中国科学院高能物理研究所
Institute of High Energy Physics

Space Charge Induced Collective Modes and Beam Halo in Periodic Channels

Chao Li, Qing Qin

Key Laboratory of Particle Acceleration Physics and Technology, Institute of High Energy Physics, Beijing 100049, China

R. A. Jameson

Institut für Angewandte Physik, Goethe Uni Frankfurt, Frankfurt-am-Main, Germany

Lichao@ihep.ac.cn. Supported by the   by the Ministry of Science and Technology of China under Grant No. 2014CB845501

Outline

1. Review of study in collective mode.
2. Collective instability – structure resonance.
3. Numerical results of the unstable collective mode with TOPO code.
4. Halo mechanism – resonance between particle and collective mode.
5. Summary

In this talk we consider the space charge effect in rms matched beams only. Rms mismatched beams are beyond of the scope of this talk.

Collective mode - a long story to tell

- **Real progress was made since the KV distribution beam was proposed (I. M. Kapchinskij and V. V. Vladimirskij, 1959)**
- **Rms envelope equation and the method of equivalent beam (Sacherer, 1971)**
- **Instability of oscillation modes of 2D *constant focusing* channel (R. L. Glustern, 1970)**
  - **Solve the perturbed Vlasov-Poisson equation simultaneously;**
  - **The perturbed potential is assumed as the form of hyper-geometric function in *constant focusing* channel.**
- **Instability of oscillation collective modes of 2D *periodic focusing* channel (I.Hofmann, et. al, 1983)**
  - **Solve perturbed Vlasov-Poisson equation simultaneously;**
  - **Perturbed potential is assumed as the form of polynomial for different orders collective mode;**
  - **Jacobi matrix of the perturbed Vlasov-Poisson equation is used to depict the stability.**
- **Instability of mismatched envelope oscillation (J. Struckmeier, et. al, 1984)**
  - **The envelope instability gives the same stop band as 2^{nd} order even mode in ingo's work (1983).**

Recent discussion on collective mode: 2nd order mode/4th order mode when $\sigma \sim 90$

- **The balance between 2nd and 4th order collective mode was discussed at 2009.**

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 12, 054204 (2009)

Fourth order resonance of a high intensity linear accelerator

D. Jeon,[1,*] L. Groening,[2] and G. Franchetti[2]
[1]*SNS, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*
[2]*GSI, Darmstadt, Germany*
(Received 15 January 2009; published 29 May 2009)

It is discovered that, for a high intensity beam, the $4\sigma = 360°$ (or $4\nu = 1$) resonance of a linear accelerator is manifested through the octupolar term of space charge potential when the depressed phase advance per cell σ is close to and below 90° but no resonance effect is observed when σ is just above 90°. To verify that this is a resonance, a frequency analysis is performed and a study of resonance crossing from above and from below the resonance is conducted. It is observed that this fourth order resonance is dominating over the better known envelope instability and practically replacing it. The simulation study shows a clear emittance growth by this resonance and its stop band. A proposal to GSI was made to perform an experiment to measure the stop band of this resonance using the UNILAC. The experiment confirmed this resonance and will be published in a separate paper.

DOI: 10.1103/PhysRevSTAB.12.054204 PACS numbers: 29.27.Bd, 41.75.−i

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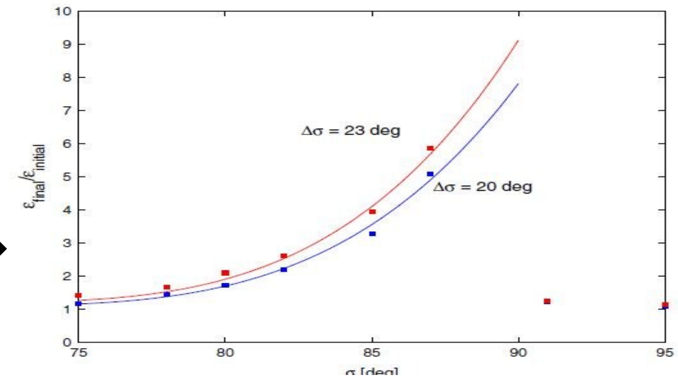

Phys. Rev. ST Accel. Beams 12, 054204 (2009)

- **First measurement of a space charge structure resonance at GSI at 2009.**

PRL 102, 234801 (2009) PHYSICAL REVIEW LETTERS week ending 12 JUNE 2009

Experimental Evidence of the 90° Stop Band in the GSI UNILAC

L. Groening, W. Barth, W. Bayer, G. Clemente, L. Dahl, P. Forck, P. Gerhard, I. Hofmann, M. S. Kaiser, M. Maier, S. Mickat, and T. Milosic
GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstrasse 1, D-64291 Darmstadt, Germany
D. Jeon
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830, USA
D. Uriot
CEA IRFU, Service des Accélérateurs, de Cryogénie et de Magnétisme, F-91191 Gif-sur-Yvette, France
(Received 16 March 2009; revised manuscript received 5 June 2009; published 12 June 2009)

In a particle accelerator with a periodic structure beam space charge force may excite resonant beam emittance growth if the particle's transverse phase advance approaches 90°. A recent simulation study with the PARMILA code [D. Jeon *et al.*, Phys. Rev. ST Accel. Beams **12**, 054204 (2009)] has shown the feasibility of measuring the stop band of this fourth order resonance in the GSI Universal Linear Accelerator UNILAC and proposed its experimental verification, which is reported here. Measurements of transverse phase space distributions behind a periodically focusing structure reveal a fourfold symmetry characteristic of fourth order resonances as well as a resonance stop band above $\sigma_0 = 90°$ per focusing cell. These experimental findings agree with results from three different beam dynamics simulation codes, i.e., DYNAMION, PARMILA, and TRACEWIN.

DOI: 10.1103/PhysRevLett.102.234801 PACS numbers: 29.27.Bd, 41.75.−i, 41.85.−p

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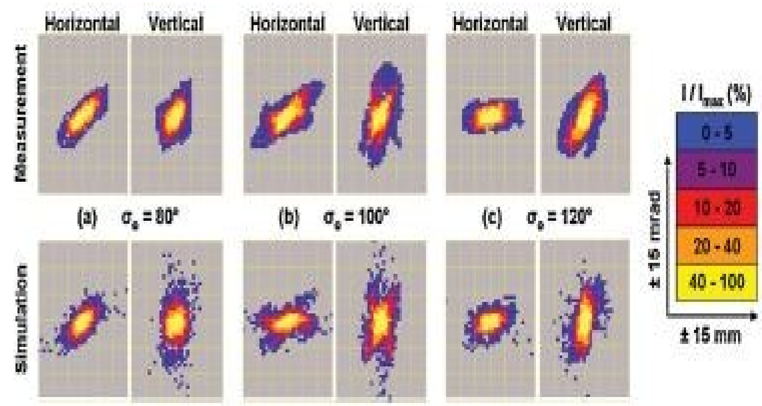

Phys. Rev. Lett. 102, 234801 (2009)

Recent discussion on collective mode: 2nd order mode/4th order mode when $\sigma \sim 90$

- **The width of stop band is explained in 2014. Phys. Rev. ST Accel. Beams, 17, 124202 (2014)**

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 17, 124202 (2014)

Envelope instability and the fourth order resonance

Chao Li* and Ya Liang Zhao
Key Laboratory of Particle Acceleration Physics and Technology, Institute of High Energy Physics, Chinese Academy of Sciences, 19(B) Yuquan Road, Beijing 100049, People's Republic of China
(Received 21 September 2014; published 19 December 2014)

The well-known envelope instability or the second order even collective mode [I. Hofmann, *Phys. Rev. E* 57, 4 (1998)] and the fourth order resonance $4\sigma = 360°$ due to the nonlinear space charge effect in high intensity beams have been studied previously. A wide stop band around 15° is found in a pure periodic focusing channel. In addition, it is illustrated that the fourth order resonance dominates over the envelope instability and practically replaces it in the stop band [D. Jeon *et al.*, *Phys. Rev. ST Accel. Beams* 12, 054204 (2009)]. In this paper, for a continuous beam with remarkable space charge, our 2D self-consistent particle-in-cell simulation work with the code TOPOPIC shows these two kinds of effects respectively in a periodic focusing defocusing (FD) channel. For a fixed tune depression $\eta = 0.8$, a stop band with a width of almost 15° is also demonstrated. Moreover, it is confirmed that analytical results of the rms envelope instability diagram are a valid tool to interpret the width of the stop band. Emittance growth rates in stop band are also well explained. It is found that, for a nearly rms matched beam, the emittance growth in the stop band is almost proportional to the saturation time of the nonlinear instability of the envelope, which happens in a quick manner and takes only a few FD cells. In contrast, the fourth order resonance is independent of rms matching and will be accompanied by beam evolution as "a long term effect" once the related mechanism is excited.

DOI: 10.1103/PhysRevSTAB.17.124202 PACS numbers: 41.75.-i, 29.27.Bd, 29.20.Ej

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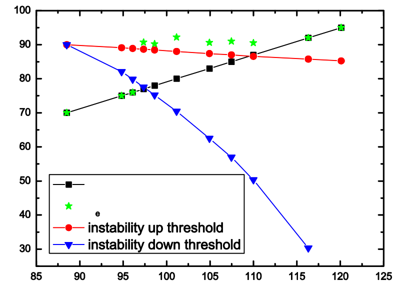
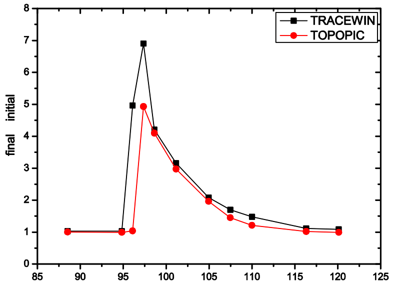




- **2nd order stop band is actually a coincidence of a 4th order structure resonance.**
- **Similar phenomenon when σ~60, Phys. Rev. Lett. 115, 204802 (2015).**

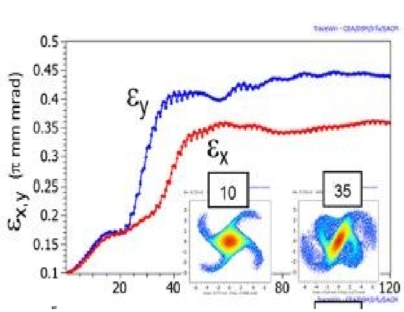
PRL 115, 204802 (2015) PHYSICAL REVIEW LETTERS week ending 13 NOVEMBER 2015

Space-Charge Structural Instabilities and Resonances in High-Intensity Beams

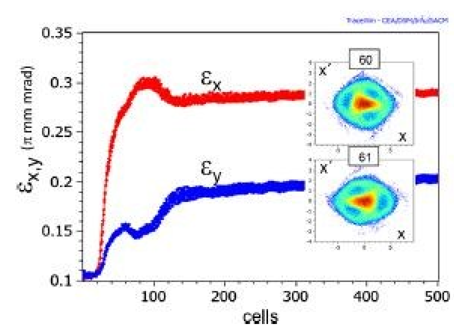
Ingo Hofmann[1,2,*] and Oliver Boine-Frankenheim[1,2]
[1]*GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstrasse 1, 64291 Darmstadt, Germany*
[2]*Technische Universität Darmstadt, Schlossgartenstrasse 8, 64289 Darmstadt, Germany*
(Received 19 June 2015; published 10 November 2015)

The existence of a structural resonance stop band caused by space charge in high-current beams, where the resonance frequency is associated with 90° phase advance per focusing period, is well known and alternatively referred to in the literature as envelope instability or as fourth-order resonance. We show, however, that this stop band is actually a coincidence of a structural fourth-order resonance and the much stronger envelope instability as competing mechanisms—depending on the time scale and initial matching. A similar complexity of behavior—dependent on the distribution function—is also found between a third-order instability and a sixth-order resonance in a 60° stop band. We claim that these findings are a generic property of high-intensity beams in periodic focusing; they also allow a reinterpretation of the 90° linear accelerator stop band previously observed experimentally at the UNILAC accelerator.

DOI: 10.1103/PhysRevLett.115.204802 PACS numbers: 41.75.Ak, 29.20.Ej, 29.27.Bd



90 collective mode



60 collective mode

Motivation of our work

- Clarify the mechanism of the collective mode analytically.
- Verify the effect of the collective modes numerically with self-consistent simulation.
- Show the effect of collective mode on particles – ”beam halo”.

2 Collective mode and structure resonance

Mathematical description of collective mode instability

Solve the perturbed Vlasov-Poisson equation simultaneously, self-consistent process

KV beam distribution:
$$f_0 = \frac{N}{\pi^2}\delta(x^2 + p_x^2 + y^2 + p_y^2 - 1)$$

Perturbed KV beam distribution:
$$f_1 = \delta f_0 = \frac{N}{\pi^2}\delta'(x^2 + p_x^2 + y^2 + p_y^2 - 1)$$

Perturbed KV Hamiltonian:
$$H = H_0 + H_1 = \frac{1}{2\beta_x}(p_x^2 + x^2) + \frac{1}{2\beta_y}(p_y^2 + y^2) + \frac{1}{\epsilon}V(x,y,s)$$

It has to meet
$$f(\mathrm{x},\mathrm{y},\mathrm{z},p_x,p_y,p_z) = \mathrm{f(H)} \qquad \frac{df}{dt} = 0$$

Thus, the perturbation of f and potential V has to satisfy:

$$\frac{Df_1}{Ds} = \{\frac{\partial}{\partial s} + \frac{1}{\beta_x}[p_x\frac{\partial}{\partial x} - x\frac{\partial}{\partial p_x}] + \frac{1}{\beta_y}[p_y\frac{\partial}{\partial y} - y\frac{\partial}{\partial p_y}]\}f_1 \qquad \Delta V = -\oint f_1 dp$$

$$= 2\frac{N}{\pi^2\epsilon}[p_x\frac{\partial V}{\partial x} + p_y\frac{\partial V}{\partial y}]\delta'(x^2 + p_x^2 + y^2 + p_y^2 - 1).$$

I. Hofmann, L. J. Laslett, L. Smith, and I. Haber, Part. Accel. 13, 145 (1983).

Mathematical description

Assuming the space charge potential perturbation inside

$$V_i = \sum_{m=0}^{n} A_m(s) x^{n-m} y^m + \sum_{m=0}^{n-2} A_m^{(1)}(s) x^{n-m} y^m + \cdots$$

space charge potential perturbation outside

$$e^{-l(\xi-\xi_0)} \cos l\zeta, \qquad e^{-l(\xi-\xi_0)} \sin l\zeta$$

With appropriate boundary condition

$$\Delta \frac{\partial V}{\partial \xi} = \frac{Q}{\epsilon} \int \left(\frac{\partial}{\partial \psi_{x'}} + \frac{\partial}{\partial \psi_{y'}}\right) V[\cos\zeta \cos(\psi_{x'} - \psi_x), \sin\zeta \cos(\psi_{y'} - \psi_y); s']ds'$$

$$I_{j;k,l} = \int_0^s A_j(s') \sin[k(\psi_{x'} - \psi_x) - l(\psi_{y'} - \psi_y)]ds';$$

$$\frac{1}{C_{k,l}(s)} \frac{d}{ds}\left[\frac{1}{C_{k,l}(s)} \frac{dI_{j;k,l}}{ds}\right] + I_{j;k,l} = -\frac{1}{C_{k,l}(s)} A_j.$$

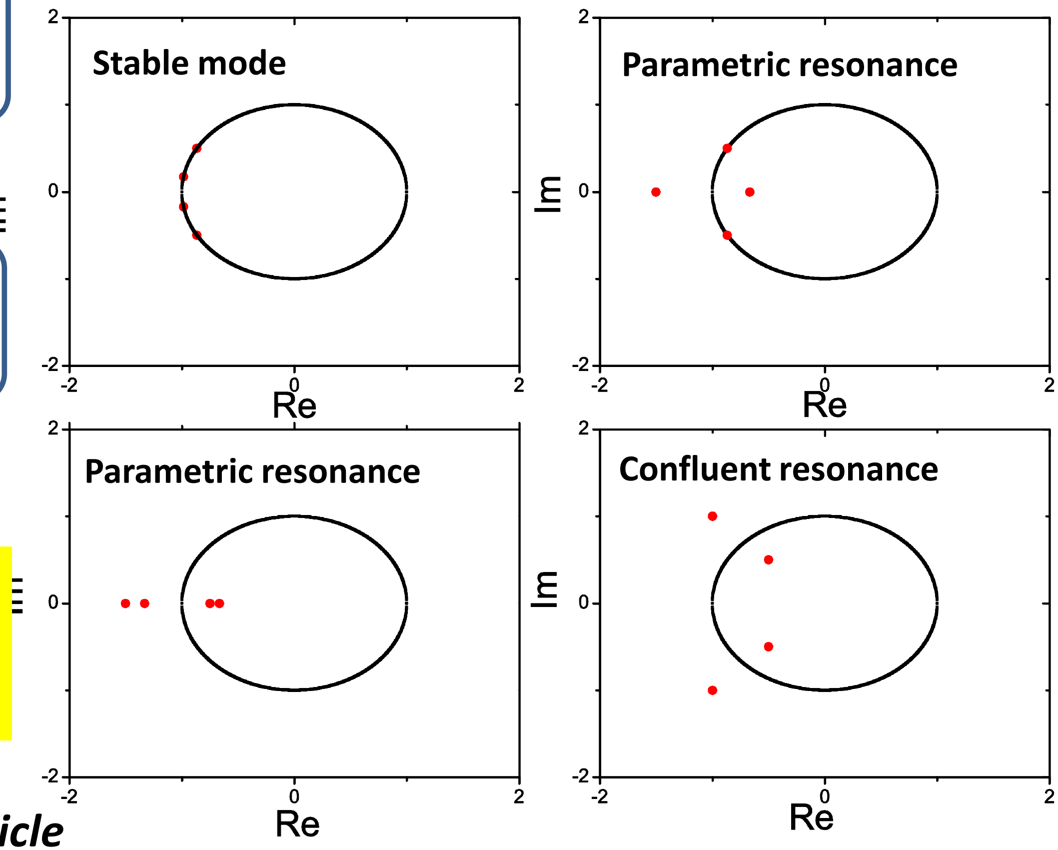
$I_{j;k,l}$: integral of the discontinuity of the surface electric field from period to period due to perturbation. The beam collective instability is decided by the Jacobi of $I_{j;k,l}$.

Unstable mode: perturbation increases exponentially

$$\frac{1}{C_{k,l}(s)}\frac{d}{ds}\left[\frac{1}{C_{k,l}(s)}\frac{dI_{j;k,l}}{ds}\right]+I_{j;k,l}=-\frac{1}{C_{k,l}(s)}A_j.$$

$$I_{j;k,l}(s+L)=\mathrm{M}(L)I_{j;k,l}(s)$$

With the form of Matthews equation, the stability of the system is decided by the eigenvalues of map M(L)



Ref. R. Dilao, *Nonlinear dynamics in particle accelerators*, Vol. 23 (World Scientific, 1996)

Possible location of the eigenvalues

Physics mechanism of unstable mode – structure resonance!

- With the classic perturbation theory, the stability of perturbation is used to represent the stable characteristics of the whole system.
- The unstable collective mode lies in structure resonance, which is composed of *parametric resonance* and *confluent resonance*

$$\Phi_{j;k,l} = n * 180$$ *Termed as parametric resonance*

$$\Phi_i + \Phi_j = n * 360$$ *Termed as confluent resonance*

In the following numerical simulation, we mainly discuss the case where $\sigma < 90$. For simplicity, we use $n\sigma \sim 180$ to approximately represent the nth order of structure resonance.

3 Simulation results with TOPO code

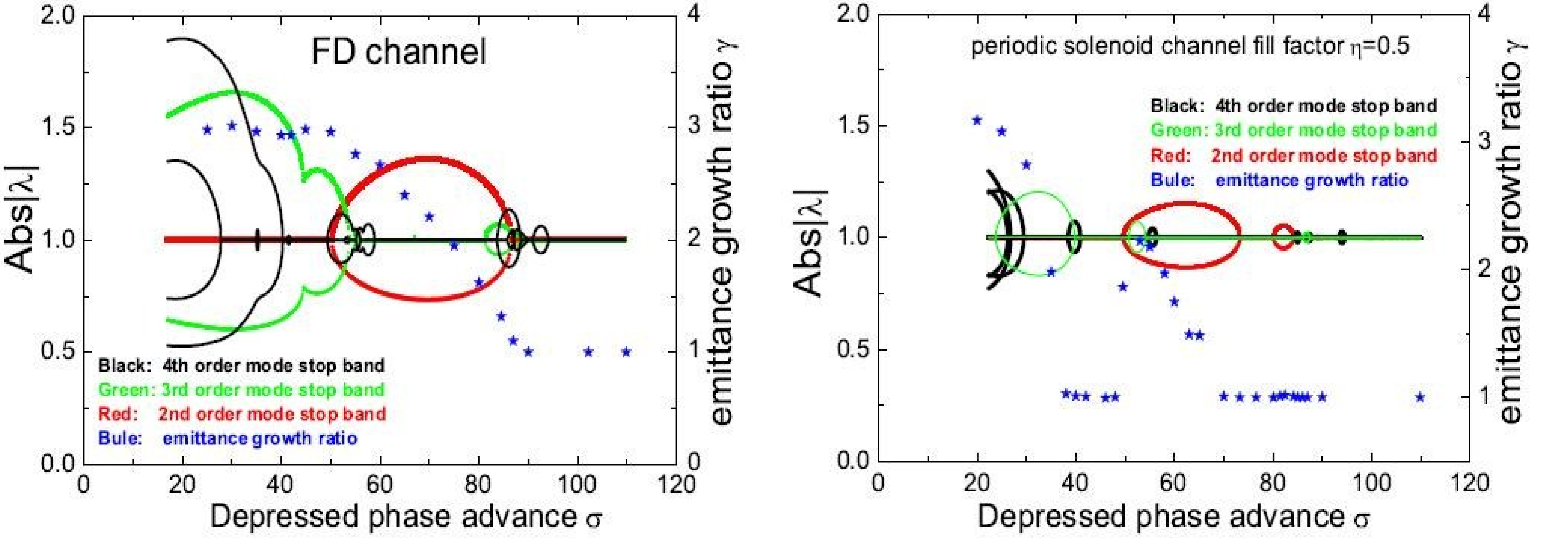
TOPO – Trace Of Particle Orbits

Techniques used in TOPO:

- Under development;
- T-code;
- Moving cuboid meshes for space charge calculating;
- First weighting methods;
- Symplectic integrator;
- Poisson solver with FFT;
- Lorentz transformation (electric fields are multiplied by the factor of $1/\gamma^2$);

Numerical results with code TOPO

- **Emittance growth ratio in 200 periods, eigenvalues of the 2nd, 3rd and the 4th order modes in FD and periodic solenoid channel when $\sigma 0 = 110$, KV initial beam.**

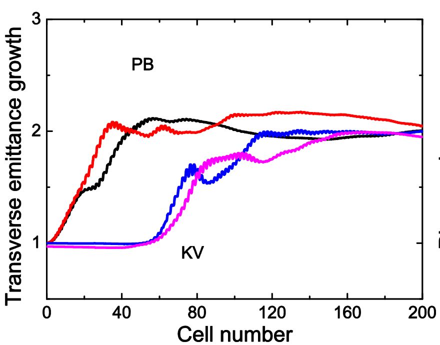


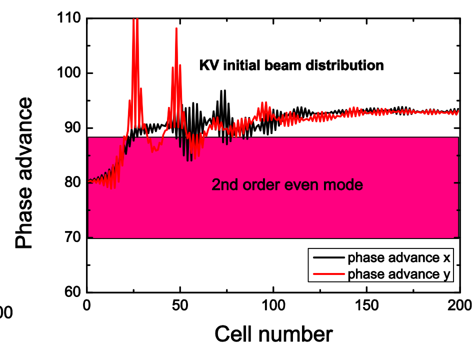
- **The broadened collective stop bands predict well the areas where the rms emittance growth take place.**

mixed 2nd / 4th order stop band: $2\sigma\sim180/4\sigma\sim360$

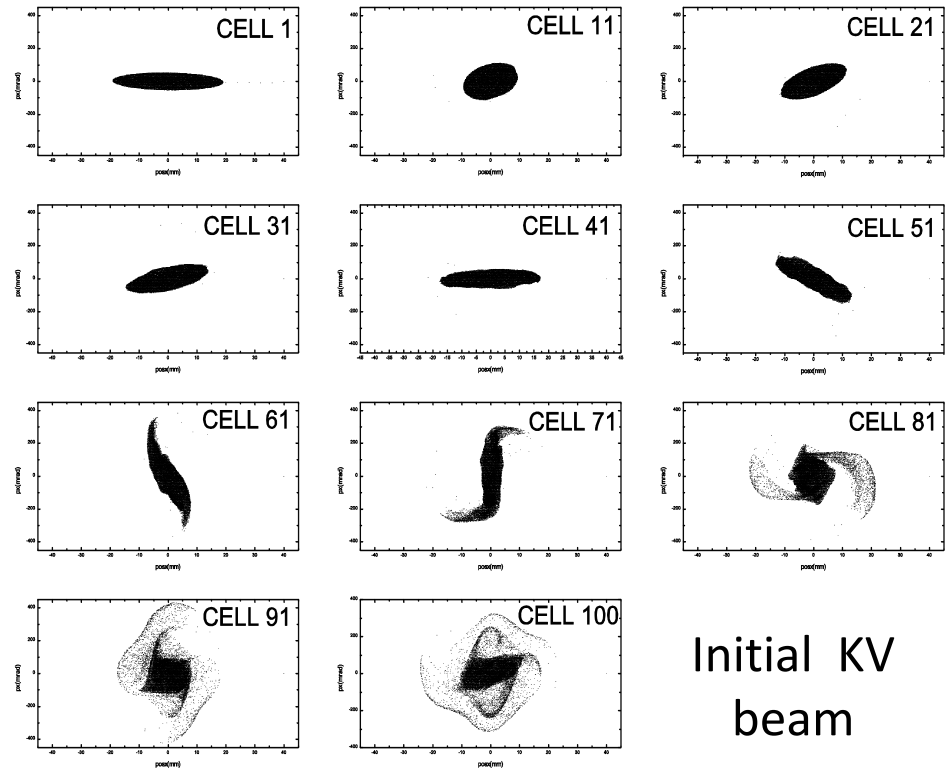
$\sigma 0 = 110, \sigma\sim80,$ FD

$$V_2 = A_0(s)x^2 + A_2(s)y^2$$





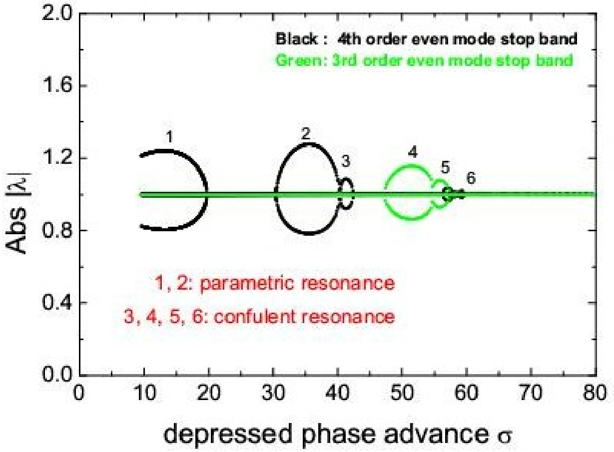
Emittance and phase advance evolution.



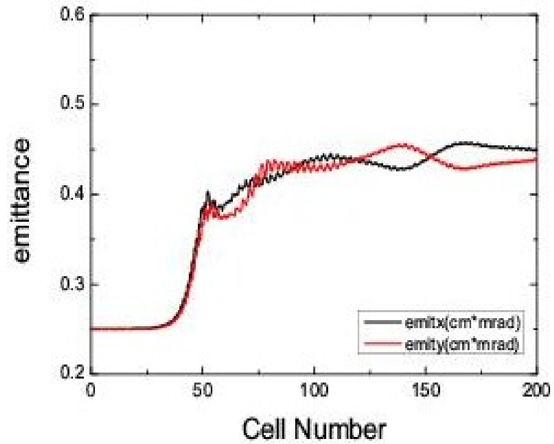
Initial KV beam

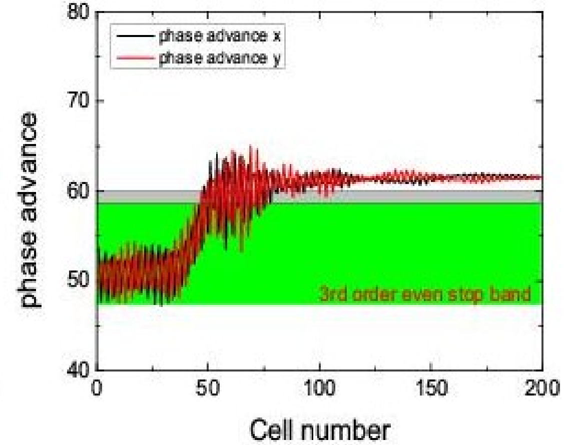
- **Both the 2nd and 4th order resonances are excited, in different time scales.**
- **Beam gets out of the stop band from the up-theshold, where space charge becomes less important .**
- **The beam gets a "local equilibrium" state finally, and it is a compromise between structure resonance and damping from density nonlinearity.**

3^{rd} order collective mode: $3\sigma\sim180$

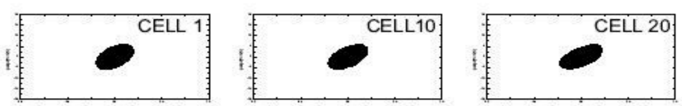


Stop bands pitches $\sigma0=80$

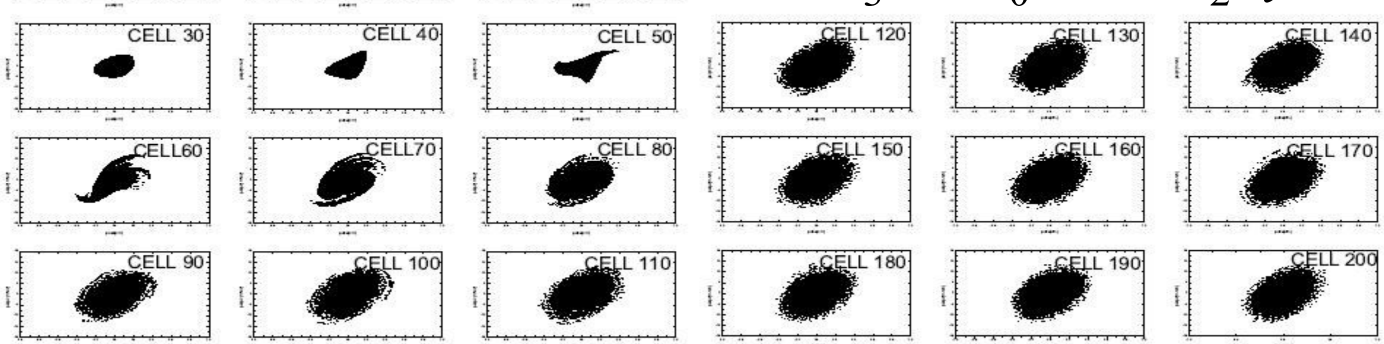




Emittance and instantaneous phase advance evolution with initial KV beam $\sigma0=80, \sigma=50.$



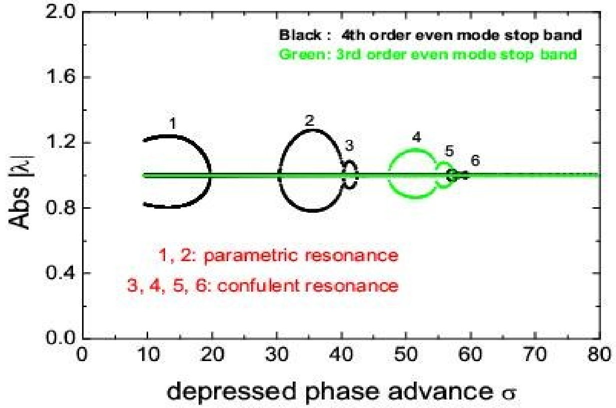
$$V_3 = A_0x^3 + A_2xy^2 + A_0^{(1)}x$$



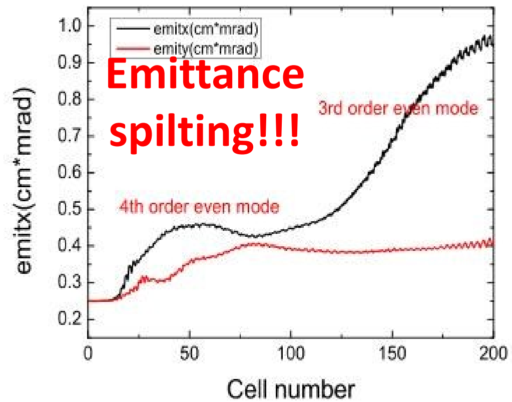
$\sigma0=80$
$\sigma\sim50$
FD

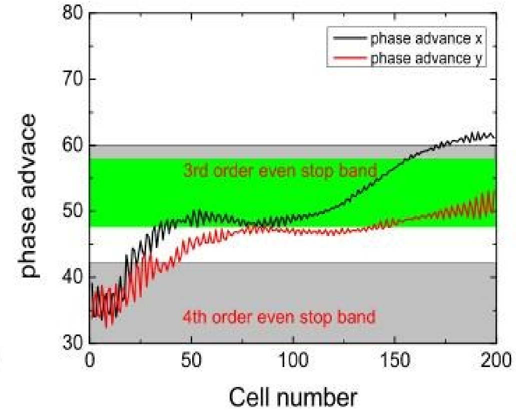
x-px phase space evolution along the FD channel, initial KV beam

4^{th} / 3^{rd} order mode: $4\sigma \sim 180$ / $3\sigma \sim 180$



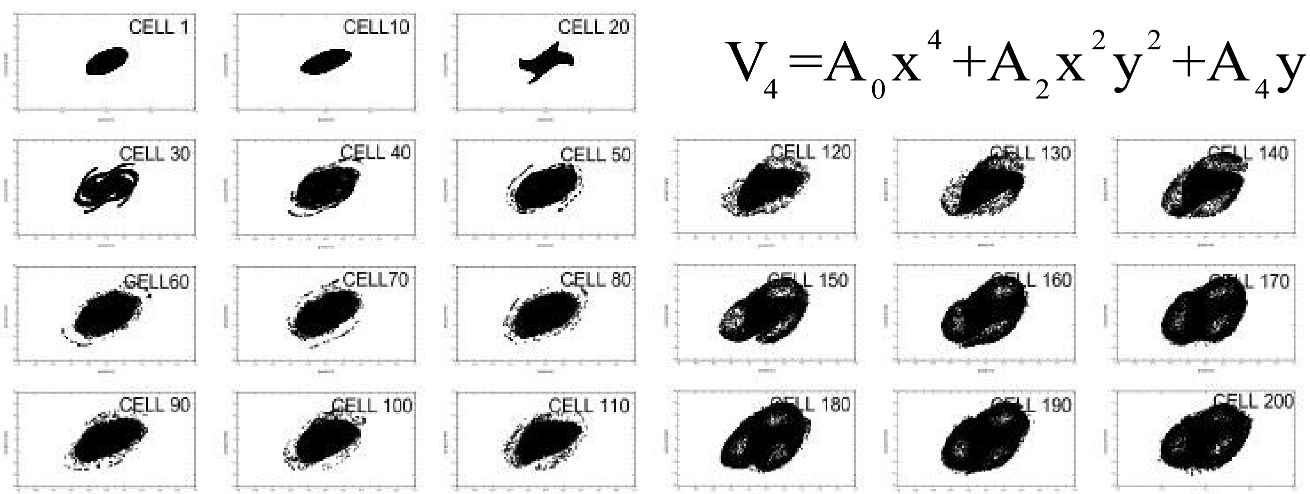
Stop bands pitches $\sigma 0 = 80$





Emittance and instantaneous phase advance
With initial KV beam, $\sigma 0 = 80$, $\sigma = 35$

$$V_4 = A_0 x^4 + A_2 x^2 y^2 + A_4 y^4 + A_0^{(1)} x^2 + A_2^{(1)} y^2$$

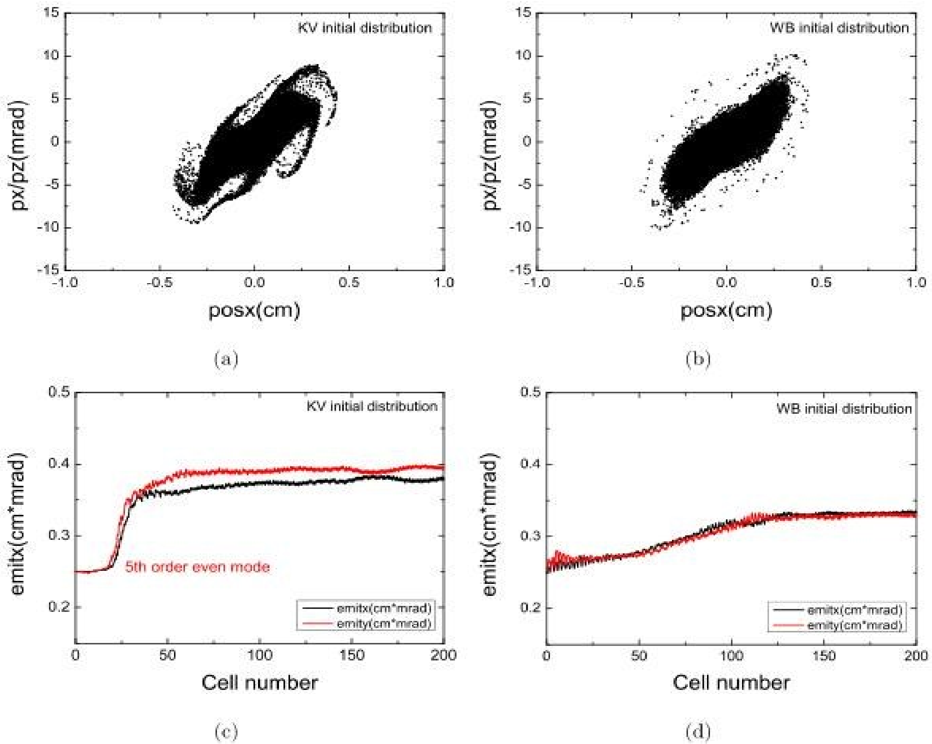


$\sigma 0 = 80$
$\sigma \sim 35$
FD

x-px phase space evolution along the FD channel

5^{th} order mode:$5\sigma\sim180$

$\sigma0 = 80,\ \sigma\sim25,$ FD, period 30



Phase space profile, and emittance evolution curve for initial KV and WB distribution in FD channel.

- **The 5^{th} orde collective mode shows clearly if initial beam is KV beam, but damped out by the nonlinear dengsity profile in WB beam case.**
- **KV initial case has a larger emittance growth compared with the WB initial case-this is why higher order resonance are usually ignored.**

Summary of the collective mode

- **Show complex time dependent process and phase space transport behind the usual steady state presentation of this subject.**
- **Nonlinear space charge plays roles as resonance driving force and source of nonlinearity damping.**
- **Resonance will be self-detuned, beam will be self-adapted to an "equilibrium" state, where space charge will be less important—nonlinear saturation effect. The final beam would be a compromise of structure resonance and nonlinear damping.**
- **Higher order mode (*n*>4): could be easily damped out by the nonlinear profile distribution, that is the reason why they are usually ignored.**
- **"Beam halo" would be a side effect of the collective mode.**

4 Interaction between particle motion and collective mode

Halo mechanism – resonance between particle and collective mode

The perturbed potential $V_n = \sum_{m=0}^{n} A_m(s) x^{n-m} y^m$

Single particle Hamiltonian $H(x, p_x, y, p_y; s) = \frac{1}{2}(K_x(s)x^2 + p_x^2) + \frac{1}{2}(K_y(s)y^2 + p_y^2) + V_n(x, y; s)$

Action – Angle frame $H(\Phi_x, J_x, \Phi_y, J_y; \theta) = \nu_x J_x + \nu_y J_y + \sum_{p,k} A_{p,k}(\theta)\cos(p\Phi_x + k\Phi_y)$

In Fourier expression $V_n = \sum_{p,k} V_{p,k}(\theta) = \sum_{p,k}\sum_{l} G_{p,k;l} e^{i(p\phi_x + k\phi_y - l\theta + p\chi_x - k\chi_y)}$

Resonance condition: *p+k=l*

The resonance condition is *p+k=l*, related resonance strength is given by $G_{p;k,l}$. In this case, perturbed space charge potential V_n is treated as external field. Resonance condition is the same as that in circular machine.

Halo mechanism – resonance between particle and collective mode

In the parametric resonance stop band, the perturbed mode also oscillating with a phase shift Φ^e in one period.

$$\Phi^e = n * 180$$

The perturbed potential is modified as

$$V_n = \sum_{p,k} V_{p,k} \kappa(\theta) e^{i\Phi^e}$$

$$= \sum_{p,k} \sum_{l} g_{p,k;l} e^{i(p\phi_x + k\phi_y - l\theta + \Phi^e + p\chi_x - k\chi_y)}$$

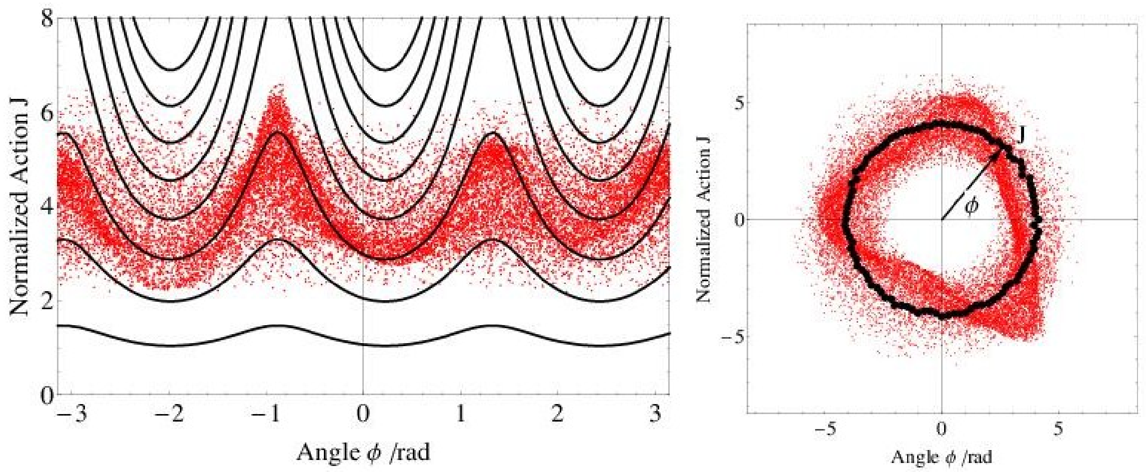
Resonance condition is modified as $p + k = l - 1/2$, when n=1.

Halo mechanism – resonance between particle and collective mode

Example: 3rd order resonance
p=3, k=0, l=1

$$H(\Phi_x, J_x, \Phi_y, J_y; \theta) = \nu_x J_x + G_{3,0;l} \cos((3\nu_x - l)\theta + \xi_{3,0;l})$$

Period 40,
$\sigma 0 = 80, \sigma \sim 50$



(a) (b)

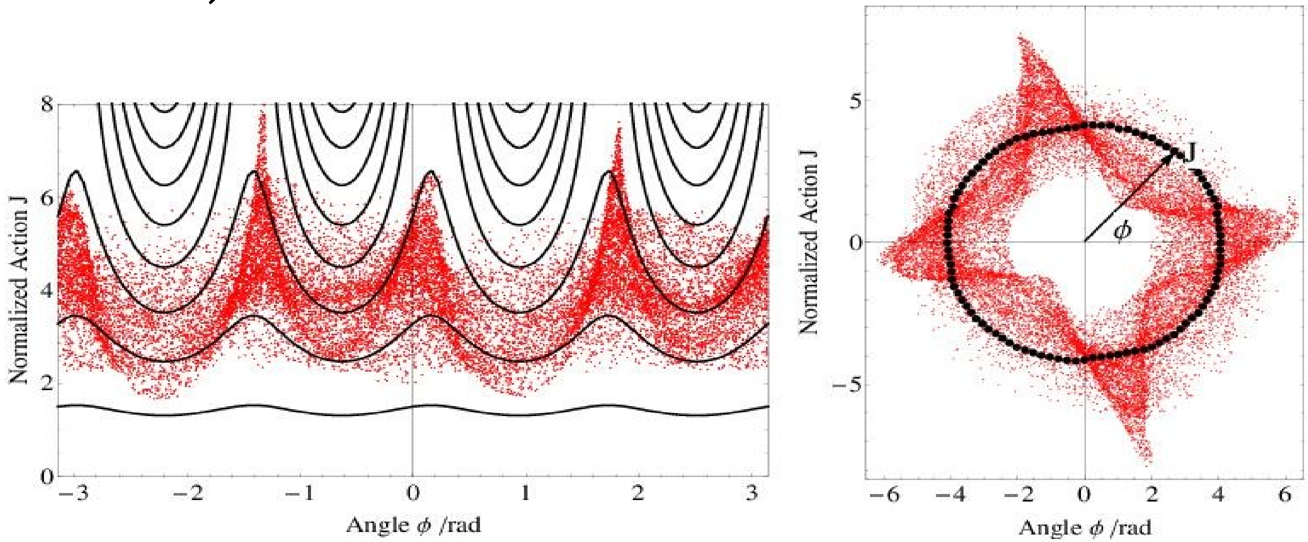
- In Cartesian and polar coordinate system for all particles at period 40. The black line in right figure represents the Hamiltonian tori obtained from analytical equation above. The black dots in the right figure represent the average action in each slice

Halo mechanism – resonance between single particle and collective mode

Example: 4th order resonance *p=4, k=0, l=1*

$$H(\Phi_x, J_x, \Phi_y, J_y; \theta) = \nu_x J_x + G_{4,0;l}\cos(4\phi_x - l\theta + \Phi_{0;4,0} + \xi_{4,0;l})$$

Period 20, $\sigma 0 = 80, \sigma \sim 35$



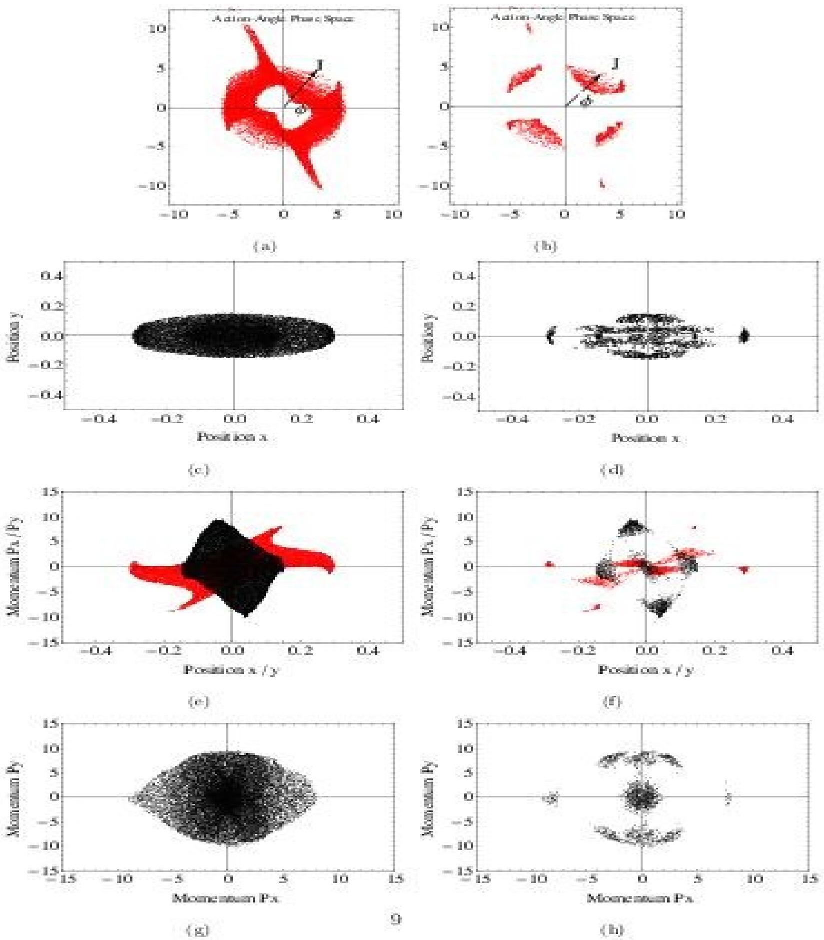
(a) (b)

Halo mechanism – resonance between single particle and collective mode

"Beam halo": particles located in the fold structures

Period 18, $\sigma 0 = 80, \sigma \sim 35$

- **The interaction between particle and collective mode well descries beam behavior.**
- **Halo could be defined as the particle whose Hamiltonian beyond certain separatrix.**
- **Halo could be hidden in the "core" in one projected plane (x-px), but they will be shown in another plane (y-py).**



Summary

- This talk only concentrates on nonlinearity from space charge in long period channel.
- Shows the complex time dependent process and phase space transport behind the usual steady state presentation.
- Resonance strongly depends on local parameters.
- Collective mode stop bands predict the area where the emittance growth take place quite well.
- If beam suffering from structure resonance, it will usually self-detuned into the safe region where the space charge will be less important.
- The final beam in a “local equilibrium” state is compromise of the structure resonance and damping from density nonlinearity.
- The higher order collective mode usually will be damped out.
- The unstable nth order collective mode is accompanied by an n-fold phase space structure.
- “Beam halo” could be grouped with particles whose actions are beyond the separatrix.
- Further study will be concentrated on collective modes in anisotropic beam in period focusing channel.

Thanks for your attentions!