ON BUNCH DIAGNOSTICS WITH USE OF SURFACE WAVES GENERATED ON PLANAR WIRE GRID*

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Abstract

Periodic structures can be used for non-destructive diagnostics of charged particle bunches. We consider structures which consist of thin conducting parallel wires. It is assumed that the structure period is much less than the typical wavelength under consideration. Therefore the influence of the structure on the electromagnetic field can be described with help of the averaged boundary conditions. We consider radiation of bunches which move along the grid but transversely to wires. Unlike previous works the bunch is assumed to have essential transversal dimensions along with definite longitudinal charge distribution. In particular we analyze the effect of reflection of the surface wave from the structure edge. For all considered situations, analytical and numerical results demonstrate that analysis of the surface waves allows estimating the size and the shape of the bunch.

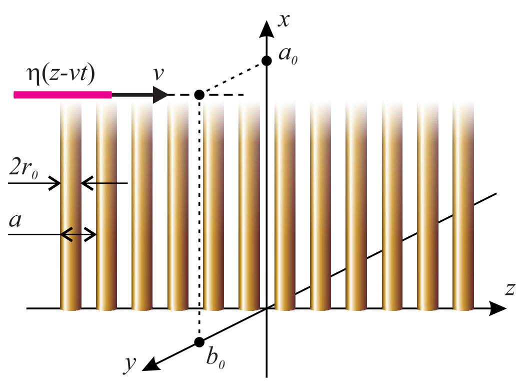

Figure 1: Bunch moving along the edge of a planar periodic wire structure.

Radiation of charges and charged-particle bunches moving in the presence of three-dimensional (wire metamaterial) and two-dimensional (planar wire grid) periodic structures comprised of parallel wires in long-wave approximation was analytically considered in a series of papers [1–4]. The results show that the charges generate non-divergent waves that propagate along wires without attenuation (if losses are negligible) with speed of light in vacuum. The spatial structure of these waves doesn't change in the process of their propagation. It can be used for determination of the bunch size [2, 4]. Previously, we mainly considered only thin bunches that have some longitudinal charge distribution and infinitesimal transversal sizes. The only exception is the analysis of radiation of bunches with finite transversal sizes moving inside unbounded three-dimensional structure [5]. In this work we analyze the radiation from "thick" bunches of cylindrical shape on the bounded planar wire grid (Fig. 1).

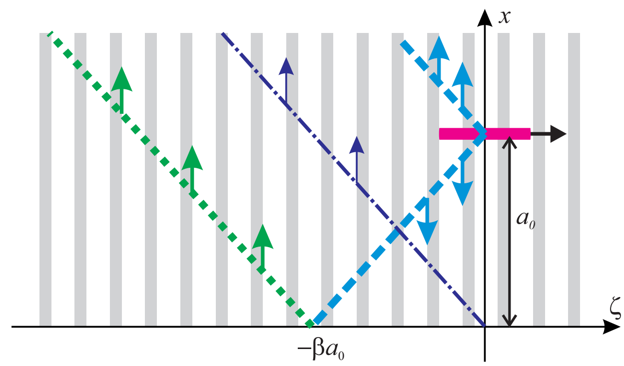

Figure 2: The structure of surface waves generated by bunch: the "infinite-like" wave (blue dashed line), the "reflected" wave (green dotted line), the wave "caught" by the edge (purple dash-dotted line). The arrows show the direction of propagation (direction of the energy flow density), $\zeta = z - vt$, $v = c\beta$.

Here we assume that the grid represents perfectly anisotropically conducting half-plane (such situation is achieved when $a \to 0$ and $a/r_0 = const$). The finite structure period and wire radius can be taken into account using averaged boundary conditions [3, 4, 6–8] (they just slightly affects the final results [3]). We suppose that the bunch moves along the edge perpendicularly to wires and, in contrast to [4, 6], its projection on the plane of structure lies in the area occupied by wires (i.e. $a_0 > 0$ accordingly to Fig. 1). Since Smith-Purcell radiation is not excited at the considered frequencies, the volume radiation is absent and the bunch generates only surface waves of three types: the "infinite-like" wave (identical to the surface wave that is generated on the infinite grid [3]), the "reflected" wave and the wave "caught" by the edge. The last wave decreases rapidly with increase of distance from the bunch trajectory to the edge, but the amplitudes of other waves do not depend on this distance. All three types of waves propagate along wires without attenuation with speed of light in vacuum c. The energy flow density of each wave is proportional to the squared electric field and directed along wires: $S_x = c|\vec{E}|^2/(4\pi)$. The field components of each wave do not change if the observation point is shifted along corresponding lines shown in Fig. 2.

If the bunch has infinitesimal transversal sizes ("thin" bunch) and uniform longitudinal charge distribution, then the components of the electric field for the "infinite-like" and

* The work is supported by the Grant of the President of Russian Federation (No. 6765.2015.2) and the Grants from Russian Foundation for Basic Research (No. 15-32-20985, 15-02-03913).
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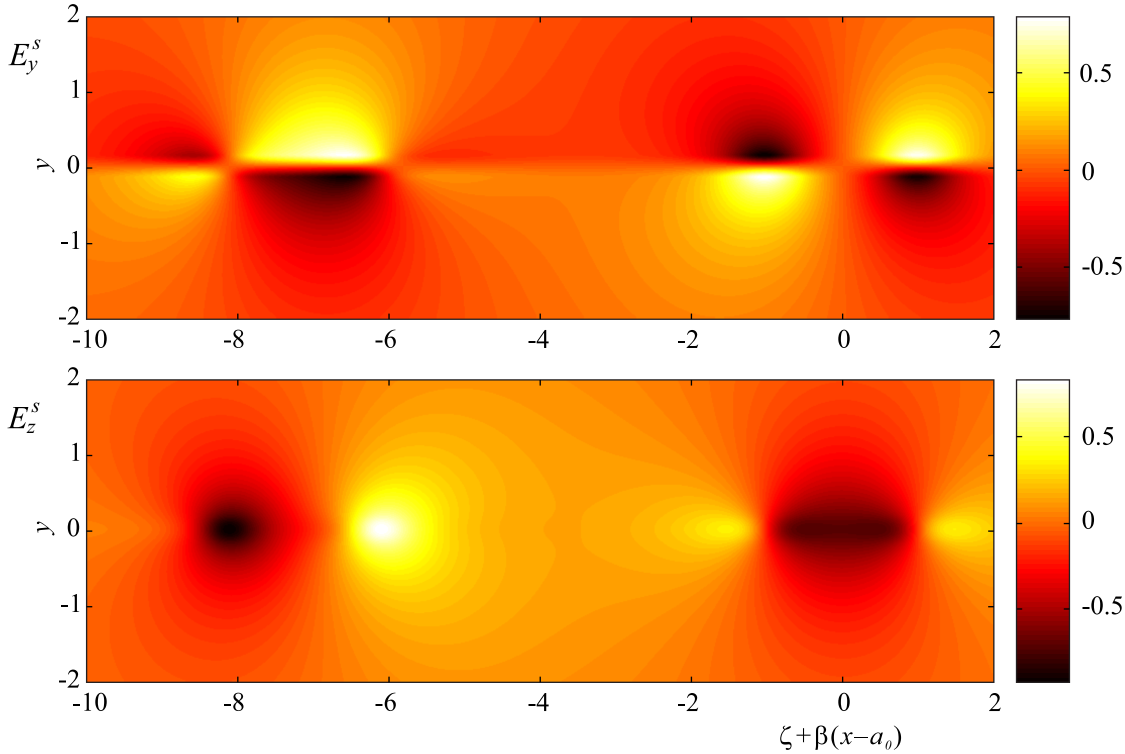

Figure 3: The field components of the surface waves generated by round cylindrical bunch moving along the edge of semi-infinite wire grid (Fig. 2) in the plane $x = const > a_0$, $\zeta = z - vt$. Wires are PEC, Gaussian units are used. Bunch length is $2\sigma = 2$ cm, thickness is $2\rho = 0.3$ cm, the charge is $q = +1$ esu ≈ 0.33 nC; $a_0 = 4$ cm and $b_0 = 0.4$ cm, $\beta = 0.9$.

"reflected" waves can be found analytically. The non-zero components of the "infinite-like" wave have form

$$\left\{ \begin{matrix} E_y^{Si} \\ E_z^{Si} \end{matrix} \right\} = q\beta \frac{\left\{ \begin{matrix} 2y_b\xi_i \operatorname{sgn} y \\ \xi_i^2 - y_b^2 - \sigma^2 \end{matrix} \right\}}{\left[y_b^2 + (\xi_i + \sigma)^2 \right] \left[y_b^2 + (\xi_i - \sigma)^2 \right]}, \quad (1)$$

where $y_b = |y| + |b_0|$, $\xi_i = \zeta + \beta|x - a_0|$, σ is the bunch half-length, q is the bunch total charge. The expressions for the "reflected" wave can be found as $\vec{E}^{Sr} = \hat{M}\vec{E}^{Si}|_{a_0 \to -a_0}$,

$$\hat{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix}, \quad \alpha = \arcsin(\beta \operatorname{sgn} y).$$

We neglect here the third wave ("caught" by the edge) because it decreases rapidly with increase of distance from the edge to the bunch trajectory.

Using results (1) we can calculate the field of the "infinite-like" and "reflected" surface waves from the "thick" cylindrical bunch having uniform charge distribution with help of the following convolution integral:

$$E_{cyl}(x, y, \zeta) = \frac{1}{\Sigma_{cyl}} \iint_{\Sigma_{cyl}} E_{thin}(x - x', y - y', \zeta)\, dx' dy', \quad (2)$$

where Σ_{cyl} is the bunch cross-section.

For numerical analysis we consider the bunch in the shape of round cylinder with length 2σ and cross-section radius ρ. In Fig. 3 the spatial distribution of the electric field and energy flow density of the surface waves in the plane orthogonal to wires are presented. In Fig. 4 we present comparison of energy flow densities of the surface waves generated by bunches with different thickness. As both figures show, the structure of the field of the surface waves contains information about both length and transversal size of the bunch.

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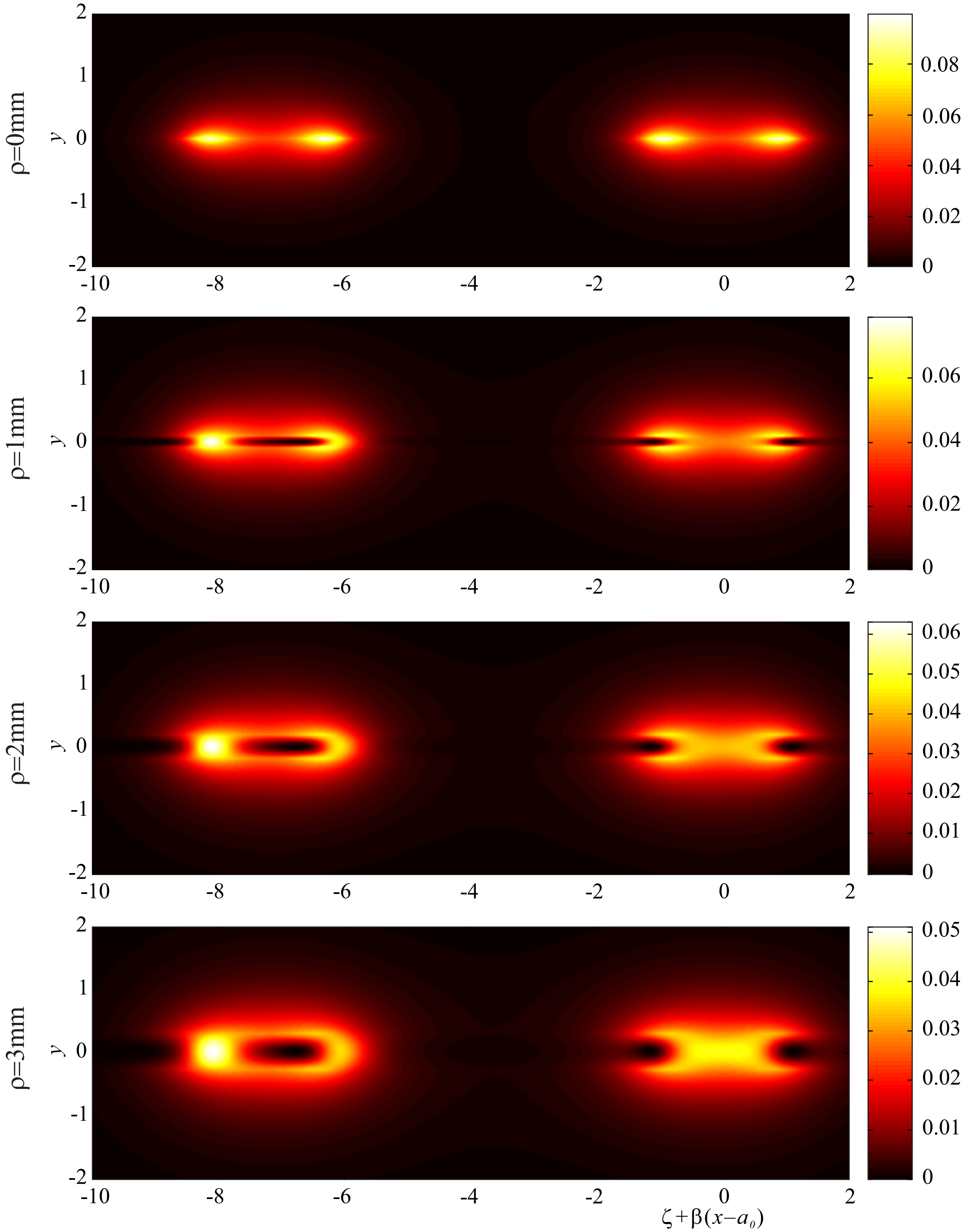

Figure 4: The energy flow density of the surface waves generated by round cylindrical bunches with different thicknesses moving along the edge of semi-infinite wire grid (Fig. 2) in the plane $x = const > a_0$, $\zeta = z - vt$. Wires are PEC, Gaussian units are used. Bunch length is $2\sigma = 2$ cm, thickness is shown near each image, the charge is $q = +1$ esu ≈ 0.33 nC; $a_0 = 4$ cm and $b_0 = 0.4$ cm, $\beta = 0.9$.

ISBN 978-3-95450-147-2