CAVITY CONTROL ALGORITHMS*

Tomasz Plawski, Curt Hovater
Jefferson Lab, Newport News VA, USA

Abstract

A digital low level radio frequency (RF) system typically incorporates either a heterodyne or direct sampling technique, followed by fast ADCs, then an FPGA, and finally a transmitting DAC. This universal platform opens up the possibilities for a variety of control algorithm implementations. The foremost concern for an RF control system is cavity field stability, and to meet the required quality of regulation, the chosen control system needs to have sufficient feedback gain. In this paper we will investigate the effectiveness of the regulation for three basic control system algorithms: I&Q (In-phase and Quadrature), Amplitude & Phase and digital SEL (Self Exciting Loop) along with the example of the Jefferson Lab 12 GeV cavity field control system.

INTRODUCTION

Typical, modern LLRF systems utilize I&Q signal processing scheme rather than Amplitude and Phase control. Once in the I&Q domain, it is the choice of the designer to use them directly or convert them into amplitude and phase and then build a feedback loop. The behaviour of these two approaches could be however quite different when used to control a detuned resonator (cavity).

AMPLITUDE&PHASE VERSUS I&Q ARCHITECTURE

Figure 1 shows both schemes: I and Q and Amplitude and Phase. This is a simple feedback with a proportional gain of G, and there is only one disturbance acting on the system, the detuning angle φ (in radians). For the amplitude and phase (AP) system, the feedback equations are

$$\mathrm{Am}(G,\varphi) = G\cos(\varphi)[\mathrm{Am}_0 - \mathrm{Am}(G,\varphi)]$$

$$\mathrm{Ph}(G,\varphi) = G[\mathrm{Ph}_0 - \mathrm{Ph}(G,\varphi)] - \varphi$$

where Am is the amplitude, Ph is the phase, Am_0 and Ph_0 are control loop set points. For the I&Q system

$$I = \cos(\varphi)[G(I_0 - I)\cos(\varphi) + G(Q_0 - Q)\sin(\varphi)]$$

$$Q = \cos(\varphi)[-G(I_0 - I)\sin(\varphi) + G(Q_0 - Q)\cos(\varphi)]$$

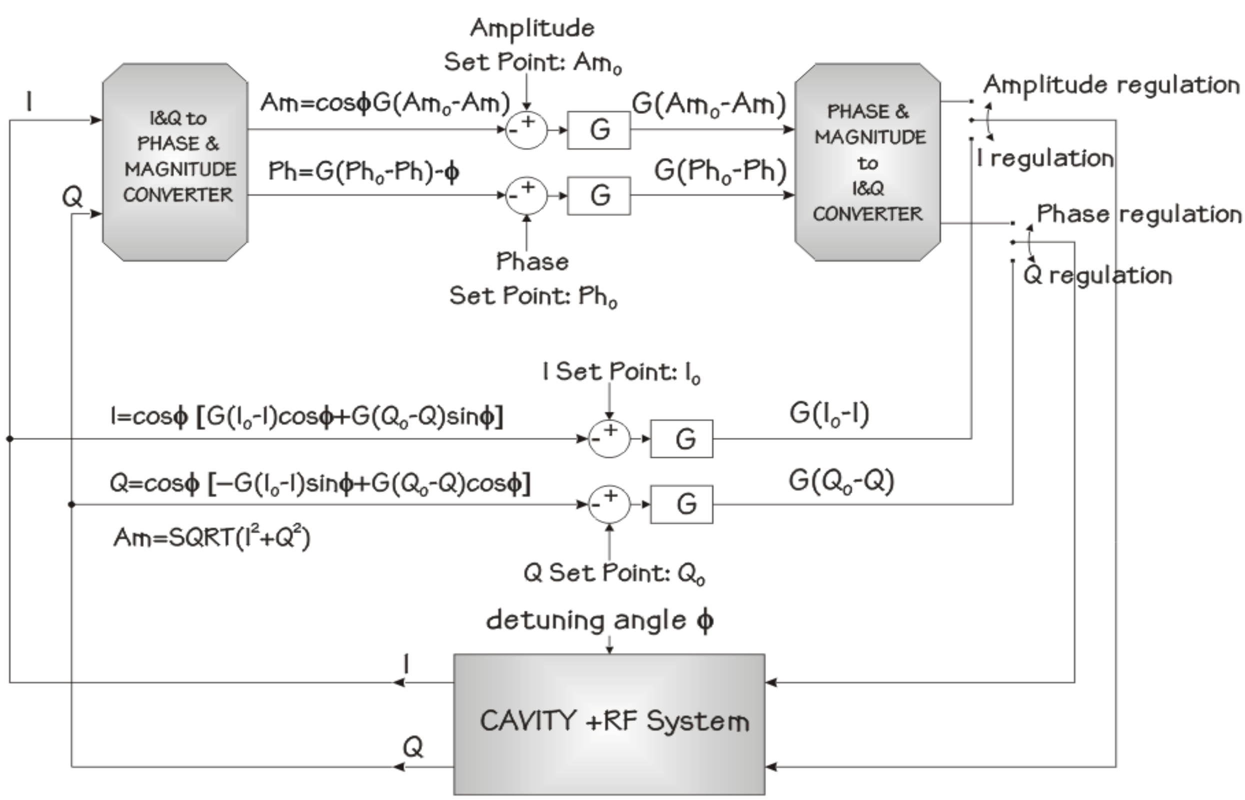

Figure 1: Amplitude&Phase versus I&Q regulation architecture.

* Notice: Authored by Jefferson Science Associates, LLC under U.S. DOE Contract No. DE-AC05-06OR23177.

Defining the relative amplitude error as a ratio of the amplitude difference (error) between the tuned and detuned system (φ=0 and $\varphi\neq$0) to the amplitude of the tuned system as a function of gain G and detuning angle φ gives

$$Am_{errorAP_r}(G,\varphi)=\frac{Am_0}{Am(G,0)}\frac{G(1-\cos(\varphi))}{(1+G)(1+G\cos(\varphi))}$$

and finally,

$$Am_{errorAP_r}(G,\varphi)=\frac{1-\cos(\varphi)}{1+G\cos(\varphi)}$$

Phase error can be calculated in a similar way

$$Ph_{errorAP}(G,\varphi)=G\frac{Ph_0}{1+G}-\frac{G\times Ph_0-\varphi}{1+G\times\cos(\varphi)}=\frac{\varphi}{1+G}$$

Calculation for the I&Q regulation is slightly more complicated. Starting with the following equations derived from the feedback equations

$$\begin{aligned}&I(1+G\cos^2(\varphi)+Q[G\cos(\varphi)\sin(\varphi)]\\&=G\cos(\varphi)[I_0\cos(\varphi)+Q_0\sin(\varphi)]\end{aligned}$$

and

$$\begin{aligned}&I(-G\cos(\varphi)\sin(\varphi)+Q[1+G\cos^2(\varphi)]\\&=G\cos(\varphi)[-I_0\sin(\varphi)+Q_0\cos(\varphi)]\end{aligned}$$

and applying Cramer's rule to this system of linear equations yields the following results:

$$I(G,\varphi)=G\cos(\varphi)\frac{I_0\cos(\varphi)+Q_0\sin(\varphi)+GI_0\cos(\varphi)}{G^2\cos^2(\varphi)+2G\cos^2(\varphi)+1}$$

$$Q(G,\varphi)=G\cos(\varphi)\frac{Q_0\cos(\varphi)-I_0\sin(\varphi)+GQ_0\cos(\varphi)}{G^2\cos^2(\varphi)+2G\cos^2(\varphi)+1}$$

Amplitude error can be calculated now as follows

$$Am_{errorIQ}=\frac{G\sqrt{I_0{}^2+Q_0{}^2}}{1+G}-\frac{G\cos(\varphi)\sqrt{I_0{}^2+Q_0{}^2}}{\sqrt{G^2\cos^2(\varphi)+2G\cos^2(\varphi)+1}}$$

and the relative amplitude error is

$$\begin{aligned}Am(G,\varphi)_{errorIQr}&=\frac{Am_{errorIQ}}{\frac{G}{1+G}\sqrt{I_0{}^2+Q_0{}^2}}\\&=1-\frac{\cos(\varphi)(1+G)}{\sqrt{G^2\cos^2(\varphi)+2G\cos^2(\varphi)+1}}\end{aligned}$$

From the same equations, the phase error is

$$\begin{aligned}Ph_{errIQ}&=\arctan\left(\frac{Q_0}{I_0}\right)-\arctan\left(\frac{Q_0-\frac{I_0\tan(\varphi)}{1+G}}{I_0+\frac{Q_0\tan(\varphi)}{1+G}}\right)\\&=\arctan\left(\frac{\tan(\varphi)}{1+G}\right)\end{aligned}$$

Now, error versus gain plots can be used to evaluate regulation performance. Figure 2 shows relative amplitude errors as a function of the gain G for a given detuning angle of 45°. The I and Q regulation (red line) needs much less gain than Amplitude and Phase (blue) for the same regulation performance. In Figure 2, the dashed line (magenta) shows the maximum allowable RMS gradient error (0.0045%) defined by the beam energy spread requirements for the 12 GeV Upgrade project [1]. The gain of 36 for the I&Q scheme will satisfy this condition while Amplitude and Phase regulation will require a gain of 920 (not shown).

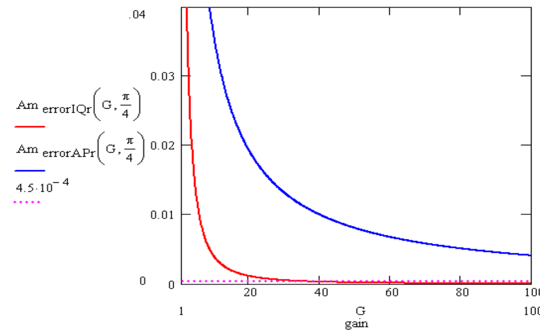

Figure 2: Relative amplitude error versus regulation gain G for I&Q and Amplitude&Phase architectures.

Figure 3 shows phase error for 45° of resonator detuning [2]. In this case I and Q regulation is slightly worse than Amplitude and Phase. The dashed (magenta) line shows the maximum tolerable 0.5° phase change of the cavity field. The gain of 80 for Amplitude and Phase regulation will satisfy above requirement while I and Q regulation requires gain of 110.

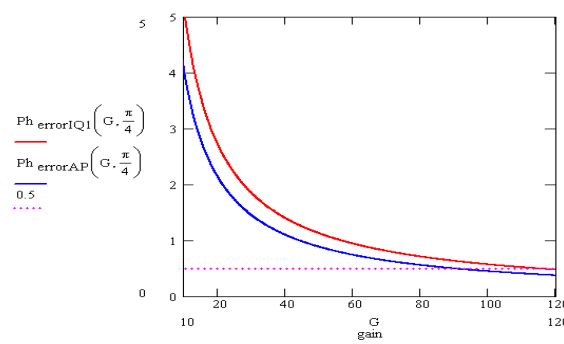

Figure 3: Phase error versus regulation gain G for I&Q and Amplitude&Phase architectures.

SEL ARCHITECTURE

Now, consider the Self Exciting Loop (SEL) type of control [3]. Figure 4 shows the SEL architecture of regulation implemented for a baseband cavity model. When switch S1 is in the upper position and switch S2 in the bottom one, the system is in the SEL mode, cavity amplitude Am is equal to Am_0 and the cavity phase Ph is rolling with the frequency, proportional to the tangent of the detuning angle φ. For operational mode (beam on), the phase loop has to be closed (S2 up). As long as cavity detuning is the only field perturbation, switch S1 can remain in the upper position.

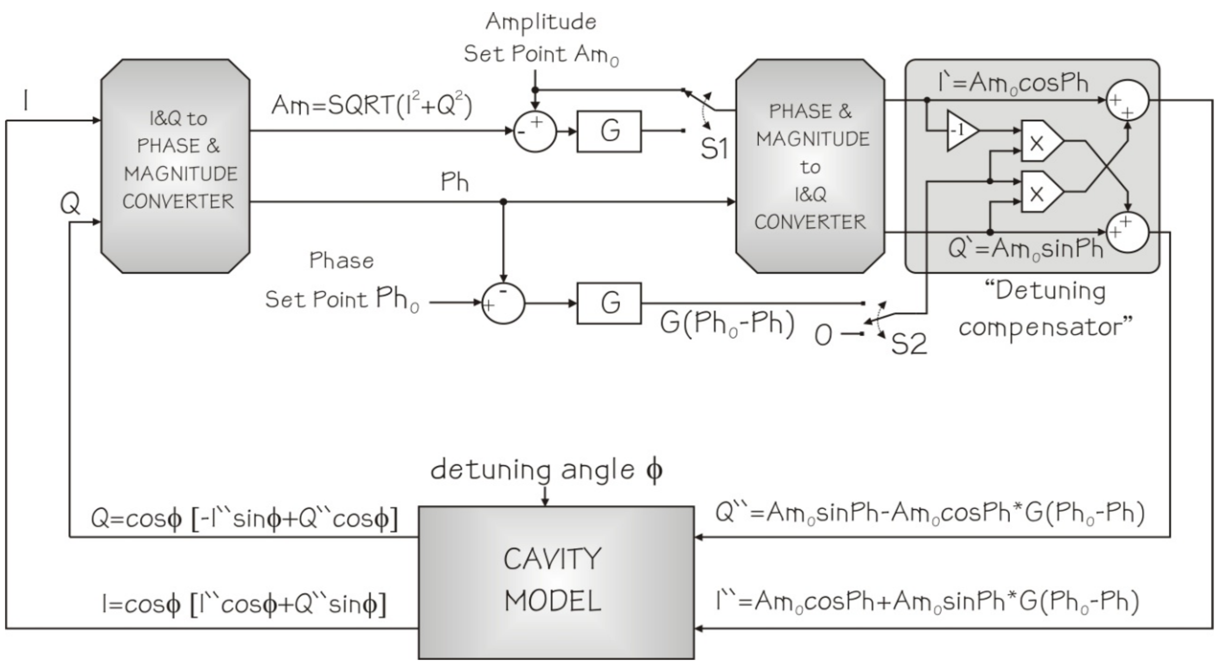

Figure 4: SEL regulation.

For the SEL system,

$$I'' = Am_0 \cos(Ph) + Am_0 \sin(Ph) G(Ph_0 - Ph)$$
$$Q'' = Am_0 \sin(Ph) + Am_0 \cos(Ph) G(Ph_0 - Ph)$$
$$I = \cos(\varphi)[I'' \cos(\varphi) + Q'' \sin(\varphi)]$$
$$Q = \cos(\varphi)[-I'' \sin(\varphi) + Q'' \cos(\varphi)]$$
$$\frac{Q}{I} = \tan(Ph) = \frac{\sin(Ph)}{\cos(Ph)}$$

From these equations we obtain

$$Ph_{error} = Ph_0 - Ph = \frac{\tan(\varphi)}{G}$$

and

$$Am = Am_0 \cos(\varphi)\sqrt{1 + G^2 (Ph_0 - Ph)^2} = Am_0$$

The cavity amplitude signal is independent of the detuning angle. In this case the "Detuning Compensator" based on the measured detuning angle counterbalances the cavity amplitude drop. Still, it is very important to remember that the cavity amplitude is not in the feedback loop so any field disturbances other than detuning will cause the amplitude to change. Figure 5 shows the phase error for 45° of cavity detuning as a function of the phase loop gain G. The dashed line (magenta) shows the maximum tolerable 0.5° phase change allowed for CEBAF cavities [1]. A gain of 110 satisfies the above requirement. The blue line shows phase error for I and Q regulation and as we see the quality of both regulations is very similar.

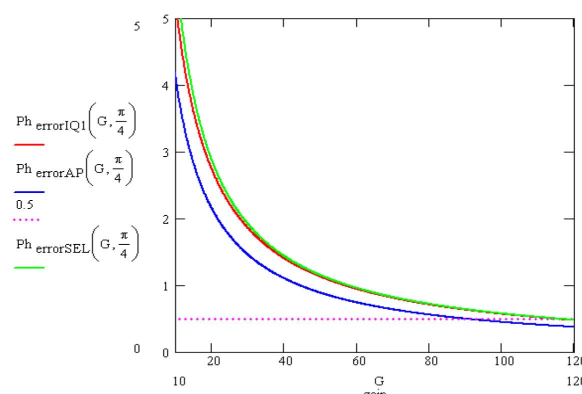

Figure 5: Phase error versus regulation gain G for I&Q Amplitude&Phase and SEL architectures.

- I&Q regulation requires more gain for a large (>40°) cavity detuning
- I&Q regulation is a preferable choice due to the direct I&Q modulation/demodulation
- SEL/Detuning Compensator system requires additional controller for disturbances other than the cavity detuning

CONCLUSION

- Analog&Phase regulation requires much more loop gain for specified amplitude stability than I&Q

REFERENCES

[1] C. Hovater, J. Musson, A. Hofler, and H. Dong, "Performance and Technical Requirements for the CEBAF Energy," JLAB-TN-05-056, 2005.

[2] J. R. Delayen and G. A. Krafft, "Comments on 12 GeV Klystrons JLAB-TN-07-029," 2007.

[3] J. R. Delayen, "Phase and Amplitude Stabilization of Superconducting Resonators," PhD, California Institute of Technology, 1978.