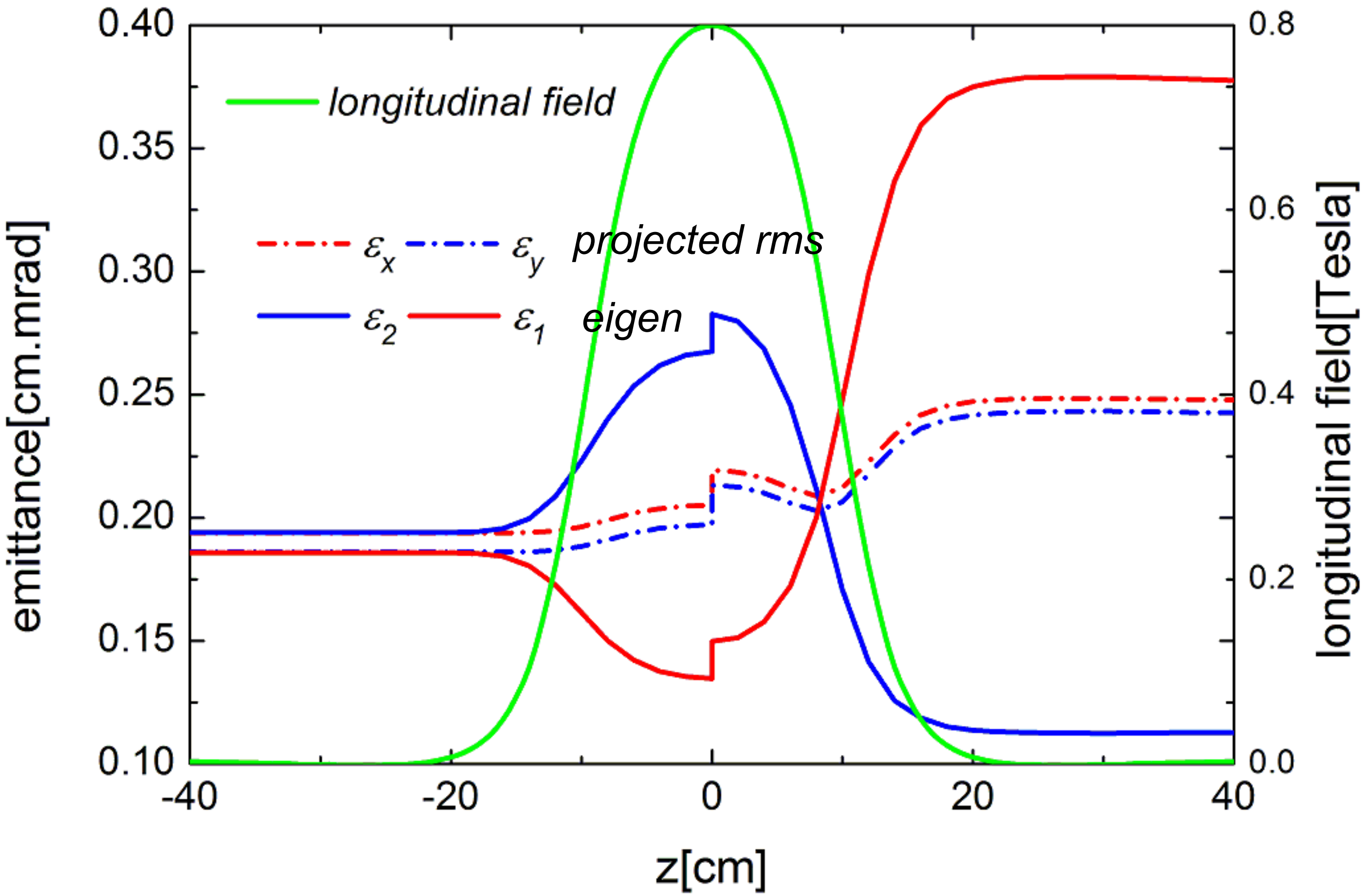
after entrance fringe field

$$\begin{bmatrix} \varepsilon\beta & 0 & 0 & ak\varepsilon\beta \\ 0 & \frac{\varepsilon}{\beta} + a^2k^2\varepsilon\beta + \Delta\varphi^2 & -ak\varepsilon\beta & 0 \\ 0 & -ak\varepsilon\beta & \varepsilon\beta & 0 \\ ak\varepsilon\beta & 0 & 0 & \frac{\varepsilon}{\beta} + a^2k^2\varepsilon\beta + \Delta\varphi^2 \end{bmatrix}$$

after exit fringe field



To Create Inter-Plane Correlations & Change Eigen-Emittances



$$t = \frac{\varepsilon_x \varepsilon_y}{\varepsilon_1 \varepsilon_2} - 1 \geq 0$$

Stripping is non-symplectic action

To quantify the inter-plane coupling

Then Remove Inter-Plane Correlations & Preserve Eigen-emittances

$$R_{triplet} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \mu \\ 0 & 0 & -\frac{1}{\mu} & 0 \end{bmatrix}$$

Quad triplet → $\overline{R} = R_r R_{triplet} R_r^T$ Skew quad triplet

See K.J. Kim PRST-AB 6, 104002 (2003)

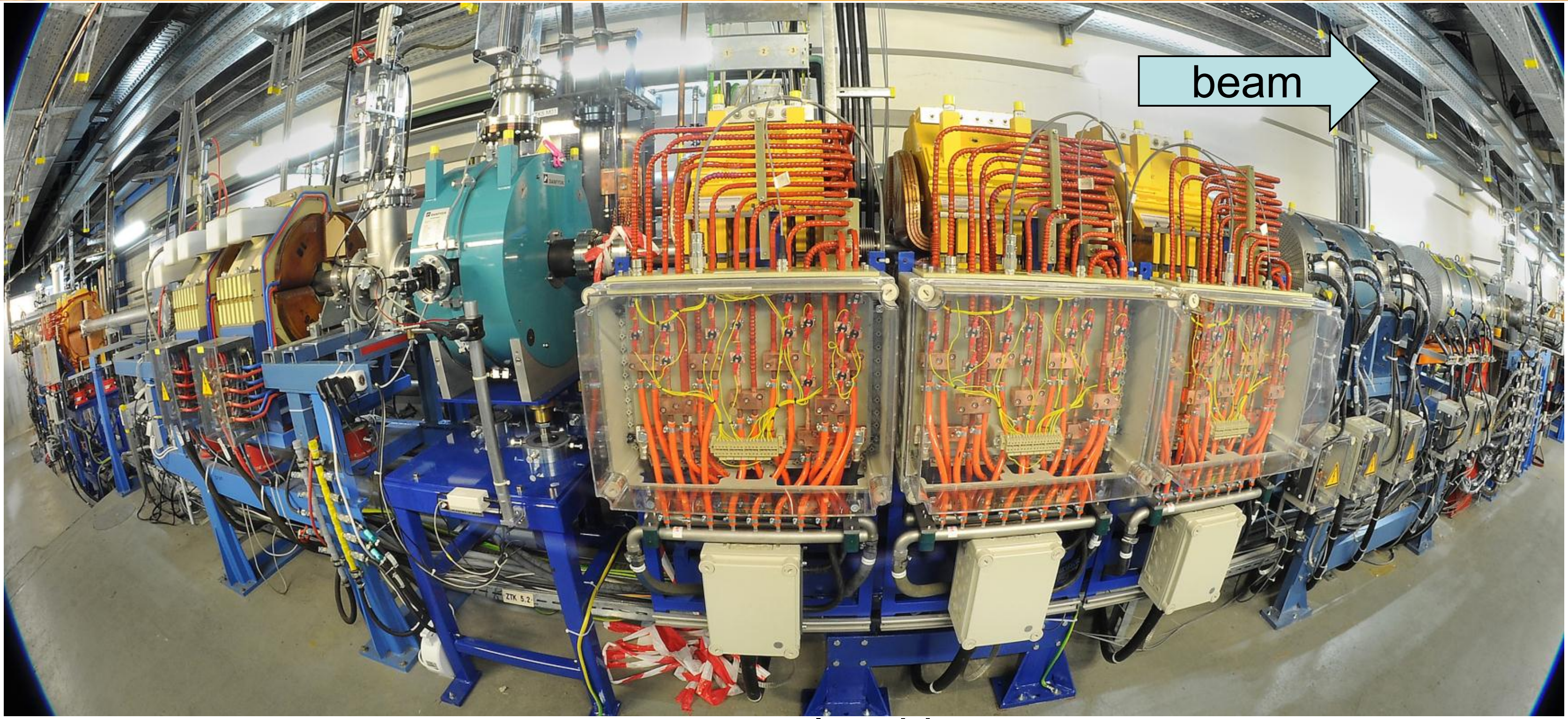
$$\mu = \beta_n = \sqrt{\frac{\varepsilon\beta}{\frac{\varepsilon}{\beta} + a^2 k^2 \varepsilon\beta + \Delta\varphi^2}}$$

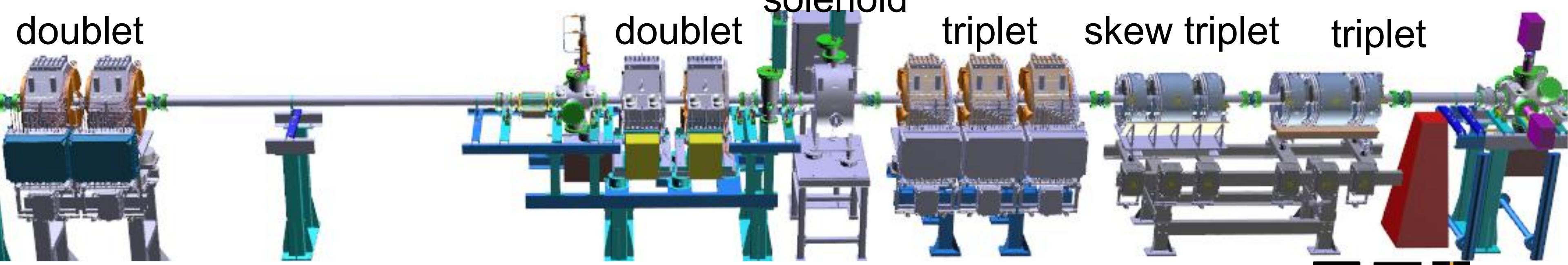
Correlations are removed!!!

angular scattering in foil

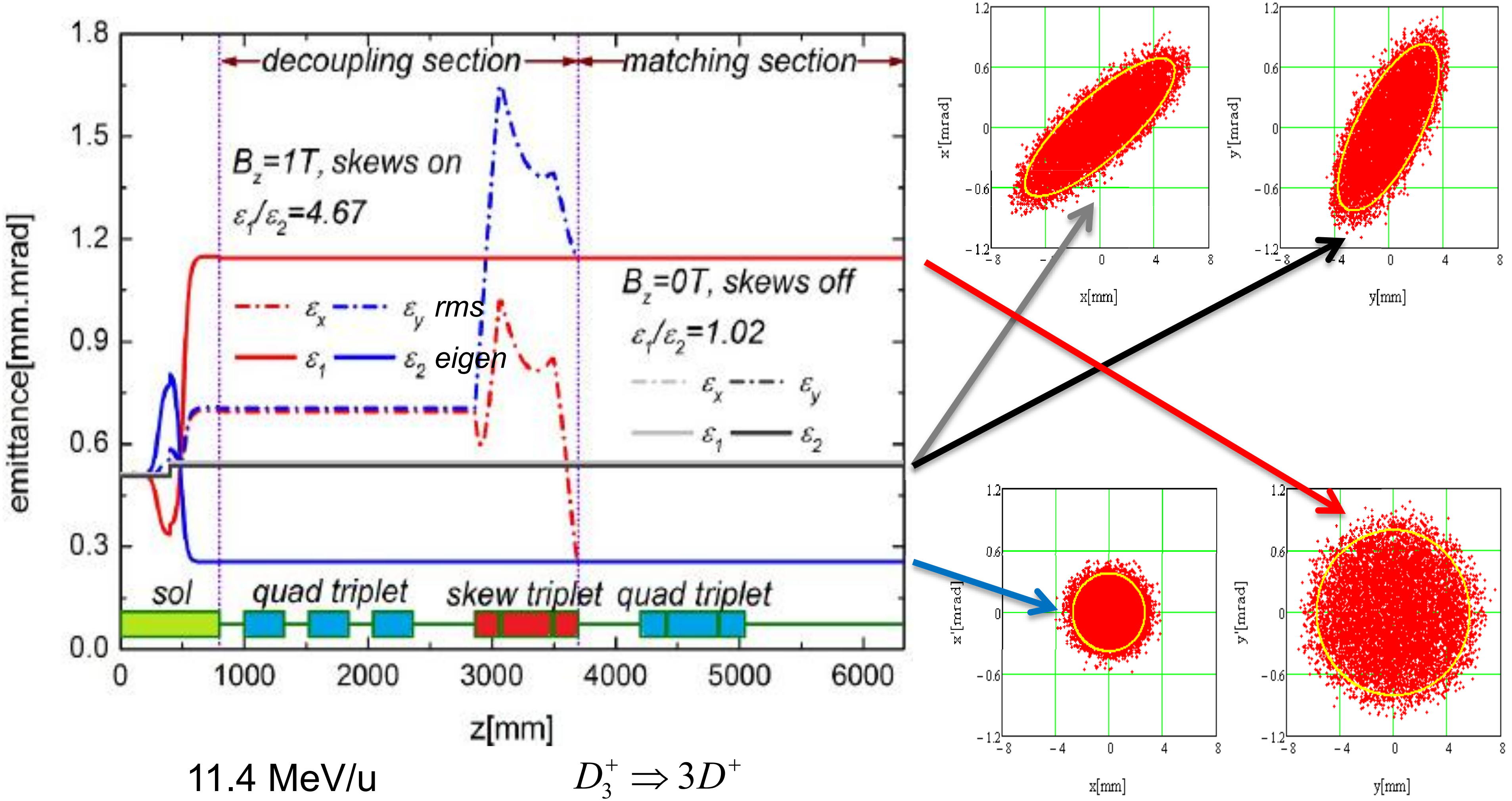
Symplectic action, the projected rms emittances are minimized to the eigen-emittance

Emittance Transfer Experiment at GSI





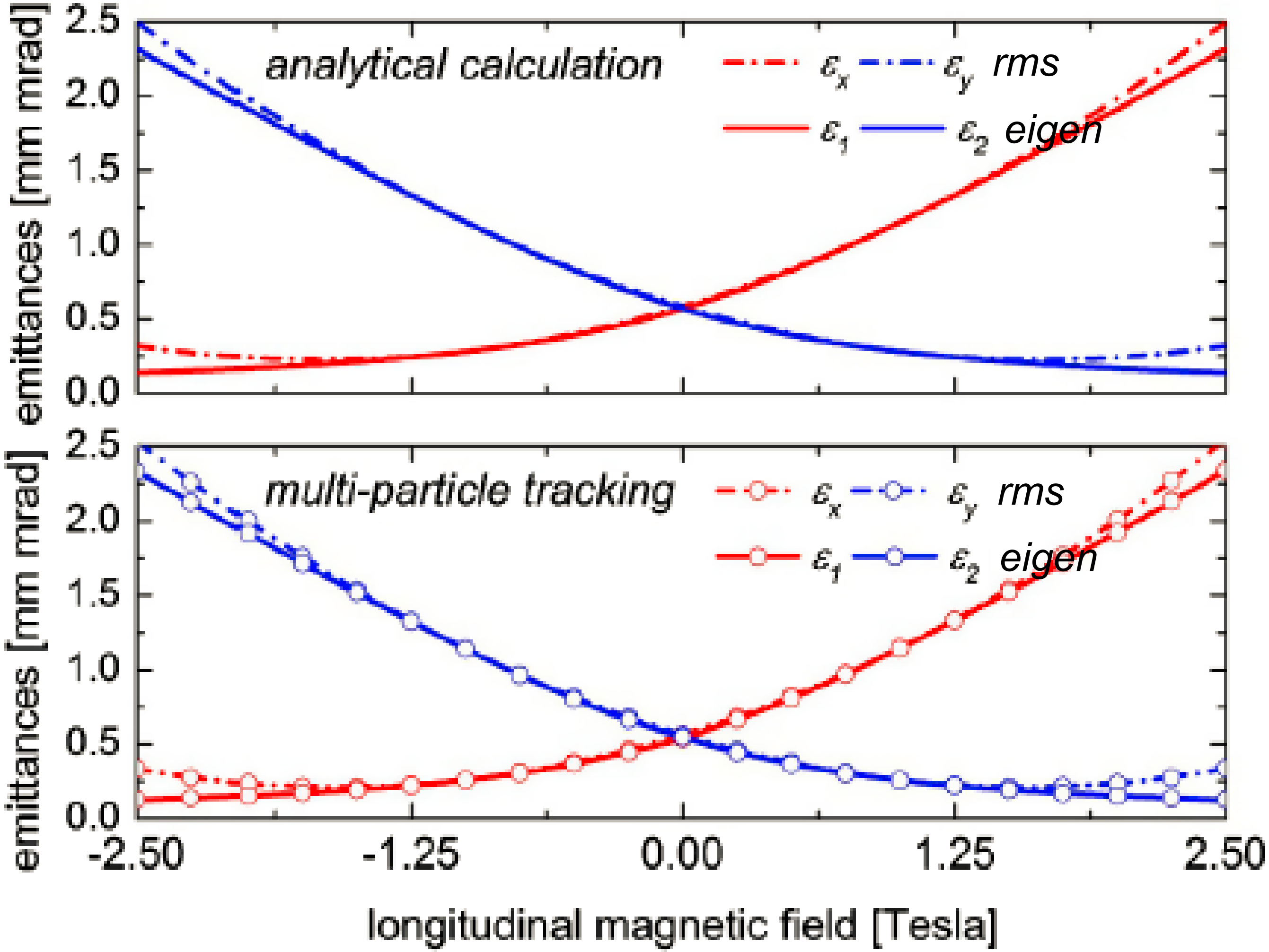
Beam Dynamics Simulations



11.4 MeV/u

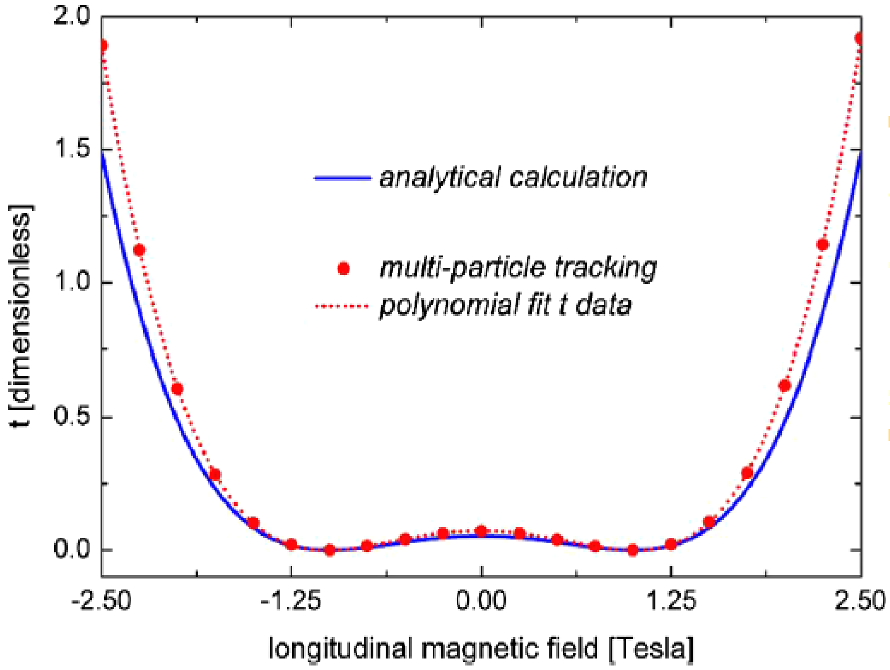
$D_3^+ \Rightarrow 3D^+$

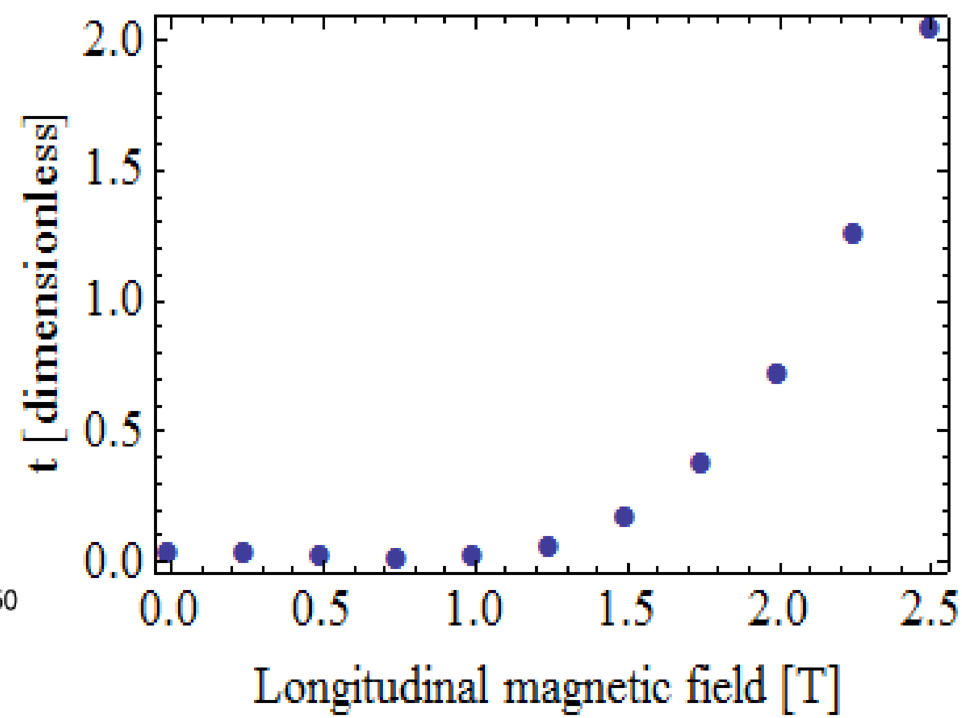
Decoupling Capability



Gradients of quads (normal and skews) are optimized for B=1.0T

Independent Confirmation



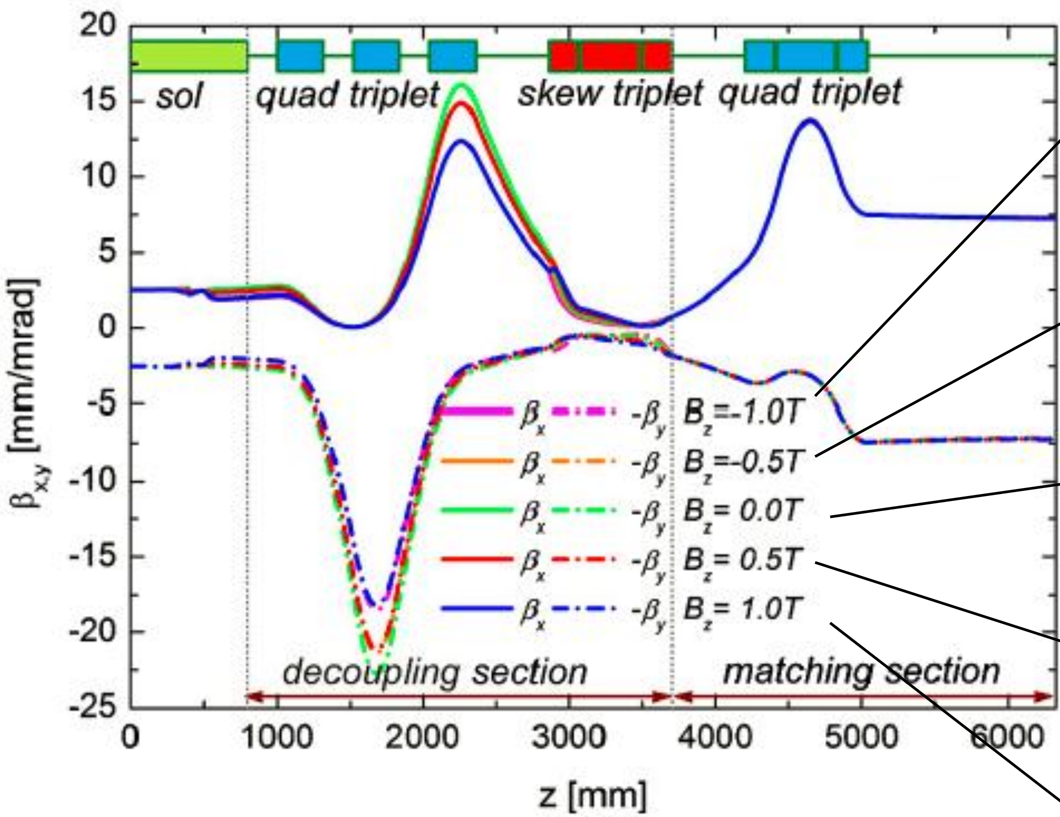


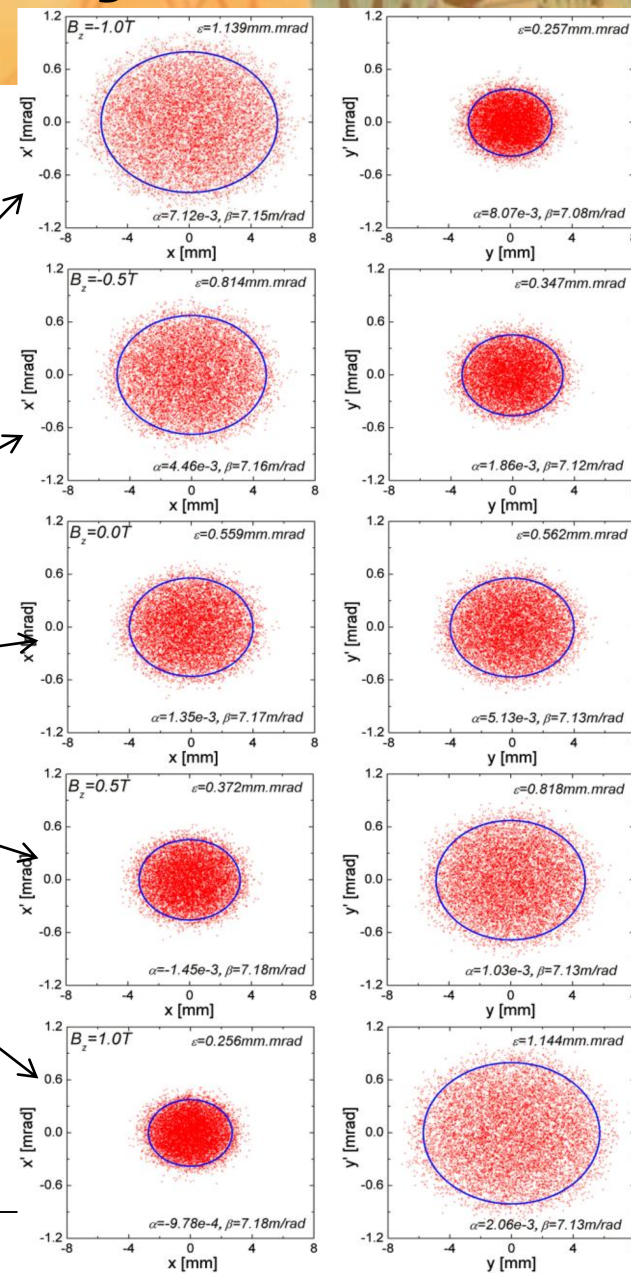
graph from C. Xiao, PRST-AB 2013

$$t = \frac{\varepsilon_x \varepsilon_y}{\varepsilon_1 \varepsilon_2} - 1 \geq 0$$

applying innovative 4d-envelope model first published in PRL 2013
H. Qin, Princeton University, USA
M. Chung, UIST, Korea

Matching Capability





- Only the longitudinal magnetic field is varied
- all (skew) quads remain constant

See C.Xiao, PRST-AB 16, 044201 (2013)

Properties of the Emtex

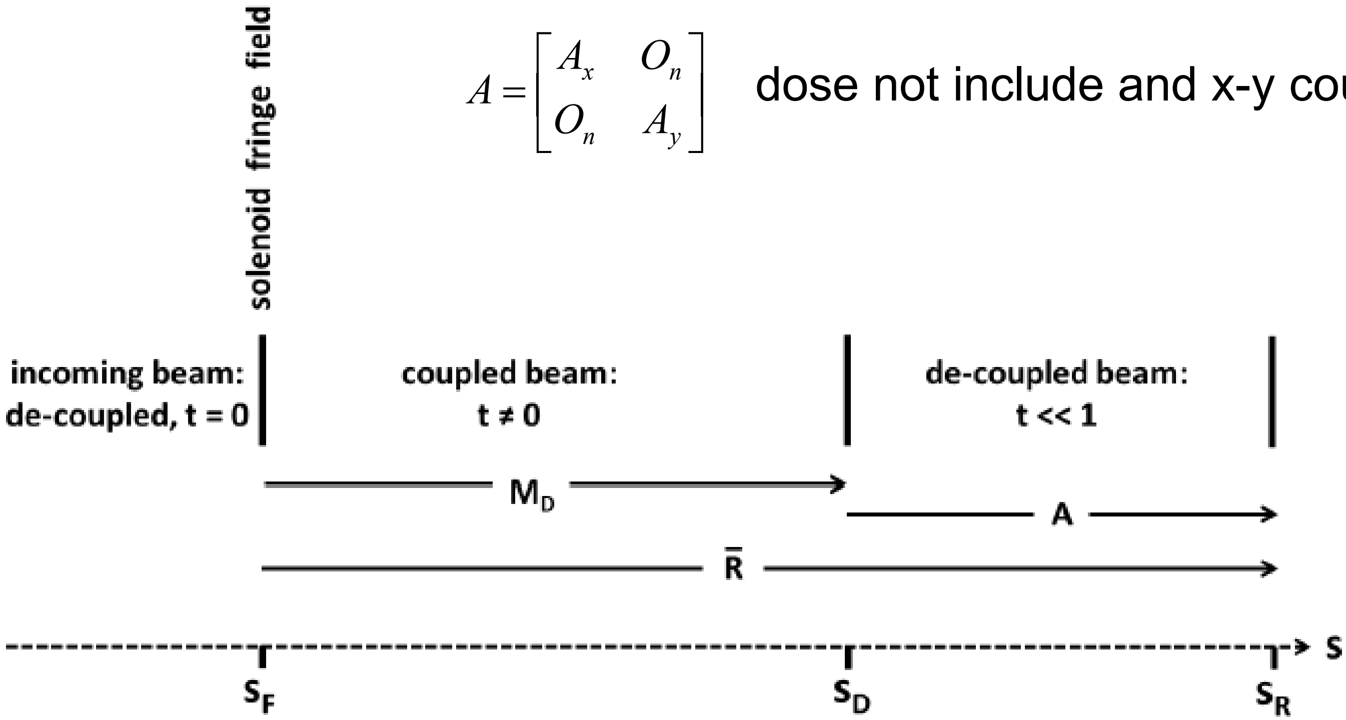
- k_0 is the assumed fringe strength, k_1 is the fringe strength actually applied for the coupling. t << 1 holds over a wide range of k_1.

- The exit Twiss parameters (beta & alpha) do not depend on the actual fringe strength k_1.

- The only quantity considerably changed through the fringe strength is the transverse rms emittance partitioning.

Decoupling in General Case

$$A = \begin{bmatrix} A_x & O_n \\ O_n & A_y \end{bmatrix}$$ dose not include and x-y coupling element



$\overline{R}$ has the properties

$M_D = A^{-1}\overline{R}$

A^{-1} does not change them since it is non-coupling

→ the properties are also intrinsic properties of M_D

See L.Groening, http://arxiv.org/abs/1403.6962v1

My special gratitude to my colleagues at GSI

Thank you for attention!