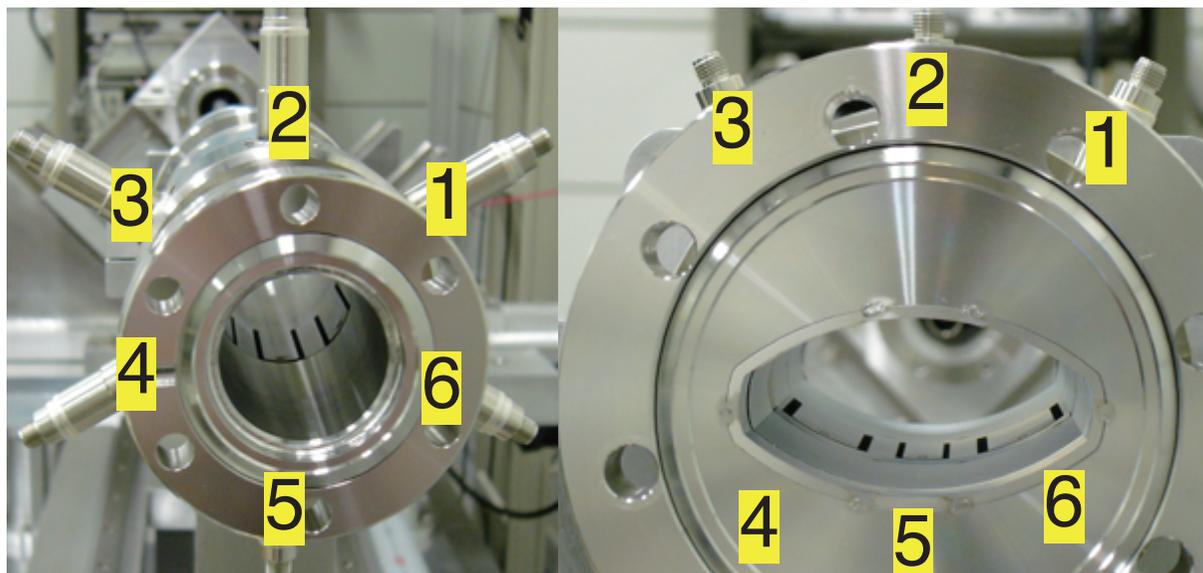


# DESIGN AND BEAM TEST OF SIX-ELECTRODE BPMS FOR SECOND-ORDER MOMENT MEASUREMENT

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- The BPM system is being upgraded to a six-electrode BPM system in the SPring-8 LINAC.
- Purpose: To enhance the beam observation during the top-up operation  
-> Measurement of second-order moments
- We developed BPMs with circular and quasi-elliptical cross-sections.
- We present the BPM designs, the principle of the multipole moment measurement, and the beam test results.



- Absolute moments, **P<sub>n</sub>**, **Q<sub>n</sub>**, are moments with respect to the duct center and obtained by taking voltage differences with normalized moments.

$$P_1 = \frac{R_{P1}}{2} \frac{V_1 - V_3 - V_4 + V_6}{V_1 + V_3 + V_4 + V_6}$$

$$Q_1 = \frac{R_{Q1}}{2} \frac{V_1 + V_3 - V_4 - V_6}{V_1 + V_3 + V_4 + V_6}$$

$$P_2 = \frac{R_{P2}^2}{2} \frac{kV_1 - 2V_2 + kV_3 + kV_4 - 2V_5 + kV_6}{kV_1 + 2V_2 + kV_3 + kV_4 + 2V_5 + kV_6}$$

$$Q_2 = \frac{R_{Q2}^2}{2} \frac{V_1 - V_3 + V_4 - V_6}{V_1 + V_3 + V_4 + V_6}$$

$$Q_3 = \frac{R_{Q3}^3}{2} \frac{KV_1 - V_2 + KV_3 - KV_4 + V_5 + KV_6}{KV_1 + V_2 + KV_3 + KV_4 + V_5 + KV_6}$$

n: Order of multipole moment

V<sub>d</sub>: Output voltage of electrode d  
(d = 1, ..., 6)

$\frac{R_{Pn}^n}{2}, \frac{R_{Qn}^n}{2}$ : Normalized moment

$R_{Pn}, R_{Qn}$ : Effective aperture radius

k, K : Geometrical Factors

Calculated : analytically (circular)  
numerically (quasi-elliptical)

|          | Circular | Quasi-elliptical |
|----------|----------|------------------|
| $R_{P1}$ | 18.69 mm | 18.03 mm         |
| $R_{Q1}$ | 32.38 mm | 54.84 mm         |
| $R_{P2}$ | 18.91 mm | 18.42 mm         |
| $R_{Q2}$ | 17.59 mm | 22.48 mm         |
| $R_{Q3}$ | 16.57 mm | 18.55 mm         |
| k        | 1.000    | 1.918            |
| K        | 1.000    | 3.056            |

- Absolute moments are complicated to compare the multipole moments taken by a screen monitor and a BPM, but relative moments are not.
- Relative moments,  $\mathbf{P}_{gn}$ ,  $\mathbf{Q}_{gn}$ , are defined as the moments with respect to the beam centroid and extracted from the absolute moments.

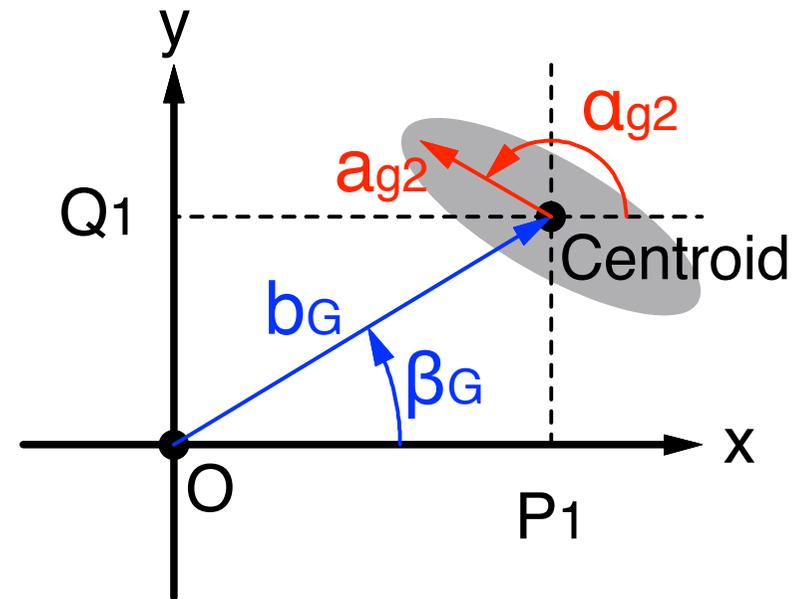
$$P_1 = b_G \cos \beta_G$$

$$Q_1 = b_G \sin \beta_G$$

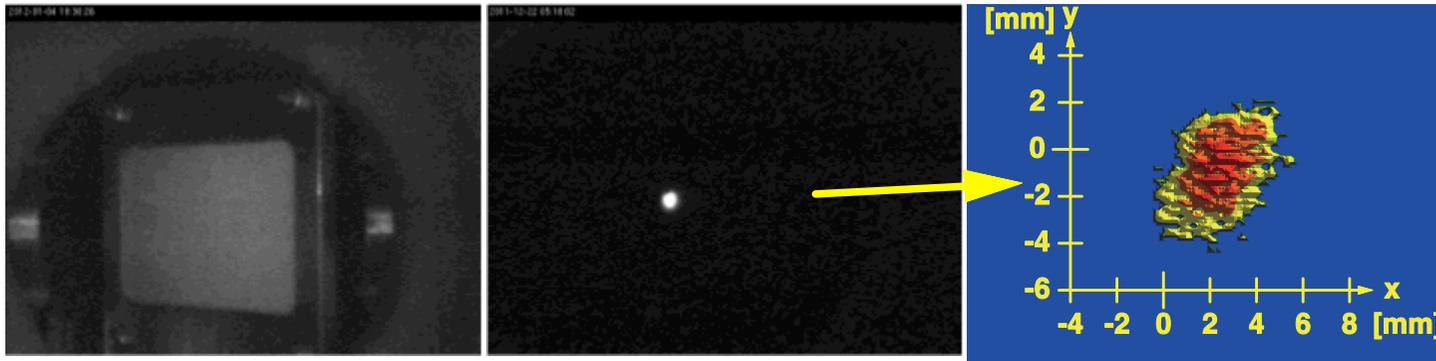
$$P_{g2} = a_{g2}^2 \cos 2\alpha_{g2} = P_2 - b_G^2 \cos 2\beta_G$$

$$Q_{g2} = a_{g2}^2 \sin 2\alpha_{g2} = Q_2 - b_G^2 \sin 2\beta_G$$

$$Q_{g3} = a_{g3}^3 \sin 3\alpha_{g3} = Q_3 - b_G^3 \sin 3\beta_G - 3b_G a_{g2}^2 \sin(\beta_G + 2\alpha_{g2})$$



- At the beam test, the relative moments were also measured by the screen monitor.
- $I(x_i, y_j)$ : Intensity distribution on the screen  
 $(x_i, y_j)$ : Position of data cell from screen center



$$\begin{aligned}
 P_1 &= 2.334 \text{ mm} \\
 Q_1 &= -0.989 \text{ mm}^2 \\
 P_{g2} &= -0.391 \text{ mm}^2 \\
 Q_{g2} &= 1.366 \text{ mm}^2 \\
 Q_{g3} &= 0.161 \text{ mm}^3
 \end{aligned}$$

$$m_{00} = \sum_i \sum_j I(x_i, y_j)$$

$$m_{10} = \sum_i \sum_j x_i I(x_i, y_j)$$

$$P_1 = \frac{m_{10}}{m_{00}}$$

$$Q_1 = \frac{m_{01}}{m_{00}}$$

$$m_{01} = \sum_i \sum_j y_j I(x_i, y_j)$$

$$m_{20} = \sum_i \sum_j x_i^2 I(x_i, y_j)$$

$$P_2 = \frac{m_{20} - m_{02}}{m_{00}}$$

$$Q_2 = \frac{2m_{11}}{m_{00}}$$

$$m_{11} = \sum_i \sum_j x_i y_j I(x_i, y_j)$$

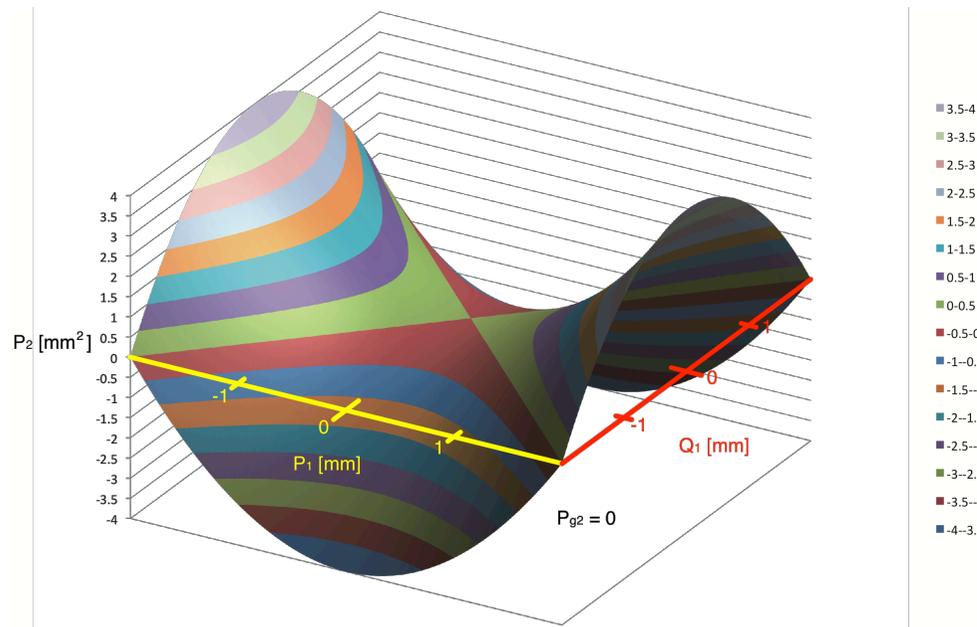
$$m_{02} = \sum_i \sum_j y_j^2 I(x_i, y_j)$$

$$Q_3 = \frac{3m_{21} - m_{03}}{m_{00}}$$

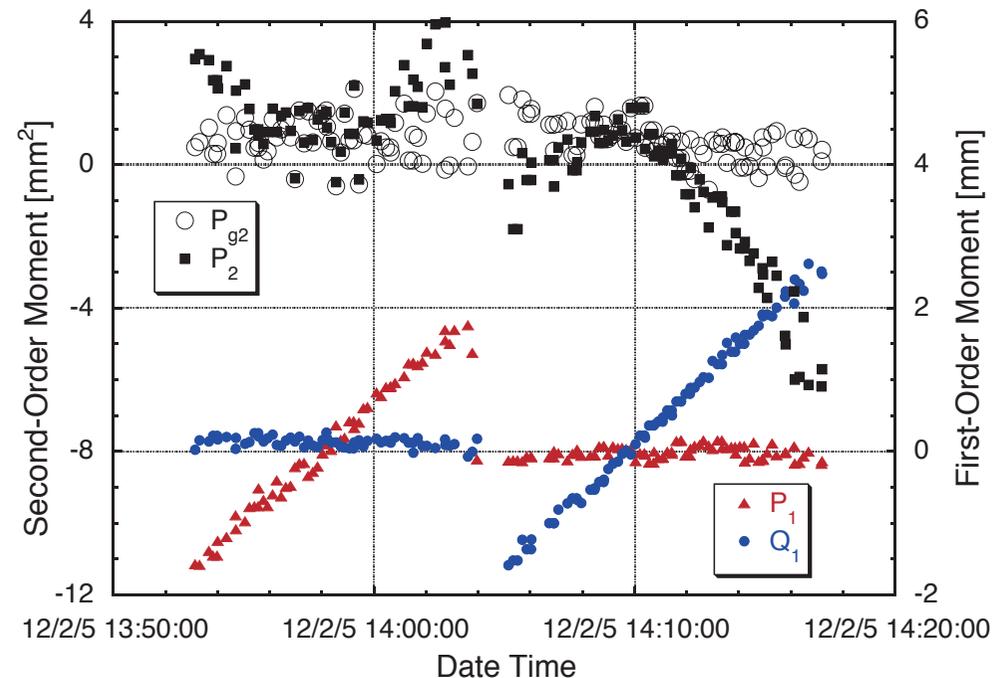
$$m_{21} = \sum_i \sum_j x_i^2 y_j I(x_i, y_j)$$

$$m_{03} = \sum_i \sum_j y_j^3 I(x_i, y_j)$$

- Entire calibration: To determine relative attenuation factors between electrode channels
- Principle:  
By means of steering magnets we change the position of the beam centroid.  
-> Absolute moments,  $P_n$ ,  $Q_n$ , vary.  
But we do not change beam distribution with respect to the beam centroid.  
-> Relative moments,  $P_{gn}$ ,  $Q_{gn}$ , do not vary.



Simulation of entire cal. for  $P_2$   
 $P_{g2}=0$  (constant),  $Q_{g2}=0$  (constant)



Beam test result of entire cal.

- We compared the second-order relative moments taken by **screen monitor (abscissa)** to those taken by **BPM (ordinate)** when we changed beam intensity distribution by changing magnetic field of upstream quadrupole magnet.
- Correlation plot apparently indicates **direct proportionality relation** with **proportionality coefficient of 1**.

### -> Important consequences

- Such BPMs with stripline-type electrodes, **normalized moments** can be **obtained accurately by analytical or maybe numerical electrostatic field calculation**.
- **Entire calibration** is **useful** as is **self-consistent method for precise calibration** to determine relative attenuation factors between electrode channels.

