ELECTROMAGNETIC TORQUE FROM LINAC RADIATION*

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Abstract

In this paper the new phenomenon of nature, called electromagnetic torque radiation from the relativistic charged particles is discussed. To begin it is shown that two well- known alternative definitions of density of angular momentum of electromagnetic field by Ivanenko-Sokolov and by Teitelboim and Villarroel give the identical integral characteristics with application of the relativistic radiation theory. And both of it yield the same results for the total power of the angular momentum, which is characterized the torque of the radiation. Then we have found that the angular distribution of torque from the Linac has the azimuthal symmetry with respect to the direction of the velocity of the particle. It is also oppositely directed to the acceleration of the particle. On the condition of the high speed the angular distribution has an expressive relativistic effect of the sharp directed radiation. With the construction of a good detectors of the torque it is possible to measure such effect.

INTRODUCTION

The hypothesis that electromagnetic waves have proper angular momentum was put forward by A. I. Sadowsky as long ago as 1897 [1]. He proposed the method of measurement for angular momentum of light based on light transmission through anisotropic crystalline plate: «…any apparatus processing linearly polarized light into circularly polarized must rotate…».

In 1935-1936, B. A. Beth in USA [2] and A. N. S. Holborn in England [3] experimentally proved that circularly polarized light has angular momentum. These precise experiments not only confirmed Sadowsky's hypothesis, but also allowed to estimate Planck's constant accurate within 10%.

Observation of considerably greater torque became possible with the emergence of lasers (see, for example, [4]). The existence of angular momentum of circularly polarized electromagnetic waves in the present time has no doubt about.

However the general definition of angular momentum of the electromagnetic field (AMEF) is argued over among physicists to this day. Contentious debates about the adequacy of the theory of angular momentum and its radiation arise from time to time [5-9].

The base of the stated here the radiation theory of AMEF is exact methods of the relativistic radiation theory of the arbitrarily moving charge [10]. As an attachment there are considered properties of the orbital and spin moments of the synchrotron radiation with the specific source of this radiation such as an electron.

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TWO ALTERNATIVE METHODS OF THE DEFINITION OF THE AMEF

It may be distinguished two alternative ways of the relativistically covariant methods for the description of AMEF originated by D. D. Ivanenko and A. A. Sokolov [11] as well as by C. Teitelboim et al. [12-14].

First from these methods was based on the principle of least action for the electromagnetic field. This principle leads to the conservation laws of the energy density of AMEF in the differential form:

$$D_\lambda \mathcal{M}^{\mu\nu\lambda} = 0. \tag{1}$$

Here

$$\mathcal{M}^{\mu\nu\lambda} = "\mathcal{A}^{\mu\nu\lambda}" + "\mathcal{T}^{\mu\nu\lambda}" \tag{2}$$

is the density tensor of the total AMEF which is interpreted at the suggestion of Ivanenko and Sokolov as the sum of the «orbital» and «spin» parts of AMEF:

$$"\mathcal{A}^{\mu\nu\lambda}" = R^\mu \mathcal{P}^{\nu\lambda}_{can} - R^\nu \mathcal{P}^{\mu\lambda}_{can}, \tag{3a}$$

$$"\mathcal{T}^{\mu\nu\lambda}" = \frac{1}{4\pi c}\left(A^\mu H^{\nu\lambda} - A^\nu H^{\mu\lambda}\right). \tag{3b}$$

In (3a) definition the tensor

$$\mathcal{P}^{\mu\nu}_{can} = -\frac{1}{4\pi c}\left(\partial_\mu A^\rho H_\rho^{\ \nu} + \frac{1}{4} g^{\mu\nu} H_{\alpha\beta} H^{\alpha\beta}\right) \tag{4}$$

is known as the canonical tensor of the momentum which is satisfied to the differential conservation low

$$D_\nu \mathcal{P}^{\mu\nu}_{can} = 0. \tag{5}$$

It should be noted that everywhere in our notations potentials and fields are located in the world point of observation with the coordinates $R^\mu = (c\tilde{t}, \mathbf{R})$, the four-dimensional vector-potential $A^\mu = (\varphi, \mathbf{A})$ and the field-strength tensor $H^{\mu\nu} = D^\mu A^\nu - D^\nu A^\mu$ with the derivative $D^\mu = \partial / \partial R_\mu$ in metric $g^{\mu\nu} = (-1,1,1,1)$. The purpose of the quotes presence in our notations $"\mathcal{A}^{\mu\nu\lambda}"$ and $"\mathcal{T}^{\mu\nu\lambda}"$ is due to the fact that the derivative $D_\lambda "\mathcal{A}^{\mu\nu\lambda}"$ is nonzero. This happens because the canonical tensor $\mathcal{P}^{\mu\nu}_{can}$ isn't symmetrical and, therefore, the separation of $\mathcal{M}^{\mu\nu\lambda}$ into «orbital» and «spin» parts isn't relativistically invariant. Another defect of the (2) decomposition is that density «spin» tensor $"\mathcal{T}^{\mu\nu\lambda}"$ isn't gauge invariant and so it can't claim to be the real observed value. In this connection widespread due to (3b) definition of the proper AMEF or «spin» of the electromagnetic field

$$\mathbf{\Pi} = \frac{1}{4\pi c}\int [\mathbf{E}\mathbf{A}] dV$$

is required in additional substantiation.

Alternative pathway was proposed by C. Teitelboim et al. [12-15] that the density tensor of the total AMEF

$$\mathcal{M}^{\mu\nu\lambda} = R^\mu \mathcal{P}^{\nu\lambda} - R^\nu \mathcal{P}^{\mu\lambda} \tag{6}$$

$$\mathcal{P}^{\mu\nu} = -\frac{1}{4}\left(H^{\mu\rho}H_{\rho}^{\ \nu} + \frac{1}{4}g^{\mu\nu}H_{\alpha\beta}H^{\alpha\beta} \right) = \mathcal{P}^{\nu\mu}, \tag{7}$$

which in contrast to $\mathcal{P}^{\mu\nu}_{can}$ is gauge invariant.

Tensor $\mathcal{P}^{\mu\nu}$ is calculated in the relativistic radiation theory of the charged particles according to that (see. [10])

$$H^{\mu\nu} = \bar{H}^{\mu\nu} + \tilde{H}^{\mu\nu}, \tag{8}$$

$$\bar{H}^{\mu\nu} = ec^2\frac{\tilde{r}^{\mu}\upsilon^{\nu} - \tilde{r}^{\nu}\upsilon^{\mu}}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^3} \sim \frac{1}{\tilde{r}^2} \tag{8a}$$

is the strength tensor of the attached fields in the near-charge zone and

$$\tilde{H}^{\mu\nu} = e\left\{ \frac{\tilde{r}^{\mu}\upsilon^{\nu} - \tilde{r}^{\nu}\upsilon^{\mu}}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^3}\tilde{r}_{\rho}\omega^{\rho} - \frac{\tilde{r}^{\mu}\omega^{\nu} - \tilde{r}^{\nu}\omega^{\mu}}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^2} \right\} \sim \frac{1}{\tilde{r}} \tag{8b}$$

is in the long-distance region which is called the radiation zone.

Here e is the charge of an arbitrary moving relativistic particle, $\upsilon^{\mu} = dr^{\mu}/d\tau$ it's four-velocity, $\omega^{\mu} = d\upsilon^{\mu}/d\tau$ it's four-acceleration of the charge the location of which is described by the trajectory radius-vector $r^{\mu}(\tau)$, τ is a proper time and

$$\tilde{r}^{\mu}(\tau, \tilde{t}) = R^{\mu}(\tilde{t}) - r^{\mu}(\tau) \tag{9}$$

is the four-dimensional vector traced from the charge to the spectator point of four-dimensional vector $R^{\mu}(\tilde{t})$.

In Teitelboim's representation the decomposition of the density of total AMEF into orbital and spin parts arise automatically according to (6) and (9)

$$\mathcal{A}^{\mu\nu\lambda} = r^{\mu}\mathcal{P}^{\nu\lambda} - r^{\nu}\mathcal{P}^{\mu\lambda} \ , \ \mathcal{T}^{\mu\nu\lambda} = \tilde{r}^{\mu}\mathcal{P}^{\nu\lambda} - \tilde{r}^{\nu}\mathcal{P}^{\mu\lambda}. \tag{10}$$

The dependence of the falling of fields strength according to its dependence on the distance $\tilde{r}$ takes a significant role in these definitions. Tensor $\mathcal{P}^{\nu\lambda}$ disintegrates on the three parts in accordance with general theory of relativistic radiation (see, for example [15])

$$\mathcal{P}^{\mu\nu} = \bar{\mathcal{P}}^{\mu\nu} + \bar{\tilde{\mathcal{P}}}^{\mu\nu} + \tilde{\mathcal{P}}^{\mu\nu}. \tag{11}$$

where

$$\bar{\mathcal{P}}^{\mu\nu} = \frac{1}{4\pi}\frac{e^2c^3}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^4}\left\{ \frac{1}{2}g^{\mu\nu} - \frac{\tilde{r}^{\mu}\upsilon^{\nu} + \tilde{r}^{\nu}\upsilon^{\mu}}{\tilde{r}_{\rho}\upsilon^{\rho}} - c^2\frac{\tilde{r}^{\mu}\tilde{r}^{\nu}}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^2} \right\} \sim \frac{1}{\tilde{r}^4} \tag{11a}$$

is the momentum density tensor of the convection field in the near zone,

$$\bar{\tilde{\mathcal{P}}}^{\mu\nu} = \frac{1}{4\pi}\frac{e^2c^3}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^3}\left\{ \frac{\tilde{r}^{\mu}\omega^{\nu} - \tilde{r}^{\nu}\omega^{\mu}}{\tilde{r}_{\rho}\upsilon^{\rho}} - \frac{\tilde{r}^{\mu}\upsilon^{\nu} + \tilde{r}^{\nu}\upsilon^{\mu}}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^2}\tilde{r}_{\lambda}\omega^{\lambda} - \frac{2c^2\tilde{r}^{\mu}\tilde{r}^{\nu}\tilde{r}_{\rho}\omega^{\rho}}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^3} \right\} \sim \frac{1}{\tilde{r}^3} \tag{11b}$$

is the momentum density tensor of the mixed fields and ,

$$\tilde{\mathcal{P}}^{\mu\nu} = -\frac{1}{4\pi}\frac{e^2}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^2}\left\{ c^2\frac{\left(\tilde{r}_{\lambda}\omega^{\lambda}\right)^2}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^4} - \frac{\omega_{\lambda}\omega^{\lambda}}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^2} \right\}\tilde{r}^{\mu}\tilde{r}^{\nu} \sim \frac{1}{\tilde{r}^2}. \tag{11c}$$

is the momentum density tensor of the radiation field in the wave zone.

In the following we will consider the AMEF only in the wave zone where in accordance with (10)-(11) it follows that

$$\tilde{\mathcal{A}}^{\mu\nu\lambda} = \frac{e^2}{4\pi c}\frac{1}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^2}\left\{ \frac{c^2\left(\tilde{r}_{\lambda}w^{\lambda}\right)^2}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^4} + \frac{w_{\lambda}w^{\lambda}}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^2} \right\}\left(r^{\mu}\tilde{r}^{\nu} - r^{\nu}\tilde{r}^{\mu}\right)\tilde{r}^{\lambda} \sim \frac{1}{\tilde{r}^2}, \tag{12a}$$

$$\tilde{\mathcal{T}}^{\mu\nu\lambda} = -\frac{e^2c}{4\pi}\left\{ \frac{\tilde{r}^{\mu}\upsilon^{\nu} - \tilde{r}^{\nu}\upsilon^{\mu}}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^2}\tilde{r}_{\rho}w^{\rho} - \frac{\tilde{r}^{\mu}w^{\nu} - \tilde{r}^{\nu}w^{\mu}}{\tilde{r}_{\rho}\upsilon^{\rho}} \right\}\frac{\tilde{r}^{\lambda}}{\left(\tilde{r}_{\rho}\upsilon^{\rho}\right)^3} \sim \frac{1}{\tilde{r}^2}. \tag{12b}$$

In terms of the definition

$$D^{\mu} = \frac{\partial}{\partial R^{\mu}} = \tilde{\partial}^{\mu} + \frac{\tilde{r}^{\mu}}{\tilde{r}_{\rho}\upsilon^{\rho}}\frac{d}{d\tau} \tag{13}$$

applying the differentiation technique of the retarded in time fields [10,12] it can be shown that

$$D_{\lambda}\mathcal{P}^{\nu\lambda} = 0, \tag{14}$$

$$D_{\lambda}(\bar{\mathcal{P}}^{\nu\lambda} + \bar{\tilde{\mathcal{P}}}^{\nu\lambda}) = 0 \tag{15a}$$

and

$$D_{\lambda}\tilde{\mathcal{P}}^{\nu\lambda} = 0. \tag{15b}$$

From this, one obtains

$$D_{\lambda}\tilde{\mathcal{M}}^{\mu\nu\lambda} = 0, \tag{16}$$

follows from

$$D_{\lambda}\tilde{\mathcal{A}}^{\mu\nu\lambda} = D_{\lambda}\tilde{\mathcal{T}}^{\mu\nu\lambda} = 0. \tag{17}$$

Thus, the decomposition of the density of the total AMEF onto orbital and spin parts now is gauge- and relativistically invariant and so we will assume Teitelboim's method as basis for our further research. And in accordance with the theory of relativistic radiation in wave zone the canonical momentum density tensor becomes symmetrical:

$$\tilde{\mathcal{P}}^{\mu\nu}_{can} = \tilde{\mathcal{P}}^{\nu\mu}_{can}$$

Hence the previously denoted defects of this method disappear! We will also use the method of D.D. Iva-nenko and A. A. Sokolov for the comparison.

RELATIVISTIC THEORY OF THE AMEF

Then in accordance with (17) we can receive orbital and spin momenta of the radiation field on basis of Gauss's integral theorem in the form

$$\tilde{L}^{\mu\nu} = \oint \tilde{\mathcal{A}}^{\mu\nu\lambda}d\sigma_{\lambda}, \ \ \tilde{\Pi}^{\mu\nu} = \oint \tilde{\mathcal{T}}^{\mu\nu\lambda}d\sigma_{\lambda}. \tag{18}$$

Here

$$d\sigma_{\lambda} = e_{\lambda}\varepsilon^2 c\,d\tau\,d\Omega_0 \tag{19}$$

is the element of spacelike hypersurface with the normal vector

$$e_{\lambda} = \frac{\tilde{r}_{\lambda}}{\varepsilon} - \frac{\upsilon_{\lambda}}{c}, \tag{20}$$

$$d\Omega_0 = \frac{\varepsilon^2}{\tilde{r}^2}d\Omega \tag{21}$$

is the solid-angle element in the rest frame of particle. As

$$\mathcal{A}^{\mu\nu\lambda}\tilde{r}_{\lambda} = 0, \ \ \mathcal{T}^{\mu\nu\lambda}\tilde{r}_{\lambda} = 0, \tag{22}$$

then the tensors $\mathcal{A}^{\mu\nu\lambda}$ and $\mathcal{T}^{\mu\nu\lambda}$ are lightlike and the whole of the radiation of AMEF move from the world line in the direction of generatrices of the light cone.

Integrating formulas (18) over the angles in the rest frame by well-known (see [10]) integrals

$$\frac{1}{4\pi}\int d\Omega_0 = 1, \ \ \frac{1}{4\pi}\int e^{\lambda}d\Omega_0 = 0,$$

$$\frac{1}{4\pi}\int e^{\lambda}e^{\nu}d\Omega_0 = \frac{1}{3}(g^{\lambda\nu} + \frac{1}{c^2}\upsilon^{\lambda}\upsilon^{\nu}), \tag{23}$$

we can receive expressions for the power of the orbital and spin AMEF radiation in the wave zone

$$\frac{d\tilde{L}^{\mu\nu}}{d\tau} = \frac{2}{3}\frac{e^2}{c^5}\omega_\rho\omega^\rho\left(r^\mu\upsilon^\nu - r^\nu\upsilon^\mu\right),$$
$$\frac{d\tilde{\Pi}^{\mu\nu}}{d\tau} = \frac{2}{3}\frac{e^2}{c^3}\left(\upsilon^\mu\omega^\nu - \upsilon^\nu\omega^\mu\right). \qquad (24)$$

It's interesting that on the method of D. D. Ivanenko and A. A. Sokolov yields the same results!

The power of the radiation of spin momentum is proportional to the Thomas's precession frequency.

THE INSTANTANEOUS INDICATRIX OF TORQUE RADIATION

To provide the estimates let's discuss the orbital part of angular momentum of the beam of N electrons. According to the Figure 1 :

$$\boldsymbol{\beta} = \left(0,0,\beta\right),\ a = a\left(\sin\alpha, 0, \cos\alpha\right), \qquad (27)$$
$$\mathbf{n} = \left(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\right)$$

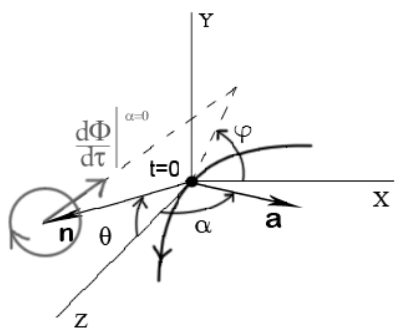

Figure 1: Instantaneous frame.

Thus, for synchrotron radiation $\left(\alpha = \pi/2\right)$

$$\frac{d\tilde{L}}{dt} = \frac{e_0^2 a}{4\pi c^2}\beta^3\oint f\left(\theta,\varphi\right)d\Omega,\ \frac{d\tilde{\Pi}}{dt} = \frac{e_0^2 a}{4\pi c^2}\frac{\beta}{\gamma^2}\oint f\left(\theta,\varphi\right)d\Omega, \qquad (28)$$

$$f\left(\theta,\varphi\right) = \frac{\gamma^2}{\left(1-\beta\cos\theta\right)^3} - \frac{\sin^2\theta\cos^2\varphi}{\left(1-\beta\cos\theta\right)^4}. \qquad (35a)$$

It is shown that instantaneous indicatrix have a relativistic large spread shape as for the AMEF radiation.

It is interesting to note that both spin and orbital indicatrises of radiation have the same space configuration. The only difference is a scale ember $\gamma^2\beta^2$.

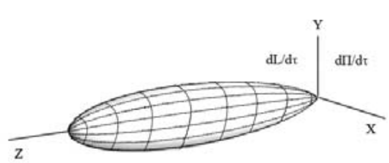

Figure 2: Relativistic instantaneous indicatrix.

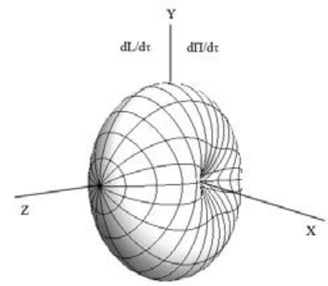

Figure 3: Nonrelativistic instantaneous indicatrix.

CONCLUSION

If we consider the Linac radiation as a synchrotron injector the maximum value of the torque would be caused by the orbital part of the angular momentum.

Summing up, it can be stated, the work presented here discovers a new theoretical basis in the relativistic theory of radiation and the spin properties of relativistic particles for the following experimental investigation.

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