THE ELECTRON BUNCH INITIAL ENERGY PROFILE ON A SEEDED FREE ELECTRON LASER PERFORMANCE*

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Abstract

A single-pass high-gain x-ray free electron laser (FEL) calls for a high quality electron bunch. In particular, for a seeded FEL, and for a cascaded harmonic generation (HG) FEL, the electron bunch initial energy profile is crucial for generating an FEL with a narrow bandwidth. After the acceleration, compression, and transportion, the electron bunch energy profile entering the undulator can acquire temporal non-uniformity. We study the effects of the electron bunch initial energy profile on the FEL performance.

VLASOV-MAXWELL ANALYSIS FOR AN INITIAL VALUE PROBLEM

The photoinjector generated electron bunch has a very small energy spread and small emittance. During the acceleration, bunch compression, and transportation, the electron bunch can acquire RF curvature, second order effect in the chicane, and collective effects, which will all lead to energy profile to be nonuniform. Further more, the electron bunch is subject to microbunching instability [1]. Thus, the electron bunch coming into the undulator can have an energy modulation. We study the energy profile nonuniformity on the free electron laser (FEL).

To analyze the start-up of a seeded FEL amplifier we use the coupled set of Vlasov and Maxwell equations which describe the evolution of the electrons and the radiation fields [2]. This approach is used as well for the Self-Amplified Spontaneous Emission (SASE) FEL [3]. We will work with a one-dimensional system analytically.

Vlasov-Maxwell Equations

We follow the analysis and notation of Refs. [3, 4, 2]. Dimensionless variables are introduced as $Z = k_w z$, $\theta = (k_0 + k_w)z - \omega_0 t$, where $k_0 = 2\pi/\lambda_0$, $\omega_0 = k_0 c$, and $k_w = 2\pi/\lambda_w$ with λ_0 being the radiation wavelength, λ_w being the undulator period, and c being the speed of light in vacuum. We also introduce $p = 2(\gamma - \gamma_0)/\gamma_0$ as the measure of energy deviation, with γ the Lorentz factor of an electron in the electron bunch, and γ_0 the resonant energy defined by $\lambda_0 = \lambda_w(1 + K^2/2)/(2\gamma_0^2)$, for a planar undulator, where the undulator parameter $K \approx 93.4 B_w \lambda_w$ with B_w the peak magnetic field in Tesla and λ the undulator period in meter. The electron distribution function is $\psi(\theta, p, Z)$ with $\psi_0(\theta, p, Z)$ describing the slow varying

unperturbed component. The FEL electric field is written as $E(t, z) = A(\theta, Z)e^{i(\theta - Z)}$ with $A(\theta, Z)$ being the slow varying envelope function.

The one-dimensional linearized Vlasov-Maxwell equations are,

$$\frac{\partial \psi}{\partial Z} + p\frac{\partial \psi}{\partial \theta} - \frac{2D_2}{\gamma_0^2}\left(Ae^{i\theta} + A^*e^{-i\theta}\right)\frac{\partial \psi_0}{\partial p} = 0, \quad (1)$$

and,

$$\left(\frac{\partial}{\partial Z} + \frac{\partial}{\partial \theta}\right)A(\theta, Z) = \frac{D_1}{\gamma_0}e^{-i\theta}\int dp\,\psi(\theta, p, Z), \quad (2)$$

where in SI units, $D_1 = ea_w n_0[JJ]/(2\sqrt{2}k_w\varepsilon_0)$, $D_2 = ea_w[JJ]/(\sqrt{2}k_w mc^2)$, with e and m being the charge and mass of the electron; $\varepsilon_0 \approx 8.85 \times 10^{-12}$ F/m being the vacuum permittivity; n_0 being the electron beam density; and $[JJ] = J_0[a_w^2/2(1 + a_w^2)] - J_1[a_w^2/2(1 + a_w^2)]$ where the dimensionless rms undulator parameter $a_w \equiv K/\sqrt{2}$. Equation (1) gives a general solution as

$$\psi(\theta, Z, \gamma) \approx \psi_0(\theta - pZ, \gamma)$$
$$+ \int_0^Z dZ'\frac{2D_2}{\gamma_0^2}A[\theta - p(Z - Z'), \gamma]e^{i[\theta - p(Z - Z')]}$$
$$\times \frac{\partial \psi_0[\theta - p(Z - Z'), \gamma]}{\partial p}. \quad (3)$$

Plugging Eq. (3) into Eq. (2), we have

$$\left(\frac{\partial}{\partial Z} + \frac{\partial}{\partial \theta}\right)A(\theta, Z) = \frac{D_1}{\gamma_0}e^{-i\theta}\int dp\,\psi_0(\theta - pZ, \gamma)$$
$$+ (2\rho)^3 e^{-i\theta}\int dp\int_0^Z dZ'A[\theta - p(Z - Z'), \gamma]$$
$$\times e^{i[\theta - p(Z - Z')]}\frac{\partial \psi_0[\theta - p(Z - Z'), \gamma]}{\partial p}, \quad (4)$$

with $(2\rho)^3 = (2D_1 D_2)/\gamma_0^3$, the Pierce parameter ρ [5, 6].

Initial Energy Imperfectness

To model an energy imperfectness in the electron bunch coming into the undulator, we assume that the initial distribution function is

$$\psi_0 = \delta[p + g(\theta_0)] = \delta[p + g(\theta - pZ)], \quad (5)$$

where $g(\theta_0)$ is a general function.

The Maxwell equation is further rewritten after performing a partial integral

$$\left(\frac{\partial}{\partial Z} + \frac{\partial}{\partial \theta}\right) A(\theta, Z) = \frac{D_1}{\gamma_0} \sum_j e^{-i\theta_j + ig(\theta_j)Z} \delta(\theta - \theta_j)$$

$$+ i(2\rho)^3 \int_0^Z dZ'(Z - Z')e^{ig(\theta)(Z-Z')} A(\theta, Z'), (6)$$

where the initial discrete radiators (electrons) are modeled as $\sum_j \delta(\theta - \theta_j)$ for the longitudinal coordinates.

To further work on Eq. (6), we now introduce the Laplace transform,

$$f(\theta, s) = \int_0^\infty dZ e^{-sZ} A(\theta, Z). \quad (7)$$

With this, Eq. (6) is now casted in the frequency domain as

$$\frac{\partial f(\theta, s)}{\partial \theta} + \left(s - \frac{i(2\rho)^3}{[s - ig(\theta)]^2}\right) f(\theta, s)$$

$$= A(\theta, 0) + \frac{D_1}{\gamma_0} \sum_j \frac{e^{-i\theta_j}\delta(\theta - \theta_j)}{s - ig(\theta_j)}, \quad (8)$$

which yields the general solution as

$$f(\theta, s) = \int_{-\infty}^\theta d\theta' e^{-s(\theta - \theta') + \int_{\theta'}^\theta \frac{i(2\rho)^3}{[s - ig(\theta'')]^2} d\theta''}$$

$$\times \left[A(\theta', 0) + \frac{D_1}{\gamma_0} \sum_j \frac{e^{-i\theta_j}\delta(\theta' - \theta_j)}{s - ig(\theta_j)}\right]. \quad (9)$$

Notice that, in the square bracket in Eq. (9), the first term $A(\theta, 0)$ characterizes the initial seed for a seeded FEL, while the second term models the Self-Amplified Spontaneous Emission (SASE) FEL. In the following, let us focus on a seeded FEL, so that the second term in the square bracket will be neglected.

Initial Sinusoidal Energy Modulation For electron bunch experienced microbunching instability, there can be an energy modulation as

$$\gamma = \gamma_0 + \varepsilon_m \sin[\omega_m(t - t_0)], \quad (10)$$

where ω_m characterizes the energy modulation. The initial distribution function is then

$$\Psi_0 = \delta[p + \eta \sin(\omega_\eta \theta_0)], \quad (11)$$

where $\eta \equiv 2\varepsilon_m/\gamma_0$ and $\omega_\eta \equiv \omega_m/\omega_0$. For such a sinusoidal modulation, we have

$$\int_{\theta'}^\theta \frac{i(2\rho)^3}{[s - i\eta \sin(\omega_\eta \theta'')]^2} d\theta'' \approx \frac{i(2\rho)^3(\theta - \theta')}{s^2}$$

$$+ \frac{2\eta(2\rho)^3[\cos(\omega_\eta \theta) - \cos(\omega_\eta \theta')]}{\omega_\eta s^3}. \quad (12)$$

Up to this stage, let us throw away the SASE term, and keep only the seed in Eq. (9).

$$f(\theta, s) \approx \int_{-\infty}^\theta d\theta' A(\theta', 0) \quad (13)$$

$$\times \quad e^{-s(\theta - \theta') + \frac{i(2\rho)^3(\theta - \theta')}{s^2} + \frac{2\eta(2\rho)^3[\cos(\omega_\eta \theta) - \cos(\omega_\eta \theta')]}{\omega_\eta s^3}}.$$

The inverse Laplace transform then gives us the FEL field envelope as

$$A(\theta, Z) = \int_c \frac{ds}{2\pi i} e^{sZ} f(\theta, s)$$

$$\approx \int_c \frac{ds}{2\pi i} e^{sZ} \int_{-\infty}^\theta d\theta' A(\theta', 0) \quad (14)$$

$$\times \quad e^{-s(\theta - \theta') + \frac{i(2\rho)^3(\theta - \theta')}{s^2} + \frac{2\eta(2\rho)^3[\cos(\omega_\eta \theta) - \cos(\omega_\eta \theta')]}{\omega_\eta s^3}}.$$

Obviously, once we know the initial seed field envelope $A(\theta, 0)$, we can obtain the seeded FEL field envelope $A(\theta, Z)$ along the undulator.

The double integral in Eq. (14) can be evaluated by first performing the contour integral to get,

$$A(\theta, Z) = \int_0^\infty d\xi A(\theta - \xi, 0) \mathcal{G}(\theta, \xi, Z, s, \eta), \quad (15)$$

with the Green function $\mathcal{G}(\theta, \xi, Z, s, \eta)$ and the corresponding phasor $\mathcal{F}(\theta, \xi, Z, s, \eta)$ defined as

$$\mathcal{G}(\theta, \xi, Z, s, \eta)$$

$$\equiv \int_c \frac{ds}{2\pi i} e^{s(Z-\xi) + \frac{i(2\rho)^3\xi}{s^2} + \frac{2\eta(2\rho)^3[\cos(\omega_\eta \theta) - \cos[\omega_\eta(\theta - \xi)]]}{\omega_\eta s^3}}$$

$$\equiv \int_c \frac{ds}{2\pi i} \exp[\mathcal{F}(\theta, \xi, Z, s, \eta)]. \quad (16)$$

The Green function can be estimated by saddle point approximation. The saddle point s_s is found from

$$\left.\frac{d\mathcal{F}(\theta, \xi, Z, s, \eta)}{ds}\right|_{s=s_s} = 0, \quad (17)$$

and the Green function is approximated as

$$\mathcal{G}(\theta, \xi, Z, s, \eta) \approx \frac{\exp[\mathcal{F}(\theta, \xi, Z, s_s, \eta)]}{[2\pi \mathcal{F}''(\theta, \xi, Z, s_s, \eta)]^{1/2}} \quad (18)$$

$$\cong \frac{-e^{i\pi/12} e^{i^{1/3}2\rho Z - \frac{i^{1/3}9(\theta - Z/3)^2 \rho}{2Z} - \frac{i2\eta\{\cos(\omega_\eta \theta) - \cos[\omega_\eta(\theta - \xi)]\}}{\omega_\eta}}}{\sqrt{2\pi Z/\rho}}.$$

For an initial Gaussian seed,

$$E(t, z = 0) = E_0 e^{-i\omega_0 t - \alpha_0 t^2} = E_0 e^{i\theta - \theta^2 \alpha_0/\omega_0^2}$$

$$\implies A(\theta, 0) = E_0 e^{-\theta^2 \alpha_0/\omega_0^2}, \quad (19)$$

where $\alpha_0 = 1/(4\sigma_{t0}^2)$ with σ_{t0} being the initial seed rms pulse duration. According to Eq. (15), the FEL pulse is

$$A(\theta, Z) = E_0 \frac{-e^{i\pi/12}}{\sqrt{2\pi Z/\rho}} e^{i^{1/3}2\rho Z - \frac{i^{1/3}9(\theta - Z/3)^2 \rho}{2Z} - \frac{i2\eta \cos(\omega_\eta \theta)}{\omega_\eta}}$$

$$\times \quad \int_{-\infty}^{\theta} d\theta' e^{-\frac{\theta'^2 \alpha_0}{\omega_0^2} + \frac{i2\eta \cos(\omega_\eta \theta')}{\omega_\eta}}$$

$$\approx \quad E_0 \omega_0 \frac{-e^{i\pi/12}}{\sqrt{2\alpha_0 Z/\rho}} \left(1 + \frac{i2\eta}{\omega_\eta} e^{-\frac{\omega_0^2 \omega_\eta^2}{4\alpha_0}}\right)$$

$$\times \quad e^{i^{1/3} 2\rho Z - \frac{i^{1/3} 9(\theta - Z/3)^2 \rho}{2Z} - \frac{i2\eta \cos(\omega_\eta \theta)}{\omega_\eta}}. \quad (20)$$

It is interesting to find that to the first order in η, in the exponential function, the microbunching energy modulation only leads to a pure phase modulation, but does not affect the power, except the small correction term $(i2\eta/\omega_\eta)\exp[-\omega_0^2\omega_\eta^2/(4\alpha_0)]$ in front of the exponential function.

Bandwidth As we find above, the first order correction is a pure phase modulation, we would like to investigate this phase modulation on the FEL coherence. Recall that, one of the most important purposes of a seeded FEL is to generate transform limited light, let us now find the FEL spectrum:

$$\tilde{E}(\omega, z) \equiv \frac{1}{\sqrt{2\pi}} \int dt E(t, z) e^{i\omega t}. \quad (21)$$

Notice that $E(t, z) \sim e^{-i\omega_0 t}$, hence the Fourier transform is defined as in Eq. (21).

First, we rewrite $E(t, z)$ to have t-dependence explicit, *i.e.*,

$$E(t, z) \approx E_0 \omega_0 \frac{-e^{i\pi/12}}{\sqrt{2\alpha_0 Z/\rho}} \left(1 + \frac{i2\eta}{\omega_\eta} e^{-\frac{\omega_0^2 \omega_\eta^2}{4\alpha_0}}\right)$$

$$\times \quad e^{ik_0 Z/k_w - 6i^{1/3} k_0 \rho Z/k_w - 9i^{1/3} k_0^2 \rho Z/(2k_w^2)}$$

$$\times \quad e^{[9i^{1/3}\rho\omega_0/(v_g k_w) - i]\omega_0 t - 9i^{1/3}\rho\omega_0^2 t^2/(2Z)}$$

$$\times \quad \left[1 - \frac{i\eta}{\omega_\eta}\left(e^{i\omega_\eta \theta} + e^{-i\omega_\eta \theta}\right)\right]$$

$$\equiv \quad \mathcal{A}(z) e^{[9i^{1/3}\rho\omega_0/(v_g k_w) - i]\omega_0 t - 9i^{1/3}\rho\omega_0^2 t^2/(2Z)}$$

$$\times \quad \left[1 - \frac{i\eta}{\omega_\eta}\left(e^{i\omega_\eta \theta} + e^{-i\omega_\eta \theta}\right)\right]$$

$$\equiv \quad \mathcal{A}(z) e^{i^{1/3}\mathcal{B}z^2/v_g^2} e^{-i\omega_0 t - i^{1/3}\mathcal{B}(t-z/v_g)^2}$$

$$\times \quad \left[1 - \frac{i\eta}{\omega_\eta}\left(e^{i\omega_\eta \theta} + e^{-i\omega_\eta \theta}\right)\right], \quad (22)$$

where $v_g = \omega_0/(k_0 + 2k_w/3)$.

Completing the integral in Eq. (21), we have

$$\tilde{E}(\omega, z) = \frac{\mathcal{A}(z)}{\sqrt{2}\mathcal{B}i^{1/3}} e^{\frac{i^{5/3}\left[(\omega - \omega_0) - \frac{2i^{4/3}\mathcal{B}z}{v_g}\right]^2}{4\mathcal{B}}}$$

$$\times \quad \left\{1 - \frac{i\eta}{\omega_\eta} e^{i^{5/3}\omega_0^2\omega_\eta^2/(4\mathcal{B})} \right. \quad (23)$$

$$\times \quad \left[e^{iz(k_0 + k_w - \omega_0/v_g)\omega_\eta - i^{5/3}(\omega - \omega_0)\omega_0\omega_\eta/(2\mathcal{B})}\right.$$

$$+ \quad \left.\left. e^{-iz(k_0 + k_w - \omega_0/v_g)\omega_\eta + i^{5/3}(\omega - \omega_0)\omega_0\omega_\eta/(2\mathcal{B})}\right]\right\}.$$

The FEL energy density is then $\mathcal{I}(\omega, z) \equiv \tilde{E}(\omega, z)\tilde{E}^*(\omega, z)$, where $\tilde{E}^*(\omega, z)$ is the complex conjugate of $\tilde{E}(\omega, z)$.

We can then compute

$$\langle \omega(z) \rangle \equiv \frac{\int \omega \mathcal{I}(\omega, z)}{\int \mathcal{I}(\omega, z)} \quad (24)$$

$$\approx \quad \omega_0\{1 + 2\eta e^{-\frac{\omega_0^2\omega_\eta^2}{4\sqrt{3}\mathcal{B}}} \sin[(k_0 + k_w - \omega_0/v_g)\omega_\eta z]\}.$$

Likewise, the standard deviation is

$$\sigma_\omega(z) \equiv \sqrt{\frac{\int \omega^2 \mathcal{I}(\omega, z)}{\int \mathcal{I}(\omega, z)} - (\langle \omega(z) \rangle)^2}$$

$$\approx \quad \frac{2\mathcal{B} - \eta\omega_0^2\omega_\eta e^{-\frac{\omega_0^2\omega_\eta^2}{4\sqrt{3}\mathcal{B}}} \cos[(k_0 + k_w - \omega_0/v_g)\omega_\eta z]}{\sqrt{2\sqrt{3}\mathcal{B}}}$$

$$\equiv \quad \frac{2\mathcal{B} - \mathcal{C}\cos[(k_0 + k_w - \omega_0/v_g)\omega_\eta z]}{\sqrt{2\sqrt{3}\mathcal{B}}}$$

$$\equiv \quad \frac{2\mathcal{B} + \mathcal{D}}{\sqrt{2\sqrt{3}\mathcal{B}}}. \quad (25)$$

Notice that, for $\eta = 0$, $\sigma_\omega(z) = \sqrt{3\sqrt{3}\rho\omega_0^2/(k_w z)}$ is the well-known rms bandwidth of the FEL Green function for a coasting electron beam [3, 7, 8].

To be explicit, we have

$$\frac{\mathcal{C}}{2\mathcal{B}} = \frac{\eta\omega_\eta k_w z}{9\rho} e^{-\frac{\omega_\eta^2 k_w z}{18\sqrt{3}\rho}}. \quad (26)$$

Recall that $\omega_\eta = \omega_m/\omega_0$, *i.e.*, the ratio of the microbunching frequency to the FEL frequency, the most serious degradation will be at the first stage, where the FEL frequency is the lowest. Also, due to the n^2 amplification, the first stage is always the most serious stage. Also notice that, the ratio in Eq. (26) is maximum at $z = 18\sqrt{3}\rho/(k_w\omega_\eta^2)$.

As an conclusion, in this paper, we study the effect on a seeded FEL performance due to an initial energy non-uniformity when the electron bunch enters the undulator.

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