THE FINE STRUCTURE FOR THE ZONE OF PARTICLE INTERACTION WITH A FINITE LENGTH PERIODIC STRUCTURE

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Abstract

Interaction of the charged particle with finite length periodic structure is considered to estimate the long range wakefield effects in the constant impedance periodic deflecting structure for the special bunch diagnostic. In each structure passband there is a narrow zone with three modes, effectively interacting with the particles. To separate these modes, both frequency domain and time domain simulations are considered and compared.

INTRODUCTION

To describe long range wakefield effects for the $m-$th mode of the cavity, both longitudinal and transversal, loss factor k^l_m and kick factor k^t_m are used.

$$k^l_m = \frac{|\int_0^L E_{zm}(r=0,z)e^{\frac{i\omega_m \cdot z}{c}}dz|^2}{4U_m}, \tag{1}$$
$$k^t_m = \frac{|\int_0^L E_{zm}(r=r_e,z)e^{\frac{i\omega_m \cdot z}{c}}dz|^2}{4r_e^2 U_m}$$

where $E_{zm}(r,z)$, ω_m and U_m are the distribution of z component, the frequency and the stored field energy of the m-th mode. For more details and explanations one can see [1], for example.

To estimate effects in a wide frequency range, one should either simulate frequencies and fields distributions in the structure, or directly simulate wakefields of the short bunch in the long structure. Below we consider some particularities of the long range wakefields in the periodic structure with finite number of periods.

PARTICLE INTERACTION WITH FINITE LENGTH PERIODIC STRUCTURE

For the traveling wave in the infinite periodic structure each field component can be represented in complex for and expanded in a series of spatial harmonics:

$$E_{fj}(r,z) = \Re E_j(r,z) - i\Im E_j(r,z) = \tag{2}$$
$$= \sum_{p\to-\infty}^{p\to+\infty} a_{jp}(r) e^{\frac{-i(\Theta_0+2p\pi)z}{d}}, \quad 0 \le \Theta_0 \le \pi.$$

where d is the structure period, Θ_0 is the phase advance per period. Suppose the structure has a finite number of periods N and appropriate (half cells) terminations. For the standing wave a discrete set of possible phase advance $\Theta_m = \frac{mpi}{N}, m = 0, 1, ..., N$ is possible and the field distribution can be described, [2], as:

$$E_{sjm}(r,z) = \sum_{p\to-\infty}^{p\to+\infty} 2a_{jp}(r)cos(\frac{(\Theta_m+2p\pi)z}{d}) \tag{3}$$

with the same coefficients $a_{jp}(r)$ as in appropriate traveling wave representation (2). For such field representation of the standing wave let us consider the weighting function $S(\Theta_{mp})$, $\Theta_{mp} = \Theta_m + 2p\pi$, which describes interaction of relativistic $\beta = 1$ particle with p-th spatial harmonics in the field expansion (3):

$$S(\Theta_{mp}) = \int_0^L cos(\frac{\Theta_{mp}z}{d})e^{\frac{i\omega_m z}{c}}dz = \tag{4}$$
$$= \frac{1}{2}\{\frac{e^{iL(\frac{\omega_m}{c}+\frac{\Theta_{mp}}{d})}-1}{i(\frac{\omega_m}{c}+\frac{\Theta_{mp}}{d})} + \frac{e^{iL(\frac{\omega_m}{c}-\frac{\Theta_{mp}}{d})}-1}{i(\frac{\omega_m}{c}-\frac{\Theta_{mp}}{d})}\}.$$

The functions $\Re S(\Theta)$, $\Im S(\Theta)$ and $|S(\Theta)|$ are plotted in

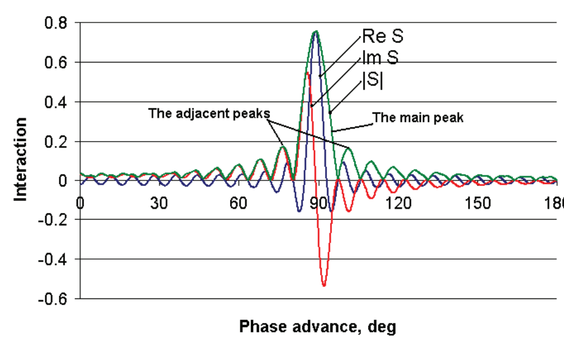

Figure 1: The functions $\Re S(\Theta)$, $\Im S(\Theta)$ and $|S(\Theta)|$ for the first monopole passband of the LOLA IV deflecting structure, $N = 46$

Fig. 1 for the first monopole passband of the LOLA IV [3] deflecting structure.

The maximal value $\Re S(\Theta_s)_{max} = |S(\Theta_s)|_{max} = \frac{L}{2} = \frac{Nd}{2}$ corresponds to the synchronous interaction $\frac{\omega_m}{c} = \frac{\Theta_s}{d}$ and linearly increases with the increasing number of cells in the structure. The width of the main peak $|S(\Theta)|$ (the distance in Θ between points $|S(\Theta)| = 0$ is $d\Theta = \frac{4\pi}{N}$ and for longer structure the main peak is narrowed. Because the separation between standing wave modes at Θ axis is of $\frac{\pi}{N}$, all time three or four modes belong to the main peak. The adjacent peaks in $|S(\Theta)|$, see Fig. 1, are placed at the distance $\delta\Theta = \pm\frac{3\pi}{N}$ with respect to the main peak $\Theta = \Theta_s$. The width of adjacent peaks is $\frac{2\pi}{N}$ - twice less, as compared to the width of the main peak. The maximal values of the

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adjacent peaks are $|S(\Theta_s \pm \delta\Theta)| = \frac{Nd}{3\pi}$ and also linearly rise with the increasing number of cells in the structure. But, with respect to the main peak, the adjacent peaks are lower in $1.5\pi \approx 4.7$ times. In the expression for k^l_m, k^t_m the values (1) the weighting function $S(\Theta)$ is in the second power and all modes, which do not belong to the $\pm\frac{2\pi}{N}$ vicinity of the synchronous point, will have at least one order lower k^l_m, k^t_m values.
In the interaction of the particle with the field in the periodical structure the synchronism has the effect of a band pass filter - only three modes with the phase advance $\Theta_m = \Theta_s \pm \frac{2\pi}{N}$ in the vicinity of the synchronous point can have a significant k^l_m, k^t_m values.
One can directly check with (4), that total loss or kick factor of modes owned by the main peak, is approximately twice as for synchronous interaction, regardless to the precise modes position with respect Θ_s, $\sum k^{l,t}_m \approx 2k^{l,t}_s$.

FREQUENCY DOMAIN APPROACH

The method, essentially based on the properties of the structure periodicity, was developed to estimate in the wide frequency range k^l_m and k^t_m values of the Transverse Deflecting Structure (TDS) for special beam diagnostic in the European XFEL. XFEL TDS has sections of the constant impedance deflecting structure with $N = 16$ and $N = 46$. The TDS prototype for $N = 14$ is considered in [4].
At first, considering only one period of the structure, we simulate with traveling wave regime in wide frequency range frequencies ω_n and complex fields distributions E_{zn} of all modes for the reference set of the phase advance $\Theta_n = \frac{n\pi}{N_1}, n = 0, 1, ..., N_1$. To simplify simulations, symmetry properties of the structure should be used actively. Defining for the m-th mode three intermediate functions,

$$I^{ee,eo,oo}_m(\Theta_n) = \int_0^{d/2} \begin{matrix} \Re E_{zm} cos \\ \Re E_{zm} sin \\ \Im E_{zm} sin \end{matrix} (\frac{\omega_m z}{c}) dz, \quad (5)$$

and taking into account (3), we can express the integral in (1) as:

$$\frac{k^l_m}{U_M} = (\frac{I^{ee}_m(\Theta_m)\{1 + (-1)^m cos(N\phi_m) + 2S^{cc}_m\}}{U_m} + \quad (6)$$
$$+\frac{I^{eo}_m(\Theta_m)(-1)^m sin(N\phi_m) - 2I^{oo}_m(\Theta_m)S^{ss}_m}{U_m})^2 +$$
$$+(\frac{I^{ee}_m(\Theta_m)\{(-1)^m sin(N\phi_m) + 2S^{cs}_m\}}{U_m} +$$
$$+\frac{I^{eo}_m(\Theta_m)\{1 - (-1)^m cos(N\phi_m)\} + 2I^{oo}_m(\Theta_m)S^{sc}_m}{U_m})^2$$

where the coefficients

$$S^{cc,cs,sc,ss}_m = \sum_{n=1}^{N-1} \begin{matrix} cos \\ cos \\ sin \\ sin \end{matrix} (n\Theta_m) \begin{matrix} cos \\ sin \\ cos \\ sin \end{matrix} (n\phi_m) \quad (7)$$

are discrete analogs of the weighting function $S(\Theta_{mp})$ in (4) with the same filtering properties. The intermediate functions in (5) are slow functions of the phase advance Θ_n and for intermediate Θ_m values one can apply smooth interpolation. Similar expression can be obtained for kick factor values k^t_m, (1).
The relation (6) describes the general case of interaction. In the case of synchronism this formula is more simple:

$$k^l_{m_{max}} = \frac{N^2(I^{ee}(\Theta_s) + (-1)^{n_b} I^{oo}(\Theta_s))^2}{U_m}, \quad (8)$$

where $n_b = \frac{\omega_s \pi}{\omega_0 \Theta_0}$ is the number of Brillouine zone for synchronous particle - wave interaction. The relation (8) provides also the upper estimation for k^l_m, k^t_m values in each passband of the structure.
The results of this method were compared with k^l_m, k^t_m values, obtained from the direct frequencies and fields distribution for the lowest monopole and dipole passbands in TDS sections with $N = 16$ and $N = 46$ and one can see comparison in Fig. 2.
The method has been applied to estimate k^l_m, k^t_m values

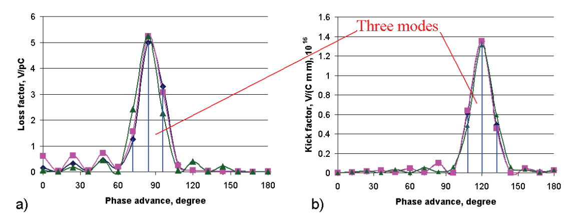

Figure 2: The loss factor k^l_m values calculated from direct field distributions simulations (blue) and from field reconstruction (magenta) for the TDS with $N = 16$ cells (a) and $N = 46$ (b). The green triangles are for the full end cell terminations.

in the XFEL TDS sections and results to be published in details in another papers. For example, in Fig.3 the TDS dispersion diagram for monopole modes in wide frequency range.
The frequency resolution of our method doesn't depend

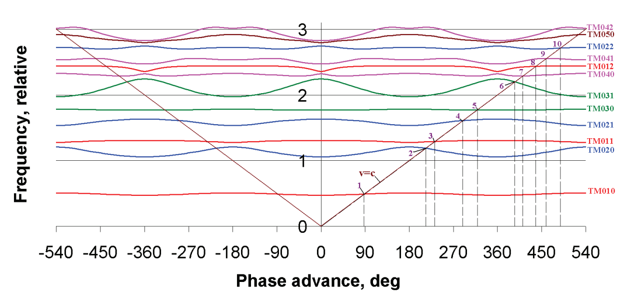

Figure 3: The diagram of interaction for the particle $v = c$ with monopole modes in the TDS XFEL structure.

both on the structure type and number of cells in the structure. All time for each passband three or four modes are allocated with maximal k^l_m, k^t_m values for this passband.

TIME DOMAIN SIMULATIONS

For more confidence, the special set of simulations with our method has been performed for LOLA IV structure and results for $N = 104$ were compared with results of time domain simulations [5].
In Fig. 4 the longitudinal wake potential, calculates with PS CST [6] for Gaussian bunch with $\sigma = 6mm$ is shown at the distance $s = 500m$ for the LOLA IV structure with $N = 16$. One can clearly see the beating with the length $\lambda_b \approx 25m$, corresponding to the frequency of $\frac{c}{\lambda_b} \approx 12MHz$. It is exactly the frequency difference between neighbor modes at the first monopole passband in LOLA IV for $N = 16$ in the vicinity of synchronous point $\Theta_s \approx 96^o$. In general case the modes separation in frequency for vicinity of synchronous point is $\delta\omega = \frac{\omega_s \beta_g \pi}{N\Theta_s}$, where β_g is the group velocity.
To extract the wake impedance from wake potential with the Fourier transform, highlighting the contributing modes, transform resolutions should be much better than $\delta\omega$.

To extract contributing modes reliably, the wake potential should be calculated at the distance $s > \frac{2\pi c}{\delta\omega} \sim \frac{cN\Theta_s}{\pi f_s \beta_g}$. Another case the information in wake potential is not sufficient for reliable modes separation. In Fig.5 the plots are shown for wake impedance, extracted with sufficient Fourier transform resolution from longitudinal wake potential, calculated at the distances 5m, 25m and 50m.

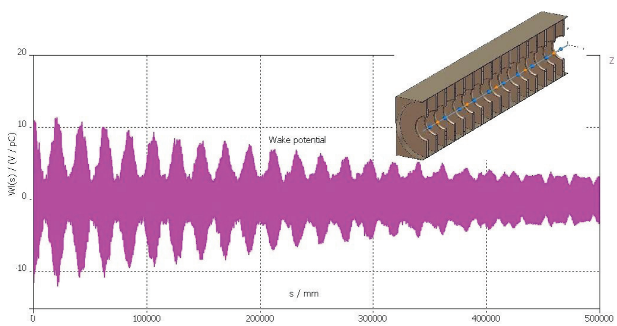

Figure 4: The longitudinal wake potential in LOLA IV structure ($N = 16$) at the distance of 500 m.

With simulations at short distance with time domain approach one get the total value of the wake impedance for the total passband, without separation to contributing modes.

SUMMARY

In the interaction of particle with finite length periodical structure at the phase advance scale in each structure passband there is the zone of effective interaction in the vicinity of synchronous interaction. The width of this zone decreases with increasing number of periods in the structure, but all time four or three modes at each passband belong to such zones. The total value of loss or kick factors for modes owned by each such zone is approximately twice as for synchronous interaction, regardless to precise modes placements inside zone.
Frequency domain approach, essentially use periodicity properties of the structure, developed to estimate long range wakefields of the structure. The method resolution doesn't depends on the structure type and number of periods in the section, all time showing the fine structure for the zones of effective interaction at each passband.
With time domain simulation the modes separation is also possible, but for long structures with small group velocity requires wake potential calculation at long distance and results treatment with high Fourier transform resolution.

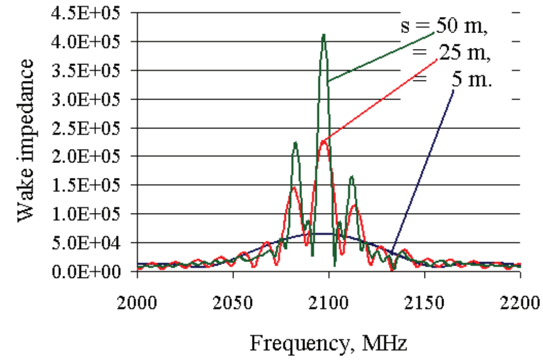

Figure 5: The calculated wake impedance of monopole modes in LOLA IV structure for simulations $s = 5m, 25m$ and $50m$, $N = 16$

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