AN ANALYTICAL CAVITY MODEL FOR FAST LINAC-BEAM TUNING*

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Abstract

Non-axisymmetric RF cavities can produce axially asymmetric acceleration fields. Conventional method using numerical 3-D field tracking to address this feature is time-consuming and thus not appropriate for on-line beam tuning applications. In this paper, we develop analytical treatment of non-axisymmetric RF cavities. Multipole models of cavities are derived using realistic 3-D field in both longitudinal and transverse dimensions. Then, beam dynamics formulism is established. Finally, special case of FRIB quarter-wave resonators are calculated by the model and benchmarked against 3-D field tracking to ensure the efficiency and accuracy of the model.

INTRODUCTION

Non-axisymmetric RF cavities, such as quarter-wave resonators (QWR), half-wave resonators (HWR) and spoke cavity, can produce dipole and quadrupole terms in transverse direction, and can cause beam steering and deformation. Up to now, no model can handle this problem except for multi-particle tracking. So, we aim at building longitudinal and transverse model for non-axisymmetric RF cavities to solve this problem. The model will be implemented into on-line beam tuning application[1], therefore the basic requirement is accuracy and efficiency.

LONGITUDINAL DYNAMICS

For linac acceleration, keeping track of kinetic energy and phase evolution would be important. Particle tracking by iteration is the traditional way, while revealing lack of efficiency when applying to online beam tuning. To solve the problem, the thin lens model is implemented, describing the energy gain and phase advance after a RF cavity, which can be considered as a drift-kick-drift model as below[2][3]:

$$\begin{cases} W_f = W_i + qV_0 T(k)\cos\phi_i - qV_0 S(k)\sin\phi_i \\ \phi_f = \phi_i + \dfrac{qV_0}{2W_i} k\left[T'(k)\sin\phi_i + S'(k)\cos\phi_i\right] \end{cases} \quad (1)$$

where W_i and W_f stands for initial and final kinetic energy and φ_i and φ_f are initial and final particle phase. V_0 are the static electric voltage. T, T', S and S' are the transit-time factors calculated from numerical electric field data. In order to analyse the error introduced by constant beta assumption during drift section, two kinds of thin lens model is developed comparatively for multi-gap cavities. 1) One gap kick model treats the whole cavity as one thin accelerating gap. 2) Multi-gap kick model treats each accelerating gap separately.

Comparison with Simulation

Facility of Rare Isotope Beams (FRIB)[4] linac segment is implemented for comparison between model and numerical calculation. FRIB linac segment one, which consists of three β=0.041 QWR cryomodules and eleven β=0.085 QWR cryomodules, accelerates ions from 0.5MeV/u to 16.6MeV/u. By adding cavity driven phase onto absolute particle phase, we obtain the initial phase of the certain particle, which can be described as: $\varphi_i = \varphi_{abs} + \varphi_{drive}$; The driven phase is set so that the synchrotron phase is -30 deg. After setting the driven phase, linac segment acceleration simulation is performed via 4 different methods, namely, IMPACT[5] tracking (used as reference), particle tracking, 2-gap kick model, 1-gap kick model. For 2-gap kick model, we can achieve the phase precision of 0.31% when divided by 2π, and kinetic energy gain prediction precision of 0.042% of the 16.1MeV/u kinetic energy gain (figure 1). The numerical electric field data comes from CST[6] simulation.

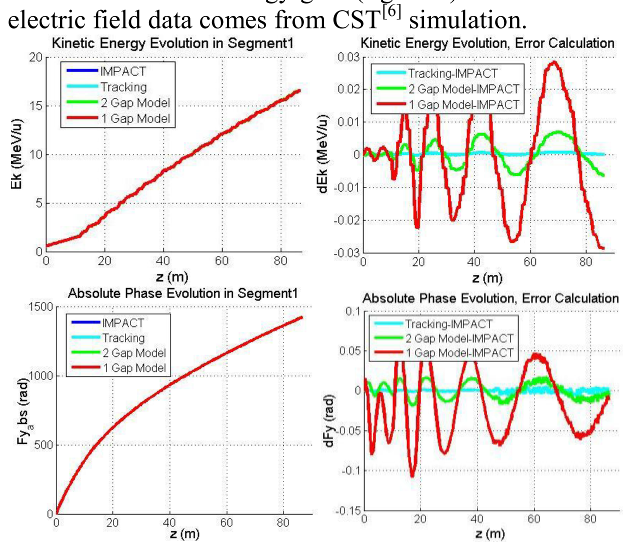

Figure 1, Kinetic energy and Absolute phase evolution and error calculation; a) Kinetic Energy evolution; b) Difference between different models and IMPACT simulation in Kinetic energy prediction; c) Absolute phase evolution; d) Difference between different models and IMPACT simulation in absolute phase prediction.

TRANSVERSE DYNAMICS

Traditional treatment of transverse RF cavity field usually only consists of focusing electric field terms. However, early studies already show that field dipole terms and quadrupole terms exist in non-axisymmetric RF

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cavities[7][8]. Therefore, we are developing a transverse model including multipole terms derived directly from numerical data.

Field Multipole Expansion

In 3-D cylindrical coordinate system the zeroth order of φ represents focusing term, the first order represents dipole or steering term and the second order represents quadrupole term. The 2-D Taylor-Fourier series expansion is conducted for each x-y plane field. In a 2-D polar coordinate, we first transfer 2-D vector field into two 2-D scalar field by projection. Then, we expand the ρ direction into Taylor series and ϕ direction into Fourier series. The coefficient is proportional to the relative strength of a certain field mode. The process can be expressed as:

$$F_{\rho,nm}(\rho,\theta) = F_{max}\cdot\begin{bmatrix}1\\ \rho\\ \rho^2\\ \rho^3\\ \vdots\\ \rho^n\end{bmatrix}^T \cdot \begin{bmatrix} a_{00}-ib_{00} & a_{01}-ib_{01} & a_{02}-ib_{02} & \cdots & a_{0m}-ib_{0m}\\ a_{10}-ib_{10} & a_{11}-ib_{11} & a_{12}-ib_{12} & \cdots & a_{1m}-ib_{1m}\\ a_{20}-ib_{20} & a_{21}-ib_{21} & a_{22}-ib_{22} & \cdots & a_{2m}-ib_{2m}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ a_{n0}-ib_{n0} & a_{n1}-ib_{n1} & a_{n2}-ib_{n2} & \cdots & a_{nm}-ib_{nm}\end{bmatrix}\begin{bmatrix}1\\ e^{i\theta}\\ e^{i2\theta}\\ e^{i3\theta}\\ \vdots\\ e^{im\theta}\end{bmatrix} \quad (2)$$

$$= F_{max}\cdot\sum_{n,m=0}^{\infty} P_n A_{nm}\Theta_m$$

In a specific case, if the geometry of the cavity is symmetric, we can apply this condition to simplify the Taylor-Fourier series expansion. Theoretically, we should expand the field to infinite order while in practice we have to truncate it to satisfy certain precision.

For the case of FRIB β=0.085 QWR, we transfer CST numerical electric field data, which is in Cartesian coordinate system, to Polar coordinate system by bilinear interpolation. Multipole expansion results show that in coefficient matrix of E_r, the linear term is A_{21} (focusing term), A_{12} (dipole term) and A_{23} (quadrupole term). In coefficient matrix of H_r, the linear term is A_{21}(monopole term), A_{12} (dipole term) and A_{23} (quadrupole term). Then longitudinal direction is scanned and multipole term coefficient is calculated for all the x-y planes of field data (figure 2).

Inversely, we can rebuild the field by using the coefficient. Only high order and random noise still exist and the precision would be at the order of 0.1% if we truncate the expansion to 4^{th} order.

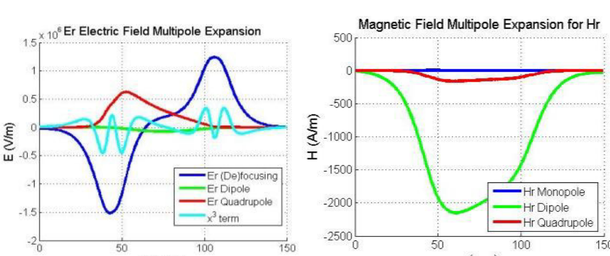

Figure 2, Result of multipole term strength $C_{max,i,j}$ curve for E_r and H_r; a) E_r multipole term strength, defined as E_{max} times multipole term coefficient, vs. longitudinal coordinate z; b) H_r multipole term strength, defined as H_{max} times multipole term coefficient, vs. longitudinal coordinate z;

Multipole Beam Kick Model

The multipole beam kick model is utilized to calculate beam kick caused by multipole terms. We start from the Lorentz force equation:

$$\Delta y' = \frac{qe\mu_0}{\gamma m_0}\int_{t_s}^{t_e} H_x(x,y,z,t)\,dt + \frac{qe}{\gamma m_0 \beta c}\int_{t_s}^{t_e} E_y(x,y,z,t)\,dt$$
$$= \Delta y'_{H,y} + \Delta y'_{E,y} \quad (3)$$

Beam kick can be caused by electric field and magnetic field. Take electric field as example. Assume the field is a sinusoidal function with time and get replaced by z as an independent variable. The expansion is done under cylindrical coordinate system, substitute the result:

$$\begin{cases} E_y(x,y,z) = E_\rho(\rho,\phi,z)\cdot\sin\phi + E_\phi(\rho,\phi,z)\cdot\cos\phi \\ E_\rho = E_{\rho,max}\sum_{j,k=0}^{n}\left(a_{\rho,jk} - ib_{\rho,jk}\right)e^{ik\theta}\rho^j \\ E_\phi = E_{\phi,max}\sum_{j,k=0}^{n}\left(a_{\phi,jk} - ib_{\phi,jk}\right)e^{ik\theta}\rho^j \end{cases} \quad (4)$$

where x=ρ·cosφ, y=ρ·sinφ. After simplification and axis transformation, we can write E_y into the following form:

$$E_y(x,y,z) = \sum_{i,j=0}^{n} E_{max}(z)\cdot c_{ij}(z)\cdot t_{ij}(x,y) \quad (5)$$

Make use of the concept of time transit factors, we can finally get the expression of vertical beam kick by electric multipole components:

$$\Delta y'_{E,y} = \frac{qe}{\gamma\beta^2 m_0 c^2}\int_{z_s}^{z_e}\sum_{i,j=0}^{n} E_{max}(z)\cdot c_{ij}(z)\cdot t_{ij}(x,y)\cdot\cos(kz+\phi_0)\,dz$$
$$= \frac{qe}{\gamma\beta^2 m_0 c^2}\cdot\sum_{i,j=0}^{n} t_{ij}(x,y)\int_{z_s}^{z_e} C_{max,ij}(z)\cdot\cos(kz+\phi_0)\,dz \quad (6)$$
$$= \frac{qe}{\gamma\beta^2 m_0 c^2}\cdot\sum_{i,j=0}^{n} t_{ij}(x,y)\cdot V_{ij}\cdot\left(T_{ij}\cos\phi_0 - S_{ij}\sin\phi_0\right)$$

Similarly, we can also get the expression of vertical beam kick by magnetic multipole components:

$$\Delta y'_{H,y} = \frac{qe\mu_0}{\gamma\beta m_0 c}\cdot\sum_{i,j=0}^{n} t_{ij}(x,y)\cdot U_{ij}\cdot\left(T_{ij}\cos\phi_0 - S_{ij}\sin\phi_0\right) \quad (7)$$

For general case, $c_{i,j}(z)$ and $t_{i,j}(z)$ would be quite complicate except for some special linear case.

Focusing: The term with i=1, j=0, stands for beam focusing term. When at this case, for y electric field, $E_{max}(z) = E_{max,\rho}(z)$, $c_{10}(z) = a_{\rho,10}(z)$ and $t_{ij} = y/\rho_{max}$; for magnetic field, $H_{max}(z) = H_{max,\varphi}(z)$, $c_{01}(z) = a_{\varphi,01}(z)$ and $t_{ij} = - y/\rho_{max}$. Using the model we can evaluate the focusing effect of the cavity.

Beam Steering: The term with i=0, j=1, stands for beam steering term, or dipole term. When at this case, for y electric field, $E_{max}(z) = E_{max,\rho}(z)$, $c_{01}(z) = b_{\rho,01}(z) = a_{\varphi,01}(z)$ and $t_{ij} = 1$; for magnetic field, $H_{max}(z) = H_{max,\rho}(z)$, $c_{01}(z) = a_{\rho,01}(z) = -b_{\varphi,01}(z)$ and $t_{ij} = 1$. Using the model we can derive the dipole kick and predict beam steering for FRIB β=0.085 QWR. The result is then benchmarked with particle tracking (figure 3a).

Quadrupole Terms: The term with i=1, j=2, stands for quadrupole term. When at this case, for y electric field, $E_{max}(z) = E_{max,\rho}(z)$, $c_{12}(z) = -a_{\rho,12}(z) = b_{\varphi,12}(z)$ and $t_{ij} =$

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y/ρ_{max}; for magnetic field, $H_{max}(z) = H_{max,\rho}(z)$, $c_{12}(z) = b_{\rho,12}(z) = a_{\varphi,12}(z)$ and $t_{ij} = y/\rho_{max}$. Using the model we can derive the quadrupole kick for FRIB β=0.085 QWR and then benchmarked against particle tracking (figure 3b).

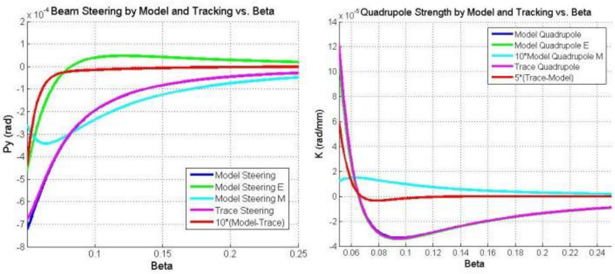

Figure 3, Prediction of beam steering and quadrupole strength by Model and Tracking vs. beta, φ_0=-π/6; a) Blue: beam steering by model, green: electric field steering, cyan: magnetic field, magenta: particle tracking, red: 10 times of error between model and tracking. b) Blue: quadrupole strength by model, cyan: electric field quadrupole, magenta: 10 times magnetic field quadrupole, green: particle tracking, red: 10 times of error between tracking and model.

Comparison with Simulation

After multipole expansion of FRIB β=0.085 QWR FRIB cavity, we derived all the linear terms including focusing term, dipole term, quadrupole term in transverse electric and magnetic fields. Then, we develop the transverse drift-kick-drift thin lens model and compare the result with particle tracking simulation. The thin lens model includes a series of multipole kicks located at their own components' electric (magnetic) centre separated by a series of drift spaces.

We implement the method to the second gap of FRIB β=0.085 QWR cavity, and the multipole thin lens components can be seen in table 1. Units: mm for position, V for V_0 and A for U_0. In order to check the correctness of this thin lens model, the result is benchmarked against particle tracking simulation (figure 4). The blue circles represent initial phase space and red triangle represent final phase space calculated by real field tracking after one β=0.085 QWR cavity. Green squares shows thin lens model with focusing components only, which tilt both x and y phase space eclipse. Blue stars represent the phase space after adding steering term into the thin lens model. There is no steering effect in x direction, while in y direction the steering effect is around -0.245 mrad. The RMS error of blue stars for x, x', y, y' is 1.3%, 23%, 1.2%, 8.4%. Red crosses represent the phase space after adding quadrupole term into the thin lens model. Then the x and y phase space tilt to an opposite direction and get closer to particle tracking. The RMS error of red crosses for x, x', y, y' becomes 0.034%, 3.7%, 0.29%, 3.6%. The model shows good agreement with particle tracking while the calculation speed is about 200 times faster.

Table 1, Optics line of transverse thin lens model

Type	Position	T	S	V0(U0)
E Defocusing	41.4319	0.9806	0	0.1203
E Quad. Pole	61.7142	-0.9530	0	0.0739
H Dipole	72.0426	0.8921	0	4.84e-4
E Dipole	77.4354	0.9530	0	0.0103
E Focusing	104.1544	-0.9650	0	0.1200

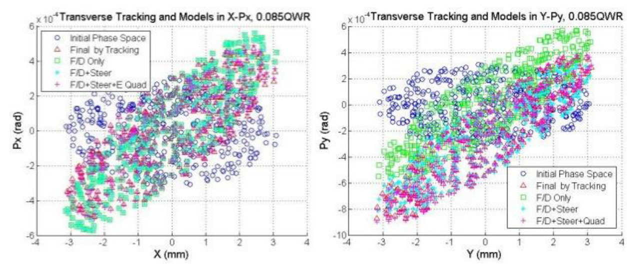

Figure 4, X-Px phase space and Y-Py phase space of a KV distribution beam using tracking and models; a) X-Px phase space when including different field terms and benchmark against particle tracking; b) Y-Py phase space when including different field terms and benchmark against particle tracking.

CONCLUSION

A longitudinal and transverse model designed for on-line beam tuning application which needs both precision and speed has been established for non-axisymmetric RF cavities and has been checked with the case of FRIB QWR. The procedure of building both longitudinal and transverse model is universal and can be easily extended to other cavities such as half-wave resonators, crab cavities and spoke cavities. The model can achieve better precision in transverse direction than traditional model where beam steering and quadrupole terms are usually omitted. An enhancement of 200 times in calculation speed has been seen when using model in transverse direction and a faster speed can be expected if database such as TTF factors are made beforehand.

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