MULTIPOLE EXPANSION OF THE FIELDS IN SUPERCONDUCTING HIGH-VELOCITY SPOKE CAVITIES *

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Abstract

Multi-spokes superconducting cavities in the high-beta regime are being considered for a number of applications. In order to accurately model the dynamics of the particles in such cavities, knowledge of the fields off-axis are needed. We present a study of the multipoles expansion of the fields from an EM simulation field data for two-spokes cavities operating at 325 MHz, $\beta = 0.82$, and 500 MHz, $\beta = 1$.

INTRODUCTION

In recent years there has been considerable progress in the optimization and characterization of accelerating multi-spoke cavities in the high-velocity regime ($\beta_0 = v_0/c > 0.6$). Multi-spokes cavities have several advantages over the more traditional multi-cell elliptical cavities [1, 2], such as increased longitudinal acceptance, lower superconducting surface resistance and therefore lower heat load allowing for operation at 4K, and a cavity diameter on the order of half the rf wavelength, which allows for lower operating frequencies for a given size.

The lack of cylindrical symmetry of the multi-spokes cavities might result in the presence of non-negligible higher order multipole components of the operating mode. In order to study the impact of the cavity fields on the beam dynamics, it is necessary to characterize off-axis fields. To this end, we present a study of the multipoles expansion of the fields for the operating (fundamental) cavity mode for spokes operating at 325 MHz, $\beta = 0.82$, and 500 MHz, $\beta = 1$.

MULTIPOLAR RF KICKS AND FIELD EXPANSION

It has recently been demonstrated that surface fitting of field data around a virtual cylinder in the proximity of the beamline can be used to compute high-order transfer maps, interior fields, and non-linear beam dynamics including high-order multipole effects [3, 4].

From the interpolation of the electromagnetic fields on the surface of a virtual cylinder of radius R through the length of the cavity in a region with no charge, a general complete solution to the fields can be found. The accelerating electric field can then be expressed as:

$$E_{acc}(r,\phi,z) = E_z e^{i\omega t} = E^{(0)}_{acc-c} + \left(\sum_{n=1}^{\infty} r^n \left(E^{(n)}_{acc-c} cos(n\phi) + E^{(n)}_{acc-s} sin(n\phi)\right)\right) e^{i\omega t} \quad (1)$$

Where E_{acc} is the accelerating electric field along the beamline direction z and ω is the rf frequency. If the field $E_{acc}(R,\phi,z)$ is known, then the coefficients $E^{(0)}_{acc-c}$, $E^{(n)}_{acc-c}$ and $E^{(n)}_{acc-s}$ can be found via inverse Fourier integration of the surface fitted field.

Inserting this series into the Panofsky-Wenzel theorem we may then calculate the transverse change in momentum (kick) of the particles as a series of multipolar RF kicks:

$$\Delta p_\perp = \left(\frac{e}{\omega}\right) \int_{-\infty}^{+\infty} (-i)\nabla_\perp E_{acc}|_{t=z/v_z} dz \quad (2)$$

To get an idea of the impact of the nonlinear effects of the higher order field multipoles, it is useful to compare the strength of the multipolar RF-kicks with that of magnetic multipoles [7]. It is usual to express the magnetic potential and static magnetic kick to incoming particles as a Taylor series expansion [5, 6]. In magnets, the a_n and b_n terms are determined by the orientation of the magnet, such that a_1 corresponds to a vertical bend of the particle trajectory, b_1 to a horizontal bend, and in general a_n correspond to skew oriented magnets and b_n to normal oriented magnets. For simplicity we will continue to use this nomenclature. Obviously, the b_0 or monpole term is not dependent on the orientation of the cavity and corresponds to the accelerating voltage across the cavity. Following this idea, we find:

$$\begin{aligned} b_0 &= \int_{-\infty}^{+\infty} E^{(0)}_{acc-c}(z)dz, \ \propto V_{acc} \\ b_n &= \int_{-\infty}^{+\infty} \frac{ni}{\omega} E^{(n)}_{acc-c}(z)dz, (n = 1, 2, ...) \\ a_n &= \int_{-\infty}^{+\infty} \frac{ni}{\omega} E^{(n)}_{acc-s}(z)dz, (n = 1, 2, ...) \end{aligned} \quad (3)$$

Additionally, the rf multipole strength depends of the rf phase of particles traversing the cavity; so we can express them as complex numbers, where the real part corresponds to particles on crest, and the imaginary part corresponds to particles off-crest.

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RF CAVITIES

We are in the process of design and characterization of a 325 MHz, $\beta = 0.82$ cavity; the optimization procedure, multipacting, and higher-order mode analyses have been presented elsewhere [8, 9, 10]. The 3D electromagnetic fields were obtained using CST Microwave Studio® 2012 (CST MWS). Table 1 lists the cavity's relevant RF parameters.

Table 1: RF Parameters of the 325 MHz cavity (At E_{acc} = 1 MV/m and reference length $\beta_0\lambda$)

Parameter		Units
Frequency	325	MHz
β_0	0.82	
R/Q	629	Ω
$(R/Q)*(QR_s)$	99200	Ω^2
E_p/E_{acc}	2.8	
B_p/E_{acc}	5.3	mT/(MV/m)
Bp/Ep	1.9	mT/(MV/m)
Energy Content	0.445	J

In order to get a better understanding of the beam dynamics, we present in Fig. 1 a plot of the transverse voltage (in the x and y directions) as a function of transverse offset from the design beamline. It is clear that focusing would occur in one direction while defocusing would occur in the other. This behavior is characteristic of quadrupole fields, so we may conclude that the dominant higher order multipole is a quadrupole. The transverse voltage due to higher-order multipoles is less than 1% with respect to the accelerating voltage. An open picture of the cavity with the electric surface fields is shown as well.

The results of the calculation of the multipole strength numbers are presented in table 2, up to the octupole terms. The numbers have been normalized to simulation field data such that V_{acc} = 1 V; notice that the deviation from this value gives us a measure of the error in our surface fitting calculation.

Table 2: RF Multipole components, normalized from simulation field data to $V_{acc} = 1V$, of the 325 MHz, $\beta = 0.82$ cavity.

Multipole component		Units
V_{acc}	0.89	V
V_x	0	V
V_y	0	V
b_2	-0.01 - 1.09 i	nT
a_2	0.003+0.22 i	nT
b_3	0	nT/m
a_3	0	nT/m
b_4	23.8 - 128.7 i	nT/m^2
a_4	-9.9 + 53.3 i	nT/m^2

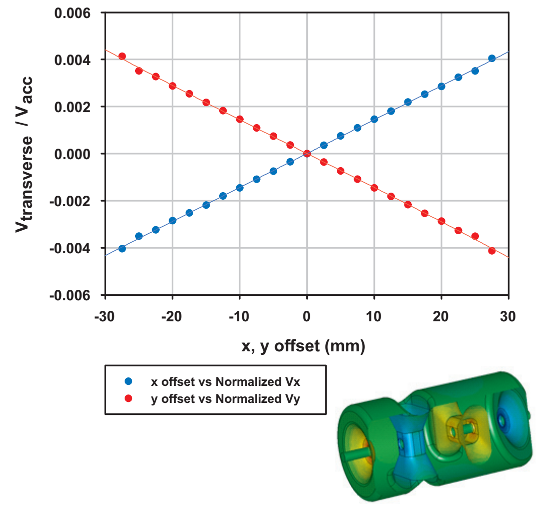

Figure 1: Normalized transverse voltage as a function of transverse offset, for the 325 MHz, $\beta = 0.82$ cavity. A CST MWS picture of the surface fields of the open cavity is shown for reference.

We have designed cavities operating at an rf frequency of 500 MHz, β = 1, optimized with racetrack-shaped spokes. It has recently been observed that there may be geometries in which the quadrupole effects can be minimized, specifically ring-shaped spokes, which exhibit more symmetry about the beamline [11]. Table 3 presents the relevant RF parameters for a cavity with racetrack shaped spokes, and for a cavity with ring spokes. This latter geometry has not been fully optimized yet for high shunt-impedance and/or low surface-fields, however it already exhibits promising rf characteristics.

Table 3: RF Parameters of the 500 MHz cavities (At E_{acc} = 1 MV/m and reference length $\beta_0\lambda$)

Parameter	Racetrack spokes	Ring spokes	Units
Frequency	500	500	MHz
β_0	1	1	
R/Q	725	737	Ω
$(R/Q)*(QR_s)$	124000	118000	Ω^2
E_p/E_{acc}	3.5	3.9	
B_p/E_{acc}	5.4	7.1	mT/(MV/m)
Bp/Ep	1.6	1.8	mT/(MV/m)
Energy Content	0.158	0.155	J

Picures of the cavities with racetrack-shaped spokes and ring-shaped spokes are presented in Figs. 2 and 3 respectively. Also presented are a plot of the transverse voltage field with respect to spatial offset from the design beamline. It is clear that there is a noticeable reduction in the

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quadrupole strength, and an increase in effects from higher order multipoles, as evinced by the slightly oscillatory behavior. The same can be concluded from the multipole strength numbers presented in table 4, while we observe that the octupole term has indeed increased.

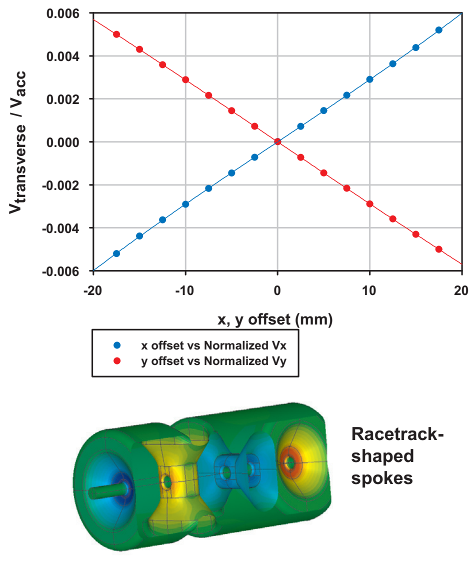

Figure 2: Normalized transverse voltage as a function of transverse offset, for the 500 MHz, $\beta = 1$ cavity with wide racetrack-shaped spokes. A CST MWS picture of the surface fields of the open cavity is shown for reference.

Table 4: RF Multipole components, normalized from field simulation data to $V_{acc} = 1V$, of the 500 MHz, $\beta = 1$ cavities.

Multipole component	Racetrack-shaped spokes	Ring-shaped spokes	Units
V_{acc}	1	1	V
V_x	0	0	V
V_y	0	0	V
b_2	-0.05 - 1.44 i	-0.002 - 0.66 i	nT
a_2	0.003+0.22 i	0 + 0.09 i	nT
b_3	0	0	nT/m
a_3	0	0	nT/m
b_4	304.5 +42.4 i	543.5 - 85.1 i	nT/m^2
a_4	-9.9 + 53.3 i	-146.6 + 22.8 i	nT/m^2

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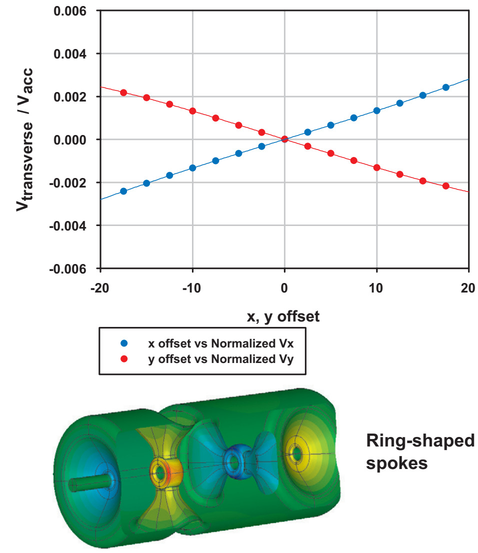

Figure 3: Normalized transverse voltage as a function of transverse offset, for the 500 MHz, $\beta = 1$ cavity with ring-shaped spokes. A CST MWS picture of the surface fields of the open cavity is shown for reference.

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