The BPM system is being upgraded to a six-electrode BPM system in the SPring-8 LINAC.

Purpose: To enhance the beam observation during the top-up operation -> Measurement of second-order moments

We developed BPMs with circular and quasi-elliptical cross-sections.

We present the BPM designs, the principle of the multipole moment measurement, and the beam test results.
Absolute moments using six-electrode BPM

- Absolute moments, \( P_n, Q_n \), are moments with respect to the duct center and obtained by taking voltage differences with normalized moments.

\[
P_1 = \frac{R_{P1} V_1 - V_3 - V_4 + V_6}{2 \left( V_1 + V_3 + V_4 + V_6 \right)}
\]
\[
P_2 = \frac{R_{P2}^2 kV_1 - 2V_2 + kV_3 + kV_4 - 2V_5 + kV_6}{2 \left( kV_1 + 2V_2 + kV_3 + kV_4 + 2V_5 + kV_6 \right)}
\]
\[
Q_1 = \frac{R_{Q1} V_1 + V_3 - V_4 - V_6}{2 \left( V_1 + V_3 + V_4 + V_6 \right)}
\]
\[
Q_2 = \frac{R_{Q2}^2 V_1 - V_3 + V_4 - V_6}{2 \left( V_1 + V_3 + V_4 + V_6 \right)}
\]
\[
Q_3 = \frac{R_{Q3}^3 KV_1 - V_2 + KV_3 - KV_4 + V_5 + KV_6}{2 \left( KV_1 + V_2 + KV_3 + KV_4 + V_5 + KV_6 \right)}
\]

- \( n \): Order of multipole moment
- \( V_d \): Output voltage of electrode \( d \) (\( d = 1,\ldots,6 \))
- \( R_{Pn}^n, R_{Qn}^n \): Normalized moment
- \( R_{Pn}, R_{Qn} \): Effective aperture radius
- \( k, K \): Geometrical Factors

<table>
<thead>
<tr>
<th>Circular</th>
<th>Quasi-elliptical</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{P1} )</td>
<td>18.69 mm</td>
</tr>
<tr>
<td>( R_{Q1} )</td>
<td>32.38 mm</td>
</tr>
<tr>
<td>( R_{P2} )</td>
<td>18.91 mm</td>
</tr>
<tr>
<td>( R_{Q2} )</td>
<td>17.59 mm</td>
</tr>
<tr>
<td>( R_{Q3} )</td>
<td>16.57 mm</td>
</tr>
<tr>
<td>( k )</td>
<td>1.000</td>
</tr>
<tr>
<td>( K )</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Absolute moments are complicated to compare the multipole moments taken by a screen monitor and a BPM, but relative moments are not.

Relative moments, $P_{gn}$, $Q_{gn}$, are defined as the moments with respect to the beam centroid and extracted from the absolute moments.

\[
P_1 = b_G \cos \beta_G \\
Q_1 = b_G \sin \beta_G \\
P_{g2} = a_{g2}^2 \cos 2\alpha_{g2} = P_2 - b_G^2 \cos 2\beta_G \\
Q_{g2} = a_{g2}^2 \sin 2\alpha_{g2} = Q_2 - b_G^2 \sin 2\beta_G \\
Q_{g3} = a_{g3}^3 \sin 3\alpha_{g3} = Q_3 - b_G^3 \sin 3\beta_G - 3b_G a_{g2}^2 \sin(\beta_G + 2\alpha_{g2})
\]
Screen monitor for relative moment measurement

- At the beam test, the relative moments were also measured by the screen monitor.

- \( I(x_i, y_j) \): Intensity distribution on the screen
- \((x_i, y_j)\): Position of data cell from screen center

\[
\begin{align*}
    m_{00} &= \sum_i \sum_j I(x_i, y_j) \\
    m_{01} &= \sum_i \sum_j y_i I(x_i, y_j) \\
    m_{02} &= \sum_i \sum_j y_j^2 I(x_i, y_j) \\
    m_{10} &= \sum_i \sum_j x_i I(x_i, y_j) \\
    m_{11} &= \sum_i \sum_j x_i y_j I(x_i, y_j) \\
    m_{20} &= \sum_i \sum_j x_i^2 I(x_i, y_j) \\
    m_{21} &= \sum_i \sum_j x_i^2 y_j I(x_i, y_j) \\
    m_{03} &= \sum_i \sum_j y_j^3 I(x_i, y_j)
\end{align*}
\]

\[
\begin{align*}
    P_1 &= \frac{m_{10}}{m_{00}} \\
    Q_1 &= \frac{m_{01}}{m_{00}} \\
    P_2 &= \frac{m_{20} - m_{02}}{m_{00}} \\
    Q_2 &= \frac{2m_{11}}{m_{00}} \\
    P_3 &= \frac{3m_{21} - m_{03}}{m_{00}} \\
    Q_3 &= \frac{1.366 \text{mm}^2}{m_{00}} \\
    P_g_2 &= -0.391 \text{mm}^2 \\
    Q_g_2 &= 1.366 \text{mm}^2 \\
    Q_g_3 &= 0.161 \text{mm}^3
\end{align*}
\]
Entire calibration using electron beams

- Entire calibration: To determine relative attenuation factors between electrode channels

- Principle:
  By means of steering magnets we change the position of the beam centroid.
  -> Absolute moments, \( P_n, Q_n \), vary.
  But we do not change beam distribution with respect to the beam centroid.
  -> Relative moments, \( P_{gn}, Q_{gn} \), do not vary.

Simulation of entire cal. for \( P_2 \) \( P_{g2} = 0 \) (constant), \( Q_{g2} = 0 \) (constant)

Beam test result of entire cal.
Correlation measurement

• We compared the second-order relative moments taken by screen monitor (abscissa) to those taken by BPM (ordinate) when we changed beam intensity distribution by changing magnetic field of upstream quadrupole magnet.

• Correlation plot apparently indicates direct proportionality relation with proportionality coefficient of 1.

-> Important consequences

• Such BPMs with stripline-type electrodes, normalized moments can be obtained accurately by analytical or maybe numerical electrostatic field calculation.

• Entire calibration is useful as is self-consistent method for precise calibration to determine relative attenuation factors between electrode channels.