

# DESIGN AND OPTIMIZATION OF ELECTRON BUNCH ACCELERATION AND COMPRESSION

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## Abstract

For electron bunches driving a hard x-ray free electron laser, the electron bunch high qualities should be preserved as well as possible in the acceleration and compression. For typical configuration, the electron bunch is accelerated in RF cavity and compressed in magnetic chicane. Besides the RF curvature and high-order optics terms in a chicane, the collective effects during the bunch acceleration, transportation, and compression can further distort the phase space. Among these collective effects, the coherent edge radiation dominates and governs the macroscopic bunch property. We study these effects and discuss their implication to general LINAC design and optimization.

## TWO-STAGE LINAC BUNCH COMPRESSOR SYSTEM

After the electron bunch born from the cathode, it is accelerated and compressed by bunch compressors to achieve high peak current for x-ray free electron laser (FEL) [1]. Typically, there are two bunch compressor chicanes to achieve good stability. In the following, we will use BC1 to stand for the first chicane, and BC2 for the second. The accelerator RF waveform introduces RF curvature on a finite electron bunch, which needs a harmonic cavity to linearize the longitudinal phase space. The bunch compressor also introduces second order effect. Besides, the collective effects: geometric wakefield in LINAC, and coherent radiation wakefield in a bunch compressor will also affect the bunch compression. Recently, it was identified that the coherent edge radiation (CER) is the main source of impedance to govern the electron bunch macroscopic properties [2]. To illustrate the related concepts for RF acceleration and bunch compression, we first show some details of one stage of acceleration and bunch compression.

The electron beam is first accelerated, hence  $E(z_0) = E_0(1 + \delta_0) + eV \cos[\varphi + kz_0]$ , where  $z_0$  is the longitudinal coordinate of a certain electron with respect to the reference electron,  $z_0 < 0$  is in the head, and  $z_0 > 0$  is in the tail;  $k = 2\pi/\lambda$  is the RF wavenumber for a wavelength of  $\lambda$ . We define the energy of the reference electron as  $E_1 = E_0 + eV \cos[\varphi]$ , so that the relative energy deviation of the electron with longitudinal coordinate  $z_0$  is

$$\begin{aligned} \delta &\approx \frac{E_0}{E_1} \delta_0 - \frac{2\pi eV \sin \varphi}{\lambda E_1} z_0 - \frac{2\pi^2 eV \cos \varphi}{\lambda^2 E_1} z_0^2 \\ &\equiv \mathcal{A} \delta_0 - \mathcal{B} z_0 - \mathcal{C} z_0^2. \end{aligned} \quad (1)$$

Now, the beam is chirped and sent through the chicane. In the thin lens approximation, the path length difference after the chicane reads  $\Delta s \sim [\theta_0/(1 + \delta)]^2 L_2$ , where  $L_2$  is half of the drift distance for a symmetric chicane, and  $\theta_0$  is the bending angle. Hence, after the chicane, the particle internal coordinate reads

$$z = z_0 + \delta(R_{56} + T_{566}\delta), \quad (2)$$

where in the thin lens approximation,  $T_{566} = -3R_{56}/2$ . In a 4-dipole (or 3-dipole) magnetic chicane, the high energy particle travels a short path, hence for compression, the tail particles should have higher energy than that of the head particles. For convention of the tail with coordinate  $z$  larger than that of the head, this means that the chirp slope  $-\mathcal{B} > 0$ , so that  $R_{56} < 0$ .

To find the bunch length after the chicane, we plug Eq. (1) into Eq. (2) to get

$$z \approx \mathcal{A} R_{56} \delta_0 + (1 - \mathcal{B} R_{56}) z_0 + (\mathcal{B}^2 T_{566} - \mathcal{C} R_{56}) z_0^2. \quad (3)$$

Assuming a longitudinal uniform beam, we have

$$\begin{aligned} \sigma_z^2 &\approx \left(\frac{E_0}{E_1}\right)^2 R_{56}^2 \sigma_{\delta_0}^2 + \left[ \left(1 - \frac{2\pi R_{56} eV \sin \varphi}{\lambda E_1}\right)^2 \right. \\ &\quad \left. + \left(\frac{6\pi^2 R_{56} eV}{\sqrt{5} \lambda^2 E_1}\right)^2 \left(\frac{2T_{566} eV \sin^2 \varphi}{R_{56} E_1} - \cos \varphi\right)^2 \right] \sigma_{z_0}^2, \end{aligned} \quad (4)$$

where  $\langle z_0^4 \rangle = (9/5)\sigma_{z_0}^4$ . Seen in Eq. (4), even if we choose a right  $R_{56}$  to minimize the bunch length according to the term quadratic in  $\sigma_{z_0}$ , the term quartic in  $\sigma_{z_0}$  will change the bunch length. This is brought up by the RF curvature. To remove the RF curvature effect, for LINAC Coherent Light Source (LCLS) setting, we install a X-band cavity, which is operated at the 4<sup>th</sup> harmonic of the other S-band cavities. After L0- and L1- and the LX-linac, the energy of the electron reads

$$\begin{aligned} E &= E_0(1 + \delta_0) + eV_0 \cos[\varphi_0 + kz_0] \\ &\quad + eV_1 \cos[\varphi_1 + kz_0] + eV_x \cos[\varphi_x + k_x z_0]. \end{aligned} \quad (5)$$

This leads to

$$\delta \equiv \frac{E - E_2}{E_2} \approx \mathcal{D} \delta_0 + \mathcal{E} z_0 + \mathcal{F} z_0^2 + \mathcal{G} z_0^3, \quad \text{where (6)}$$

$$E_2 = E_0 + eV_0 \cos(\varphi_0) + eV_1 \cos(\varphi_1) + eV_x \cos(\varphi_x), \quad (7)$$

$$\mathcal{E} = -\frac{e}{E_2} [kV_0 \sin(\varphi_0) + kV_1 \sin(\varphi_1) + k_x V_x \sin(\varphi_x)], \quad (8)$$

$$\mathcal{F} = -\frac{e[k^2 V_0 \cos(\varphi_0) + k^2 V_1 \cos(\varphi_1) + k_x^2 V_x \cos(\varphi_x)]}{2E_2}, \quad (9)$$

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and  $\mathcal{D} = E_0/E_2$ . Now, plug Eq. (6) into Eq. (2), we have

$$z = (1 + \mathcal{E}R_{56})z_0 + (\mathcal{F}R_{56} + \mathcal{E}^2T_{566})z_0^2. \quad (10)$$

In principle, there should be multiple solutions for  $V_x$  and  $\varphi_x$  to set the term  $\propto z_0^2$  to zero.

### COLLECTIVE EFFECT

In above, we illustrate the design concepts with emphasis on the RF curvature and second order effect in the chicane. Besides, the collective effects have to be considered, *i.e.*, the LINAC geometric wakefield, the coherent synchrotron radiation (CSR) and the coherent edge radiation (CER) in a chicane, and space charge effect [2]. It was identified that CER is the dominating effect governing the electron bunch macroscopic properties [2]. In the following, we will discuss the geometric wakefield and the CER.

#### RF LINAC Geometric Wakefield: Longitudinal Double-horn Distribution

For LCLS Linac, the wakefield is approximated as [3]

$$w(z) = \frac{Z_0 c}{\pi a^2} e^{-\sqrt{z/s_0}} \quad (11)$$

where  $s_0$  is a characteristic length, and  $a$  is the iris radius; radius. For S-band SLAC cavity,  $s_0 = 1.32$  mm, and  $a = 11.6$  mm and for X-band,  $s_0 = 0.77$  mm, and  $a = 4.72$  mm. The wakefield given in Eq. (11) is in units of  $V/c/m$ . The induced voltage along the bunch is  $V(z) = -NeL \int_{-\infty}^z w(z-z')f(z')dz'$ , where  $L$  is Linac length,  $N$  is the bunching electron population, and  $e$  is the charge of the electron.

The double-horn structure is originated from the cubic term in the  $\delta$ - $z$  plot. It is easy to check that, for LCLS design parameters, the RF waveform induced cubic term is small, while the wakefield combined with the parabolic distribution leads to the cubic term. The normalized parabolic distribution function reads  $f(z) = 3/(4\sqrt{5}\sigma_z)[1 - z^2/(5\sigma_z^2)]$ , where  $\sigma_z$  is the rms bunch length. Given this parabolic distribution, and the wake Green function in Eq. (11), the wakefield induced voltage along the bunch is

$$\begin{aligned} V_{w,2}(z) &= \mathcal{H} \left\{ \left[ 8 \left( 15 + 15\sqrt{\eta} + 6\eta + \eta^{3/2} \right) e^{-\sqrt{\eta}} \right. \right. \\ &- \left. \left. (120 - 12\eta + \eta^2) \right] s_0 \right. \\ &+ \left. 2\sqrt{5} \left[ 2(3 + 3\sqrt{\eta} + \eta) e^{-\sqrt{\eta}} + (\eta - 6) \right] \sigma_z \right\}, \\ &\equiv \mathcal{H}\bar{V}_{w,2}(z) \end{aligned} \quad (12)$$

where  $\mathcal{H} = -(3NeLZ_0cs_0^2)/(10\sqrt{5}\pi a^2\sigma_z^3)$ , and  $\eta = z/s_0$ . Note that  $z \in (0, \Delta z \equiv 2\sqrt{5}\sigma_z)$ . Hence after L2, the acceleration together with the wakefield gives

$$\begin{aligned} E_2(z) &= E_1 + eV_2 \cos\left(\varphi_2 + \frac{2\pi z}{\lambda_2}\right) + eV_{w,2}\left(z + \frac{\Delta z}{2}\right) \\ &= eV_2 \left[ E_{r,1} + \cos(\varphi_2 + k_2 z) + \epsilon \bar{V}_{w,2}\left(z + \frac{\Delta z}{2}\right) \right], \end{aligned} \quad (13)$$

where  $E_{r,1} = E_1/(eV_2)$ ,  $k_2 = 2\pi/\lambda_2$ , and  $\epsilon = e\mathcal{H}/(eV_2)$ . We have assumed that  $E_1$  is a constant due to the large acceleration in L2. Also, we have shifted the definition domain so that  $z \in (-\Delta z/2, \Delta z/2)$ .

Now, BC2 gives the following transformation  $E \rightarrow E$  and  $z \rightarrow z + R_{56,2}(E - E_2(0))/E_2(0)$ . Assume that the intrinsic energy spread is extremely small, the longitudinal phase space distribution before BC2 is

$$f(z, E) = \frac{3}{4\sqrt{5}\sigma_z} \left( 1 - \frac{z^2}{5\sigma_z^2} \right) \delta[E - E_2(z)]. \quad (14)$$

After BC2, the distribution function is transformed into

$$\begin{aligned} f(z, E) &= \frac{3}{4\sqrt{5}\sigma_z} \left\{ 1 - \frac{\left[ z - R_{56,2} \frac{E - E_2(0)}{E_2(0)} \right]^2}{5\sigma_z^2} \right\} \\ &\times \delta \left[ E - E_2 \left( z - R_{56,2} \frac{E - E_2(0)}{E_2(0)} \right) \right] \end{aligned} \quad (15)$$

Integrating out the energy, the final distribution function is

$$f(z) = \frac{3}{4\sqrt{5}\sigma_z} \left\{ 1 - \frac{\left[ z - R_{56,2} \frac{E_s(z) - E_2(0)}{E_2(0)} \right]^2}{5\sigma_z^2} \right\} \bigg|_{E=E_s(z)} \frac{\partial g(z, E)}{\partial E} \quad (16)$$

where  $E_s(z)$  is determined by  $g[z, E_s(z)] = 0$ , with

$$\begin{aligned} g(z, E) &\equiv E - E_1 \\ &- eV_2 \cos\left(\varphi_2 + k_2 \left[ z - R_{56,2} \frac{E - E_2(0)}{E_2(0)} \right]\right) \\ &- eV_{w,2} \left[ z - R_{56,2} \frac{E - E_2(0)}{E_2(0)} + \frac{\Delta z}{2} \right]. \end{aligned} \quad (17)$$

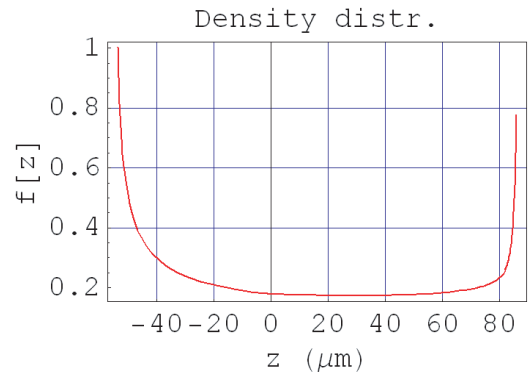


Figure 1: Double-horn structure in current profile.

With the nominal parameters [1], the longitudinal current profile after BC2 is double-horn as in Fig. 1.

#### CSR and CER Effect

We assume that the initial electron bunch distribution before BC1 is a linearly energy chirped bunch

$$f_0(z, \delta) = \frac{1}{\sqrt{2\pi}\sigma_{z_0}} e^{-\frac{z^2}{2\sigma_{z_0}^2}} \frac{1}{\sqrt{2\pi}\sigma_{\delta_0}} e^{-\frac{(\delta - h_1 z)^2}{2\sigma_{\delta_0}^2}}, \quad (18)$$

where  $\sigma_{z_0}$  and  $\sigma_{\delta_0}$  are the electron rms bunch length and rms energy spread, respectively. The first bunch compressor BC1 then introduce a transformation as  $z \rightarrow z + R_{56,1}\delta$ , which leads to  $\sigma_{z_1} = \sqrt{(1 - h_1 R_{56,1})^2 \sigma_{z_0}^2 + R_{56,1}^2 \sigma_{\delta_0}^2}$  and  $\sigma_{\delta_1} = \sqrt{h_1^2 \sigma_{z_0}^2 + \sigma_{\delta_0}^2}$ . The current profile is simply  $I_1(z) = ecN/(\sqrt{2\pi}\sigma_{z_1})e^{-z^2/(2\sigma_{z_1}^2)}$ , where  $e$  is the electron charge,  $N$  is the number of electrons in the bunch, and  $c$  is the speed of light in vacuum.

Downstream of the BC1 magnet, the wake primarily results from CER. For  $k > 0$ , the integrated resistive CER impedance in a drift space of length  $L_d$  is modeled as [2]

$$\int Z_{\text{CER}}(k) = \frac{Z_0}{2\pi} \ln \left\{ \frac{\min[L_d, \lambda\gamma^2/(2\pi)]}{\rho^{2/3}\lambda^{1/3}} \right\} \equiv Z_{\text{CER}}, \quad (19)$$

for wavelengths where the right-hand side (rhs) of Eq. (19) is positive. For typical chicane parameters ( $\gamma \sim 1000$ ,  $\rho \sim 1$  m,  $\lambda/(2\pi) \sim \sigma_z \sim 100$   $\mu\text{m}$ ,  $L_d \sim 10$  m), the integrated CER impedance is  $\sim Z_0$ . Due to the weak logarithmic dependence in Eq. (19), an integrated impedance of  $\sim Z_0$  might be a good approximation for a wide range of parameters. The voltage induced along the electron bunch is then  $V_1(z) = Z_{\text{CER},1}I_1(z)$ . This leads to a relative energy deviation of

$$\frac{eV_1(z)}{E_2} = \frac{eZ_{\text{CER},1}}{E_2} I_1(z) \equiv \varepsilon_1 e^{-z^2/(2\sigma_{z_1}^2)}, \quad (20)$$

where  $E_2$  is the electron nominal energy at the entrance of BC2, and

$$\varepsilon_1 = \frac{e^2 c N Z_{\text{CER},1}}{\sqrt{2\pi}\sigma_{z_1} E_2}. \quad (21)$$

For the LCLS low charge operation mode, the charge is 250 pC,  $\sigma_{z_1} = 100$   $\mu\text{m}$ ,  $E_2 = 4.3$  GeV, and  $Z_{\text{CER},1} \sim Z_0$ , we have  $\varepsilon_1 \approx 2.6 \times 10^{-5}$ .

Assuming that the electron bunch acquires additional linear chirp in the LINAC between BC1 and BC2, the total linear chirp on the electron bunch is then  $h_2$ . The electron bunch distribution function at the entrance of BC2 is

$$\begin{aligned} f_{1,2}(z, \delta) &\approx \frac{e^{-\frac{z^2}{2\sigma_{z_1}^2} - \left[ \delta - h_2 z - \varepsilon_1 e^{-z^2/(2\sigma_{z_1}^2)} \right]^2 / (2\sigma_{\delta_1}^2)}}{2\pi\sigma_{z_1}\sigma_{\delta_1}} \\ &\approx \frac{e^{-\frac{z^2}{2\sigma_{z_1}^2}}}{\sqrt{2\pi}\sigma_{z_1}} \delta_D \left[ \delta - h_2 z - \varepsilon_1 e^{-\frac{z^2}{2\sigma_{z_1}^2}} \right] \end{aligned} \quad (22)$$

where  $\delta_D(x)$  is the Dirac Delta-function.

The electron bunch is then transport through BC2, *i.e.*,  $z \rightarrow z + R_{56,2}\delta$ . Integrated out  $\delta$ , we have the current profile as

$$I_2(z) = \frac{ecN}{\sqrt{2\pi}\sigma_{z_2}} e^{-\frac{z^2}{2\sigma_{z_2}^2}} \left[ 1 - \varepsilon_1 \frac{2R_{56,2}z}{\sigma_{z_2}^2} e^{-\frac{z^2}{2\sigma_{z_1}^2}} \right], \quad (23)$$

where  $\sigma_{z_2} = \sigma_{z_1}/C_2$ , with  $C_2 \equiv (1 - h_2 R_{56,2})^{-1}$  being the compression factor in BC2. According to Eq. (23), the

electron current profile will be enhanced at the head or the tail when  $2\varepsilon_1 R_{56,2}/\sigma_{z_2}$  is not negligibly small. Since the quantity  $\varepsilon_1$  in Eq. (21) is positively defined, the current will be enhanced at the tail of the electron bunch. For typical machine design as for WfEL [2], this CER wakefield can cause a distortion of the longitudinal current profile as in Fig. 2, leading to an asymmetric distribution.

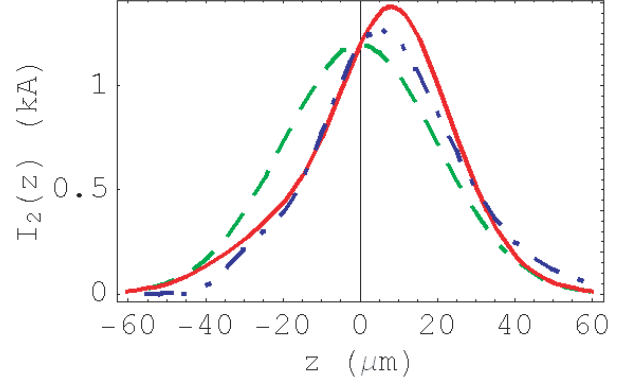


Figure 2: Illustration of the CER wakefield effect on the longitudinal current profile. The ideal case without CER effect is shown as dashed green curve, the case with the CER effect is shown as the red solid curve, and the case with the CER and also a second order curvature is shown as the blue dot-dashed curve.

To reduce the effect of the CER wakefield, we can deliberately mismatch the harmonic cavity to have some residual second order RF curvature before entering the BC1. This is to modify the distribution function in Eq. (22) to be

$$f_{1,2,m}(z, \delta) \approx \frac{e^{-\frac{z^2}{2\sigma_{z_1}^2}}}{\sqrt{2\pi}\sigma_{z_1}} \delta_D \left[ \delta - h_2 z - g_2 z^2 - \varepsilon_1 e^{-\frac{z^2}{2\sigma_{z_1}^2}} \right], \quad (24)$$

where  $g_2$  represents the residual second-order curvature effect. As an illustration, in Fig. 2, the blue dot-dashed curve is for  $g_2 = -5 \times 10^4$   $\text{m}^{-2}$ . One sees that the bunch profile can be restored to certain degree, but will lead to some ripple in the large  $z$  head and tail region.

As an conclusion, in this paper, we discuss the key design issues for a two-stage acceleration and compression system for x-ray FEL. The emphasis is on the RF curvature cancelation, chicane second order effect, the LINAC geometric wake, and CSR/CER effect.

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