

# STUDY OF PLASMA EFFECT IN LONGITUDINAL SPACE CHARGE INDUCED MICROBUNCHING INSTABILITY\*

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## Abstract

Microbunching instability ( $\mu$ BI) usually exists in the LINAC of a free electron laser facility. If it is not well-controlled, the beam quality will be seriously damaged and the machine will not operate properly. In many cases, the longitudinal space charge (LSC) is a dominant factor that generates the instability; therefore its contribution must be studied in details. The current model of the LSC impedance [1] derived from the fundamental electromagnetic theory [2] has been widely used to explain the physics of the LSC-induced  $\mu$ BI. [3] However, in the case of highly bright electron beams, the plasma effect may also play a role. In this article, the basic model of the LSC impedance including the plasma effect is constructed by solving Vlasov and Poisson equations in the 6-D phase space, and preliminary investigation is carried out to study the modification to the instability gain. The solution indicates that the  $\mu$ BI gain depends not only on the spatial information of the beam, but also on the velocity (momentum) and time information. The comparison of the gains of the  $\mu$ BI in the LINAC of Shanghai soft X-ray Free Electron Laser Facility (SXFEL) computed by various methods is given and the discrepancy is illustrated.

## INTRODUCTION

The possibility of oscillation in a plasma due to local separation of charges and the consequent restoring forces was discussed by J. D. Jackson long time ago. [4] The theory is based on a neutral plasma, which has both the positively (ion) and negatively (electron) charged components. For a charged particle beam in an accelerator, although it is not neutral in terms of charges, there is still density fluctuation due to the graininess of the individual particles — in our case, the individual electrons. Such graininess is usually smoothed out in the fluid model and ignored in most computations. In a highly intensive beam, however, it may introduce the “plasma-like” oscillation (for convenience, “plasma oscillation” is used hereafter), and must be investigated in details in order to reveal its magnitude and to discover its physics. Similar discussions in the 2-D phase space for this effects on the free electron laser have been addressed by Kim, et. al. [5]

In this article, we start our discussions in the 6-D phase space by employing Vlasov and Poisson (Gauss) equations,

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which describe the evolution of the distribution function of the electron bunch and the electric field induced by the charge distribution. We then use a method similar to Jackson's [4] to linearize the Vlasov equation and obtain the solution of the initial-value problem. The solution includes the contributions from both the perturbed and unperturbed parts of the initial distribution. As the result, the contribution from the velocity distribution is also included. From the solution, we see that the plasma oscillation introduces a “relative dielectric factor (permittivity)”  $\epsilon_r$ , and also an extra factor coming from the initial velocity distribution.

## SOLUTION OF INITIAL-VALUE PROBLEM

We carry out the investigations with the equations describing the evolution of beam distribution under the influence of the electromagnetic force. The discussion is in laboratory frame hereafter. In cylindrical coordinates system, the linearized Vlasov-Poisson equation and the Poisson (Gauss) equation can be written:

$$\frac{\partial f_1}{\partial t} + v_z \frac{\partial f_1}{\partial z} + v_\perp \frac{\partial f_1}{\partial r} - \frac{eE_z}{\gamma m} \frac{\partial f_0}{\partial v_z} + \frac{F_\perp}{\gamma m} \frac{\partial f_0}{\partial v_\perp} = 0, \quad (1)$$

$$\frac{\partial E_z}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r}(rE_r) = -\frac{e}{\epsilon_0} \int f_1 dv_z, \quad (2)$$

where  $-e$  is charge of an electron,  $f(t, \vec{r}, \vec{v}) = f_0(t, \vec{r}, \vec{v}) + f_1(t, \vec{r}, \vec{v})$ , with  $f_0$  being the unperturbed background of the beam and  $f_1$  the density perturbation due to plasma oscillation. We assume that  $f_0$ ,  $f_1$  and  $\vec{E}$  have no azimuthal dependence, which is reasonable. The Gauss's law or Poisson equation, Eq. (2), will be solved in the particles' rest frame and then Lorentz transformed to the lab frame in the next section.

Since the transverse velocity  $v_\perp$  is small, we can assume  $v_z \approx v$  and  $F_\perp \ll F_z$ . Then Eq. (1) simplifies to

$$\frac{\partial f_1}{\partial t} + v \frac{\partial f_1}{\partial z} - \frac{eE_z}{\gamma m} \frac{\partial f_0}{\partial v} = 0. \quad (3)$$

Let us focus on Eqs. (3) and (2). Following Jackson, [4] we perform Fourier transform in  $t$  and Laplace transform in  $z$  on Eq. (3), and integrate by parts to obtain

$$\int dz \left[ e^{-ikz+i\omega t} f_1(v, z, t) \right]_{t=0}^{t=\infty} + \int_{-\infty}^{\infty} dz \int_0^{\infty} e^{-ikz+i\omega t} dt \times \left[ (-i\omega + ikv)f_1 - \frac{e}{\gamma m} \frac{\partial f_0}{\partial v} E \right] = 0. \quad (4)$$

For  $\omega$  in the upper half plane, the upper limit of the first term on the right hand side of Eq. (4) vanishes as  $t \rightarrow \infty$ . We have then the solution in  $(v, \omega, k)$  space,

$$f_1(v, \omega, k) = \frac{1}{i(kv - \omega)} \left[ \Phi(v, k) + \frac{e}{m} \frac{\partial f_0}{\partial v} E(\omega, k) \right], \quad (5)$$

where

$$\Phi(v, k) = \int_{-\infty}^{\infty} dz e^{-ikz} f_1(v, z, t = 0). \quad (6)$$

Both Eq. (5) and Eq. (6) form the solution depending on the initial-value of the density perturbation. If we perform inverse Fourier transform on  $\omega$ , we will obtain the density perturbation  $f_1(v, t, k)$  at later time, which represents the time revolution of the density fluctuation. In the regular LSC theory, the density fluctuation is neglected. However, it will be taken into account under certain conditions in the following discussions.

## ELECTRIC FIELD INDUCED BY LSC

Based on the classical electromagnetic theory, the solution of Eq. (2) can be written as

$$E_z(\vec{x}) = \frac{e\lambda}{4\pi\epsilon_0} \int G(\vec{x}, \vec{x}') \rho(\vec{x}') d^3\vec{x}', \quad (7)$$

with the Green function

$$G(\vec{x}, \vec{x}') = \frac{(z - z')\gamma}{[(x - x')^2 + (y - y')^2 + (z - z')^2\gamma^2]^{3/2}}. \quad (8)$$

Here the same notations of Venturini's [1] have been used, with beam density  $\lambda\rho(x, y, z)$ , uniform linear density  $\lambda$ ,  $\rho(x, y, z) = \rho_{\perp}(x, y)\rho_z(z)$ , and normalization  $\int \rho_{\perp}(x, y) dx dy = 1$ .

In cylindrical coordinates, the Green function can be expanded as [2]

$$G(\vec{x}, \vec{x}') = -\frac{i}{\pi\gamma^2} \sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')} \int_{-\infty}^{\infty} k dk e^{ik(z-z')} \times I_m\left(\frac{kr_{<}}{\gamma}\right) K_m\left(\frac{kr_{>}}{\gamma}\right), \quad (9)$$

where  $r_{<}$  and  $r_{>}$  denotes the smaller and larger between  $r$  and  $r'$ , respectively, and  $I_m$  and  $K_m$  are the modified Bessel function of the first and second kind. Thus the longitudinal electric field  $E_z$  in the  $k$ -space becomes

$$E_z(k) = i \frac{e}{4\pi\epsilon} \frac{\lambda}{\pi\gamma^2} \sum_{m=-\infty}^{\infty} \int dV' \rho(r', \phi', z') e^{im(\phi-\phi')} \times k e^{-ikz'} I_m^{\leq} K_m^{\geq}. \quad (10)$$

In our study, the unperturbed density distribution serves as the smooth background, therefore it does not contribute to the longitudinal electric field at all. For this reason, only the perturbed one (Eq. (5)) plays a role. Assuming that the

beam is of radius  $r_b$  and has a uniform transverse distribution  $f_{\perp} = 1/\pi r_b^2$  when  $r \leq r_b$ , we write the beam distribution as  $\rho(r, \phi, z) = f_{\perp} f_1(v, z, t = 0)$ . For an observation point located on axis (only  $m = 0$  term contributes), [1] finally we obtain

$$E(\omega, k) = \frac{e\lambda}{2\pi\epsilon_0\epsilon_r\pi r_b^2} \int_W dv \times \int_{-\infty}^{\infty} dz \frac{e^{-ikz} f_1(v, z, t = 0)}{kv - \omega} [1 - \xi K_1(\xi)], \quad (11)$$

where  $\xi = kr_b/\gamma$ , and

$$\epsilon_r = k - \frac{e^2\lambda}{2\pi\epsilon_0\gamma m} \int_W \frac{\partial f_0}{\partial v} \frac{dv}{kv - \omega} \quad (12)$$

is the relative dielectric factor (permittivity). The path of integration  $W$  is from  $v = -\infty$  to  $\infty$  passing below the pole  $v = \omega/k$ . This path comes from the analytic continuity from the upper  $\omega$ -half-plane to the whole  $\omega$ -plane. Introducing the frequency of plasma oscillation in lab frame,  $\omega_p = \sqrt{e^2\lambda/2\pi\epsilon_0\gamma m}$ , Eq. (12) can be written as

$$\epsilon_r = k - \omega_p^2 \int_W \frac{\partial f_0}{\partial v} \frac{dv}{kv - \omega}. \quad (13)$$

Equation (13) is also called the dispersion relation, it is a function of the wavenumber  $k$  of the density fluctuation. Equation (11) is the expression of the longitudinal electric field induced by the LSC under the influence of density fluctuation (plasma oscillation). Apparently, it includes the contribution due to the velocity distribution of the beam.

In most of the cases, where the momentum and location of the electron are decoupled, the perturbation can also be written as  $f_1(v, z, t = 0) = f_{v1}(v, t = 0) f_{z1}(z, t = 0)$ . Equation (11) becomes

$$E(\omega, k) = \frac{e\lambda [1 - \xi K_1(\xi)]}{\epsilon_0\epsilon_r\pi r_b^2} \int_W dv \frac{f_{v1}(v, t = 0)}{kv - \omega} \times \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ikz} f_{z1}(z, t = 0) dz. \quad (14)$$

Equation (14) decouples the contributions from the beam density distribution and velocity/momentum distribution.

## INFLUENCE ON $\mu$ BI

In this section, we start our preliminary discussions on the effects of the modified LSC impedance in microbunching instability. As already known, the gain of  $\mu$ BI due to the LSC impedance (linear in beam current) reads [6]

$$G = Ck |R_{56}| \frac{I_0}{\gamma_0 I_A} \frac{|Z(k)|}{Z_0} \exp\left(-\frac{C^2 k^2 R_{56}^2 \sigma_r^2}{2\gamma_0^2}\right), \quad (15)$$

where  $C$  is the compression factor of a bunch compressor (chicane),  $R_{56}$  is the transport matrix element of the whole bunch compressor, and  $\sigma_r$  is the uncorrelated energy spread. In Eq. (15), one can see that the gain is proportional to the absolute value of LSC impedance per unit length  $Z(k)$ .

The impedance per unit length of the longitudinal space charge is defined as

$$Z(k) = -E_z(k)/I(k), \quad (16)$$

where  $I(k)$  is the Fourier transform of the beam current, i.e.,  $I(k) = ec\lambda\rho_z(k)$ . Note that in our discussion  $\rho_z(k) = (2\pi)^{-1} \int e^{-ikz} f_1(z, t=0) dz$ . Based on Eq. (14), we can derive the LSC impedance in the existence of density fluctuation:

$$Z(\omega, k) = -\frac{Z_0}{\pi\gamma\epsilon_r r_b} \frac{1-\xi K_1(\xi)}{\xi} \int_W \frac{f_1(v, t=0) dv}{kv - \omega}. \quad (17)$$

Comparing with the regular expression of the LSC impedance without density fluctuation, [1, 3] we can see that the difference comes from the relative dielectric factor  $\epsilon_r$  and the initial velocity perturbation  $f_1(v, t=0)$ . Moreover, it is also a function of the oscillation frequency  $\omega$ .

At this point, we believe that more investigations are needed to obtain the exact picture of the development of the  $\mu$ BI induced by the longitudinal space charge field. For an example, assuming the Gaussian distribution of electron velocity, and performing inverse Fourier transform on  $\omega$ , we will find the explicit time and wavelength dependence of the microbunching instability.

In the proposed Shanghai X-ray Free Electron Laser Facility (SXFEL), [7] at the exit of the LINAC injector, the momentum distribution is Gaussian-like (Fig. 1) and the current profile is Gaussian with a number of small spikes, i.e., initial modulation (Fig. 2). Although the numerical simulation and analytic solution of the  $\mu$ BI gain at the exit of LINAC are both not-so-significant, we can still see the discrepancy between them (Fig. 3).

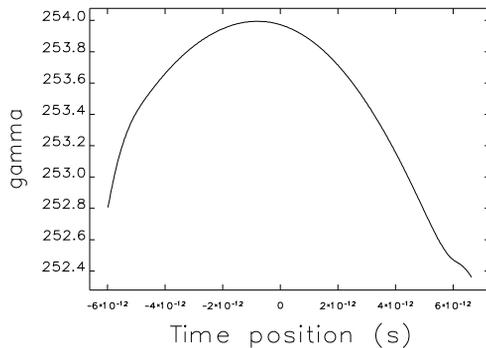


Figure 1: Beam momentum distribution at the exit of SXFEL injector.

## CONCLUSIONS

The electric field introduced by the longitudinal space charge (LSC) with density fluctuation (plasma oscillation) is studied in detail by solving the Vlasov and Poisson equations. Its influence on the LSC-induced microbunching instability ( $\mu$ BI) is carried out. The preliminary investigation implicates that the gain curve of  $\mu$ BI does not depend

only on the spatial information of the beam, but also on the beam's momentum/velocity information as well. Further study is ongoing to reveal the details of the development of the LSC-induced  $\mu$ BI.

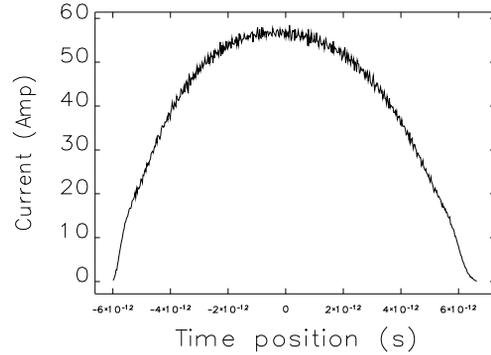


Figure 2: Current profile at the exit of SXFEL injector.

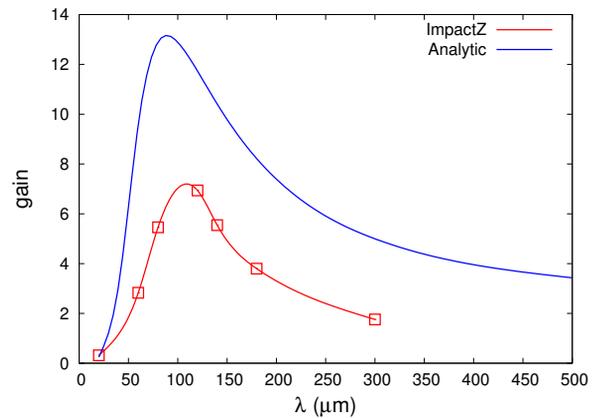


Figure 3: (color)  $\mu$ BI gain at the exit of SXFEL LINAC computed by different methods: ImpactZ simulation (red), and analytic solution (blue).

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