

# PREDICTION OF $4\nu=1$ RESONANCE OF A HIGH INTENSITY LINAC\*

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## Abstract

The  $4\nu=1$  resonance of a linac is demonstrated when the depressed tune is around  $90^\circ$ . It is observed that this fourth order resonance is dominating over the better known envelope instability and practically replacing it. Simulation study shows a clear emittance growth by this resonance and its stopband. Experimental measurement of the stopband of this resonance is proposed and conducted in 2008 using the UNILAC at GSI. This study will serve as a benchmarking and guidance for the experiment.

## INTRODUCTION

Recently many high intensity linacs have been designed or constructed like the SNS (USA) [1], J-PARC (Japan) [2], or people are trying to increase the intensity of existing linacs such as the UNILAC of GSI (Germany) [3]. For the high intensity linacs, it is the utmost goal to minimize the beam loss of halo particles by avoiding or minimizing contributions of various halo formation mechanisms. One such mechanism is the envelope instability [4]. So far the high intensity linac design such as the SNS linac has avoided the  $\sigma_{ot} = 90^\circ$  phase advance because of the envelope instability [1].

Until 1998, mismatch was the primarily studied mechanism of halo formation. Late 1998, Jeon found a case of halo formation induced by the  $2\nu_x-2\nu_y=0$  resonance from the space charge potential in the ring [5]. Further studies of halo formation and/or emittance growth by space charge and resonances are reported in [6] and by space charge coupling resonance studies of linac such as [7]. Besides these, halo formation by non-round beam was reported [8] and halo formation by rf cavity [9].

In this paper, we will report about a collaborative effort between FAIR-GSI and SNS concerning the  $4\nu=1$  resonance of a high intensity linac. We are preparing for an experiment to measure the stop-band of this resonance using the UNILAC at GSI. Numerical simulation is performed with 50 000 to 100 000 macroparticles with the PARMILA code [10]. Space charge tune shift is about  $-20^\circ$ .

## THE LINAC FOURTH ORDER RESONANCE

The study shows that the  $4\nu=1$  resonance occurs when the phase advance with space charge  $\sigma$  is slightly lower than  $90^\circ$  for a linac just like a ring through the space charge octupole potential for a variety of beams including Gaussian, waterbag, etc.. For the phase advance with

space charge  $\sigma > 90^\circ$ , no resonance effect is observed, as shown in Fig. 6.

### Crossing the Resonance from Below $90^\circ$

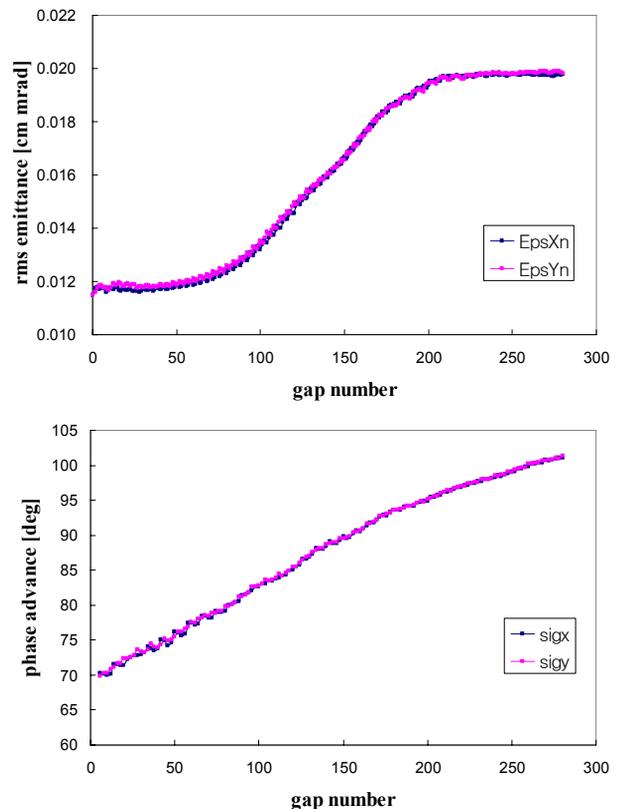


Figure 1: Top plots display rms emittance vs gap number and bottom plots the phase advance with space charge when the beam crosses the resonance from below.

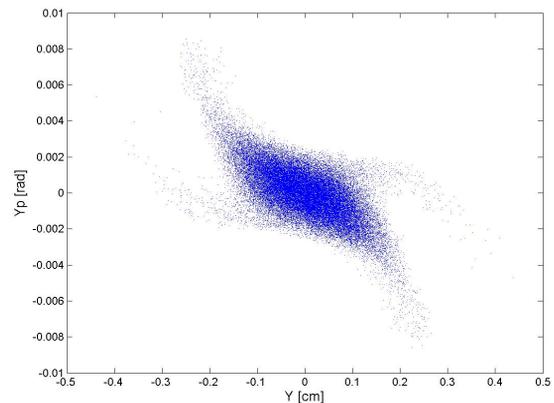


Figure 2: Plot of the beam distribution in Y phase space at the 96<sup>th</sup> gap. Transport of beam particles along the separatrices is observed.

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We performed a simulation of beam crossing the resonance from below  $90^\circ$ . Fig. 1 shows the normalized rms emittance growth as the beam crosses the  $4\nu=1$  resonance together with the variation of the depressed phase advance  $\sigma$ . When the depressed phase advance with space charge  $\sigma$  reaches about  $75^\circ$ , emittance starts to grow. The initial beam is a well matched Gaussian beam to the linac and the emittance growth is solely due to the resonance crossing. The same phenomenon is observed with waterbag beam.

When the beam crosses the resonance from below  $90^\circ$ , the stable fixed points move from afar to the origin and beam particles are transported along the separatrices, not captured by the stable islands. This is well illustrated in Fig. 2.

### Crossing the Resonance from Above $90^\circ$

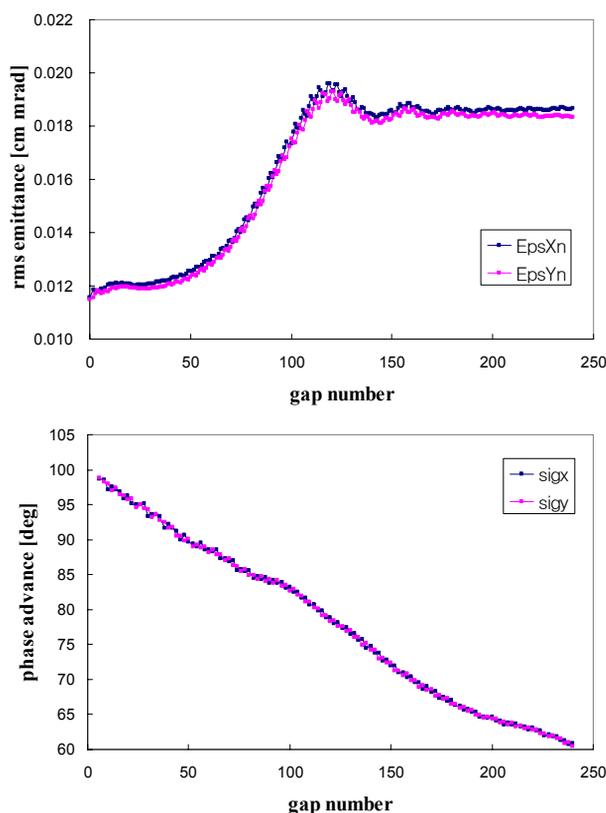


Figure 3: Top plots display rms emittance vs gap number and bottom plots the phase advance with space charge, when the beam crosses the resonance from above.

We performed a simulation of beam crossing the resonance from above  $90^\circ$ . Fig. 3 shows the normalized rms emittance growth as the beam crosses the  $4\nu=1$  resonance along with the variation of the depressed phase advance  $\sigma$ . As the depressed phase advance  $\sigma$  crosses  $90^\circ$ , emittance starts to grow. The initial beam is a well matched Gaussian beam to the linac. For  $\sigma > 90^\circ$ , there is no resonance effect.

When the beam crosses the resonance from above  $90^\circ$ , stable fixed points move away from the origin to afar.

Unlike rings, tune change rate is not slow enough for adiabatic capture of beam particles by the stable islands. So beam particles are not entirely captured as illustrated in Fig. 4.

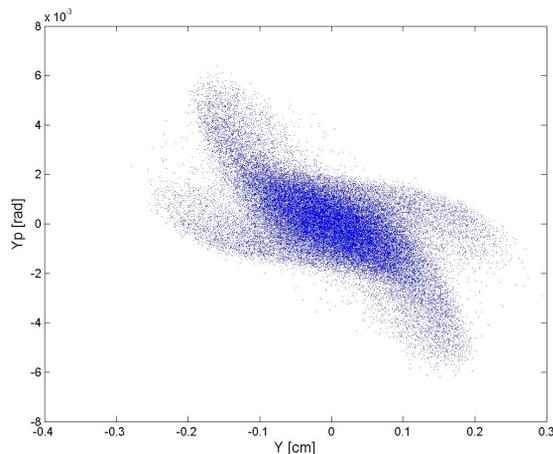


Figure 4: Plot of the beam distribution in Y phase space at the 88<sup>th</sup> gap.

### Measurement of the Stopband

We are preparing for an experiment to measure the stop-band of the  $4\nu=1$  resonance with an emittance scanner installed right after the first DTL tank A1 of the UNILAC. Figure 5 displays the simulation of the sum of transverse rms emittances vs. zero current phase advance. About 45% of rms emittance increase is anticipated due to the resonance.

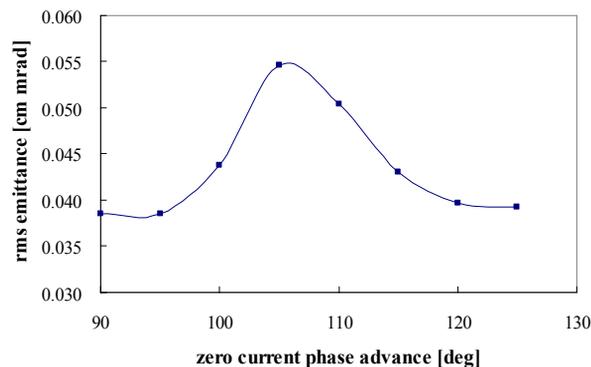


Figure 5: Plot of the sum of the normalized rms emittance  $\epsilon_x + \epsilon_y$  vs. zero current phase advance.

### Envelope Instability??

High intensity linac design has avoided the  $90^\circ$  phase advance because of the well known envelope instability. Our study indicates that the  $4\nu=1$  resonance is dominating over the envelope instability and practically replacing it. We did not observe the envelope instability during the simulation for the phase advance around  $90^\circ$  as shown in Fig. 5. For  $90^\circ \leq \sigma_0 \leq 95^\circ$ , no appreciable emittance growth is observed. It should be noted that the initial beam is well matched to the linac.

Considering this, it should be stated that the high intensity linac design should avoid  $90^\circ$  phase advance because of the  $4\nu=1$  resonance rather than the better known envelope instability. The effect of the envelope instability can actually be minimized – in theory - by nearly perfect envelope matching, whereas the  $4\nu=1$  resonance is independent of the rms matching.

### Rms Emittance vs. Phase Advance

Simulation is done to study the equilibrium rms emittance as a function of the depressed phase advance  $\sigma$ . For this simulation,  $\sigma$  is kept constant throughout the linac. Figure 6 shows plots of the ratio of the final transverse emittance over the initial emittance as a function of the depressed phase advance  $\sigma$ . Rms emittance grows as a power of 3.5 up to  $90^\circ$  and there is no emittance growth beyond  $90^\circ$ . Two cases are simulated and plotted; one is with a tune depression of  $-23^\circ$  and the other  $-20^\circ$ .

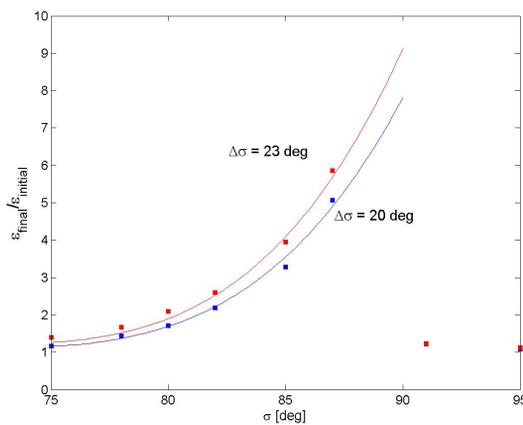


Figure 6: Plots of the rms emittance vs. the depressed phase advance  $\sigma$ . No emittance growth occurs for  $\sigma > 90^\circ$ .

### Effect of Mismatch

When strong mismatch is applied to the initial beam distribution, the effect of mismatch is manifested on top of the  $4\nu=1$  resonance, as shown in Fig. 7. Obviously the fourth order resonance remains dominant, and there is not enough time for the envelope instability to take over. In both planes, initial  $\beta_x$  and  $\beta_y$  are increased by 40%.

### CONCLUSION

The  $4\nu=1$  resonance of a linac is demonstrated through space charge potential when the depressed tune is around  $90^\circ$ . It is observed that this fourth order resonance is dominating over the better known envelope instability and practically replacing it. It needs to be rephrased that the high intensity linac design should avoid  $90^\circ$  phase advance because of the  $4\nu=1$  resonance rather than the better known envelope instability.

### ACKNOWLEDGEMENT

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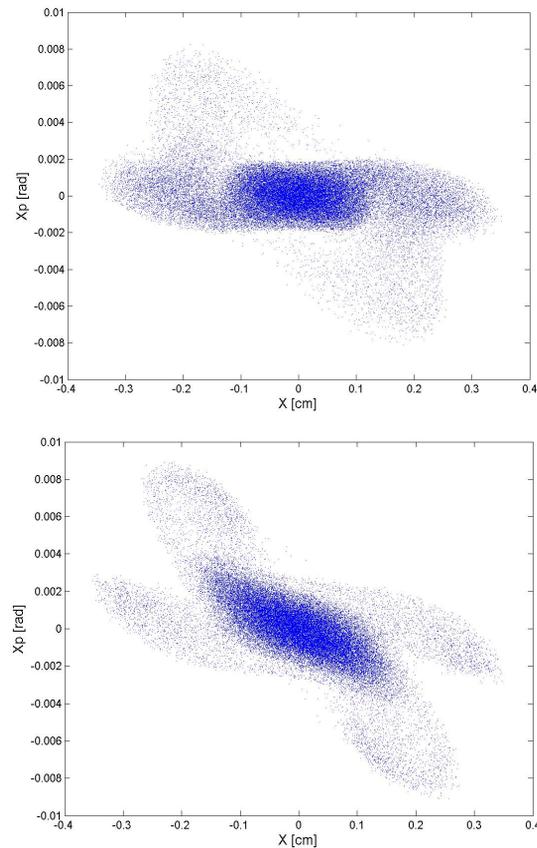


Figure 7: Plots of Beam distribution with mismatch (upper plot) and without mismatch (lower plot).

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