TWISTED STRUCTURES AND THEIR APPLICATION AS ACCELERATING STRUCTURES*

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Abstract

Normally, reactive loading is employed to construct slow-wave accelerating cavities. However, their nonuniform cross section, difficult machining, and complicated welding or brazing processes increase the total cost. Although straight hollow waveguides can only support faster-than-light propagation, twisted waveguides can support propagation at or below c. Because twisted structures have a uniform cross section in the transverse plane, they offer several potential advantages over dielectric loaded structures or other types of periodic structures. Of particular interest are twisted structures whose longitudinal cross section has been selected to resemble well-known accelerating structures, such as the disk-loaded accelerating structure and the TESLA type elliptical cavity. Comparisons are drawn between these conventional cavities and their twisted counterparts in terms of the phase velocity and dispersion relationship. The accelerating modes are found and analyzed, and R/Q's are calculated.

INTRODUCTION

One very important consideration in the effective acceleration of particle beams is matching the velocity of an electromagnetic wave to the velocity of the particles traveling slower than c.

A conventional method for slowing the wave that is more well-suited for superconducting accelerators is to use a corrugated structure with nonuniform cross section. However, such corrugated structures have drawbacks. First, manufacturing is difficult due to the necessecity of very difficult machining and welding to create the smooth finish necessary. Secondly, trapped modes can exist in the structure because of the presence of stop bands in the dispersion characteristics. Finally, such corrugated structures tend to have zero group velocity when operating at $\pi-$mode, which causes problems for mode separation.

Therefore, we consider a uniformly twisted waveguide as a potential accelerating structure, consisting of a uniform cross section twisted about the beam axis. It has long been known that such a twisted structure can be a slow wave structure, and a preliminary investigation into the feasibility of twisted guides as accelerating structures was undertaken by Kang [1]. In this paper, we show that a twisted waveguide accelerating structure can be used to circumvent problems relating to trapped modes and mode separation. Also, tuning for a twisted structure could be done for the whole structure at once.

Any non-circular cross section will generate a nontrivial shape when twisted along the center axis, which has the potential to produce slow-wave effects. However, this paper concentrates on a few simple representative cross sectional shapes in order to describe the general nature of the slow-wave accelerating characteristics: a twisted structure whose longitudinal cross section is identical to that of a disk-loaded accelerating cavity, and a twisted structure with an elliptical cavity-like longitudinal cross section.

PROPAGATION CHARACTERISTICS OF TWISTED STRUCTURES

When selecting a cross sectional shape for a twisted waveguide, an interesting choice would be one that would produce a structure with similar longitudinal cross section to an existing accelerating structure. The first structure we considered has the shape shown in Fig. 1. This figure shows that when this particular cross section is twisted, the shape of the longitudinal cross section is identical to a disk loaded structure. Table 1 gives relevant information for this structure based on simulation results.

Fig. 2 shows a CST Microwave Studio [2] simulation of the electric field in such a twisted structure subject to a periodic boundary condition. The simulation results clearly indicate a TM mode which could be useful for particle acceleration. Such a twisted analog can also be considered for the medium β SNS superconducting cavity. Although the details are not given here, Fig. 2 shows the electric fields for such a structure. A clear similarity is seen between the fields of these twisted structures and the fields typically seen in rotationally symmetric (non-twisted) corrugated accelerating cavities.

Table 1: Parameters for twisted analog of disk-loaded accelerating cavity

Parameter	Value	Unit
Frequency	2.84	GHz
Inner radius	4.13	cm
Outer radius	5.493	cm
Twist rate	89.76	Radians/m
Notch angle	1.048	Radians
Phase advance per cell	$\frac{2\pi}{3}$	Radians
Phase velocity	2.98×10^8	m/s

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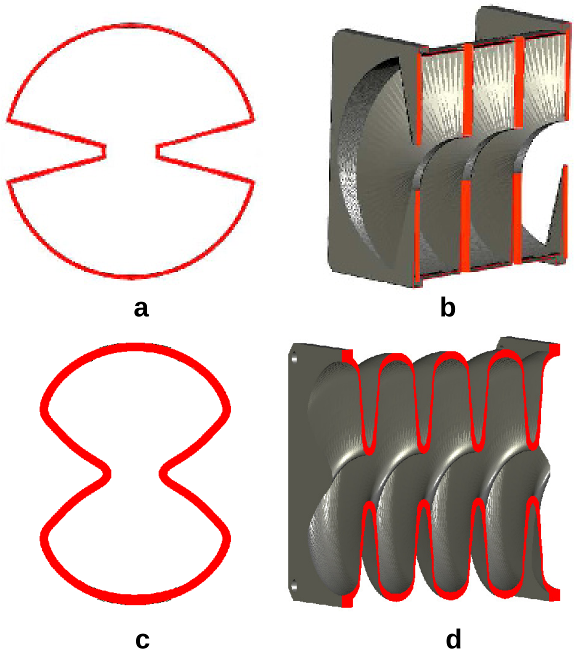

Figure 1: (a) “Notched” cross section of twisted disk-loaded analog. (b) 3D Cutaway view of twisted disk-loaded analog. (c) “Dumbell” cross section of elliptical twisted analog. (d) 3D Cutaway view of elliptical twisted analog.

This particular structure was designed to accelerate particles at relativistic velocities (i.e. electron accelerators). To do this, the dispersion properties were investigated in order to set the phase velocity equal to the speed of light c. The dispersion curves for this structure are shown in Fig. 3 for twist rates ranging from 67.3 $\frac{Rad}{m}$ to 337 $\frac{Rad}{m}$. The dispersion curves were generated using the finite-difference frequency domain method discussed in [3], and they clearly indicate that for this particular TM accelerating mode, the phase velocity of the wave decreases as the twist rate increases. At the selected twist rate of 88.76 $\frac{Rad}{m}$, the phase velocity was found to be c at the design frequency of 2.84 GHz.

TRAPPED MODES AND GROUP VELOCITY CONSIDERATIONS

One of the problems associated with reactively loaded corrugated accelerating structures is the tendency for trapped modes to develop. These occur when electromagnetic energy from the beam is deposited in a stop band where propagation in the cavity structure is not permitted.

The twisted guide is unique in that there exists a uniform straight waveguide equivalent (see [4]) to the twisted structure. A consequence of this equivalent waveguide is that the dispersion curves are continuous. In other words, in corrugated accelerating structures, it only makes sense to speak of phase constants between 0 and $\frac{\pi}{D}$, where D is the length of a corrugation (single unit cell). However, in twisted structures, the dispersion curves continue indefinitely because of the straight waveguide equivalent, meaning that the stop bands are eliminated. Therefore, the problem of trapped modes can be dealt with effectively.

Another common problem encountered with corrugated accelerating structures is vanishing group velocity near π mode. In a cavity of finite length, this normally causes modes to be very closely spaced (so exciting the π mode would likely cause other modes nearby to also be excited). However, the twisted guide does not have a point of vanishing group velocity, increasing mode spacing. Note that the “π mode” of a twisted structure does not have the same precise meaning as it does for a corrugated accelerating

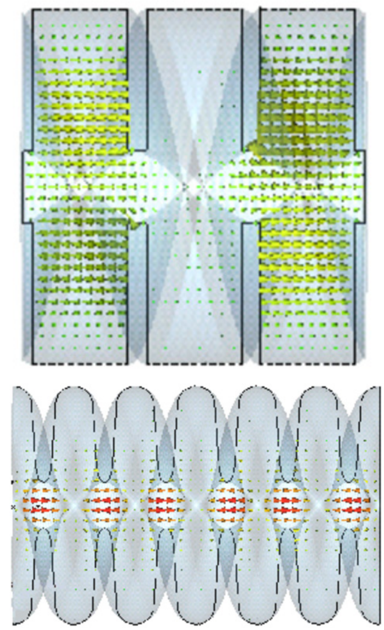

Figure 2: CST Simulation of Twisted disk loaded analog and SNS cavity analog: Electric field

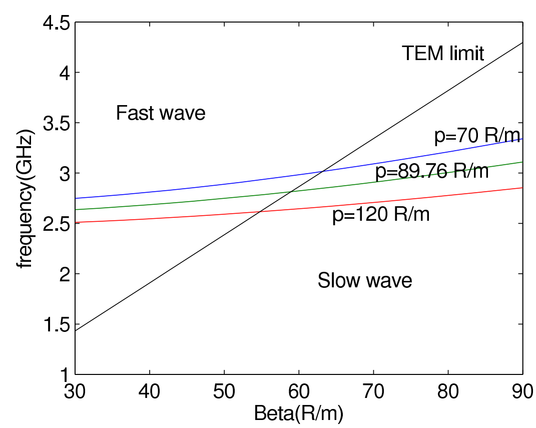

Figure 3: Dispersion diagram for twisted disk-loaded equivalent structure for several twist rates. (Twist rates in radians/meter.)

structure – we simply mean the phase variation of the electromagnetic fields per $\frac{1}{2}$ twist. The fields will vary sinusoidally along the particle axis regardless of the mode designation.

PERFORMANCE

The performance of the twisted guide was analyzed chiefly with respect to the $\frac{R}{Q}$. We used the twisted analog of the disk-loaded accelerator described in Table 1 for our investigation. The twist rate p was allowed to vary, and the frequency was readjusted at each value of p to ensure that the phase velocity remained at c. The results are shown in Fig. 4

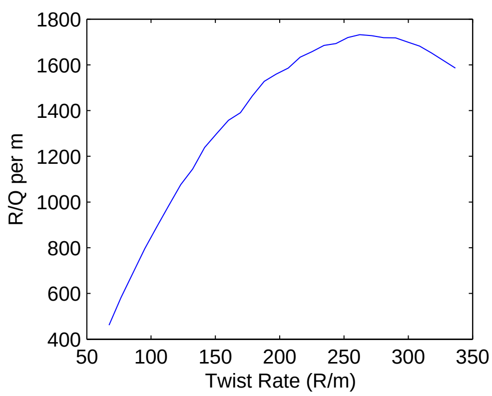

Figure 4: Simulated traveling wave $\frac{R}{Q}$ as a function of twist rate in a twisted analog of a disk-loaded accelerating structure.

This shows that the twist rate has a significant effect on $\frac{R}{Q}$. In general, the twist rate should not be chosen to maintain a geometric similarity to an existing accelerating structure (such as a disk-loaded accelerating structure), but should rather be carefully chosen to optimize performance.

An example of a twisted elliptical cavity with particle $\beta = 0.61$ was printed using a 3D Stereolithography Apparatus (SLA) and electroplated with copper on the inner surface (Fig. 5). A beadpull measurement was then performed to measure the intensity of the electric field on the axis of the guide. The results of the beadpull are shown in Fig. 6, showing sinusoidal variation of the electric field in the center of the twisted cavity.

CONCLUSION

We have examined twisted guides as an alternative to reactively loaded corrugated accelerating structures. Like corrugated structures, empty twisted waveguides can support slow wave operation. Twisted analogs to the disk-loaded accelerating structure and the TESLA-type elliptical cavity have been examined. Such twisted structures have a uniform cross section, meaning they could be easier to

Figure 5: Prototype of a twisted elliptical cavity.

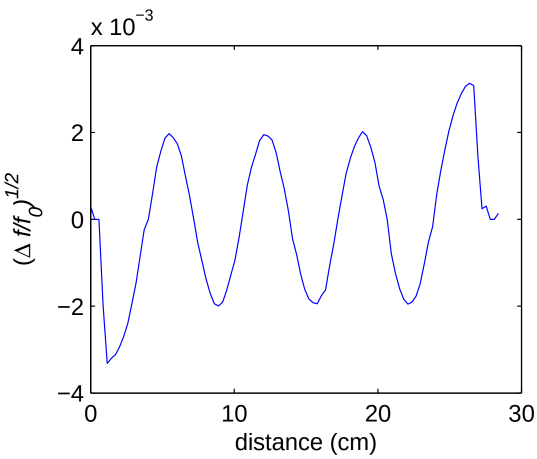

Figure 6: Measured (bead pull) results for elliptical twisted cavity prototype.

machine, eliminating complicated welding or brazing processes. The twisted structure can be tuned to a frequency as a single structure, and its field ditribution may have greater tolerance to internal dimensional error. The modes and field distributions in the twisted analogs are comparable to their nontwisted counterparts.

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