RF PHASE JITTER CONSIDERATION IN BUNCH COMPRESSION

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Abstract

Error propagation of RF phase jitter is analysed for various linac layout configurations and the sensitivity of the compression ratio due to RF phase jitter is analysed. Multiple sources of jitter have the opportunity to destructively interfere, and found to not add in quadrature. Results are compared to Elegant simulations.

INTRODUCTION

RF phase and amplitude jitter present a familiar challenge encountered at FEL facilities. Jitter minimization may be attempted, through development of finer tolerance of the output of klystrons and RF components. Here we present another approach to limiting the impact of RF jitter. This involves looking at the accelerator design, and searching for a linac design that is more resilient to jitter. To do this, we need to understand the effect that jitter has on bunch compression, energy spread, etc., and the extent of that effect due a given offset or jitter. Using this we can determine the tolerance, or allowable jitter our design can withstand. In this paper we outline the sensitivity due to phase jitter for two cases. The first is described by a single sine wave with a uniformly distributed jitter applied to the phase. The second is comprised of two sine waves, of the same frequency, with different phases, and different jitters applied to both phases.

MULTIPLE INDEPENDENT SOURCES OF PHASE JITTER

We will compare two cases. Case 1 describes the impact of RF phase jitter for an accelerating section supplied by one klystron. Case 2 has the same accelerating section but is supplied by two klystrons, and therefore have two independent sources of RF phase jitter. In case 2, the RF phases of the two sections supplied by the two klystrons can take on different phases, so long as it delivers the same effective output as case 1 (see Fig. 1).

Two sine waves bearing the same frequency but different phases, when added together can be exactly represented by a single sine wave of the form,

$$y = V_{eq}\sin(zk + \phi_{eq}) \quad (1)$$

where,

$$V_{eq} = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos(\phi_1 - \phi_2)} \quad (2)$$

and

$$\phi_{eq} = \tan^{-1}\left(\frac{V_1\sin(\phi_1) + V_2\sin(\phi_2)}{V_1\cos(\phi_1) + V_2\cos(\phi_2)}\right) \quad (3)$$

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where ϕ_1 and ϕ_2 are the phases of the two added sine waves, V_1 and V_2 are the respective amplitudes of the two sine waves, k is the RF wave number, and z is the longitudinal position.

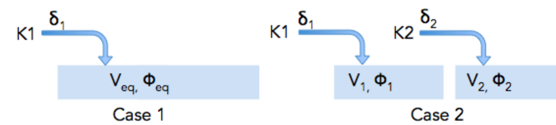


Figure 1: Schematic of two cases being compared for RF phase jitter sensitivity, where K_1, and K_2 are the klystrons providing independent sources of RF phase jitter.

Compression Ratio

The relative energy chirp can be written as,

$$\delta = h_1 z_i + h_2 z_i^2 + h_3 z_i^3. \quad (4)$$

where the first, second and third order energy chirp can be written as,

$$h_1 = \frac{k_s V_{eq}\cos\phi_{eq} + k_x V_h \cos\phi_h}{E_f}, \quad (5)$$

$$h_2 = \frac{-k_s^2 V_{eq}\sin\phi_{eq} - k_x^2 V_h\sin\phi_h}{2E_f}, \quad (6)$$

$$h_3 = \frac{-k_s^3 Veq_{eq}\cos\phi_{eq} - k_x^3 V_h\cos\phi_h}{6E_f}. \quad (7)$$

The final bunch position of an electron traversing the chicane is,

$$z_f = z_i + R_{56}\delta + T_{566}\delta^2 + U_{5666}\delta^3. \quad (8)$$

Substituting Eq. (4) into Eq. (8) and using the common approximation of truncating to first order in z_i, we define the compression ratio as,

$$C = \frac{\sigma_f}{\sigma_i} = \frac{1}{(1 + h_1 R_{56})} \quad (9)$$

where σ_i and σ_f are the electron bunch length before and after the bunch compressor.

The sensitivity of the compression ratio due to phase jitter will be proportional to $\frac{dC}{d\phi_{eq}}$ for case 1, and for case 2, $|\nabla C|$ with respect to ϕ_1 and ϕ_2:

$$|\nabla C| = \sqrt{\left(\frac{\partial C}{\partial\phi_1}\right)^2 + \left(\frac{\partial C}{\partial\phi_2}\right)^2}. \quad (10)$$

Comparison of Two Cases

For case 1, where the RF phase is $\phi_{eq} = 1.22$, the sensitivity due to jitter is $\nabla C_1 = 1.326$. For case 2, the same RF phases of $\phi_1 = \phi_2 = 1.22$ produce the same compression ratio but with the sensitivity due to jitter $\nabla C_2 = 0.938$. Therefore the ratio of the ∇C_2 to ∇C_1 is $\frac{1}{\sqrt{2}}$. That is, the improvement in the robustness against phase jitter for two independent sources of phase jitter is a factor of $\frac{1}{\sqrt{2}}$. This is always the case when $\phi_1 = \phi_2$ and $V_1 = V_2$. For n independent sources of jitter, the sensitivity in compression ratio due to to RF phase jitter is reduced by a factor $\frac{1}{\sqrt{n}}$, where the RF phases for each section are identical.

For different values of the RF phase in the different sections, the ratio of ∇C_2 to ∇C_1 can go below unity but never improves beyond $\frac{1}{\sqrt{2}}$. Figure 2 shows compression ratio and gradient functions of the compression ratios for cases 1 and 2, when $\phi_1 = \phi_2 = \phi_{eq}$ and then ϕ_2 is varied by $\Delta\phi_2$.

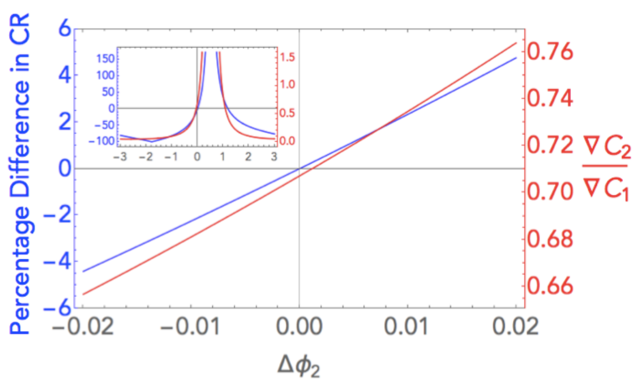


Figure 2: Variation in the CR (blue) and the sensitivity of CR due to phase jitter (red) as they vary by changing ϕ_2 by $\Delta\phi_2$. Here $\phi_1 = \phi_{eq}$, which equals ϕ_2 at $\Delta\phi_2 = 0$.

Elegant Simulations

Numerical simulations were performed using Elegant [1] to compare the two cases shown in Fig. 1. Figure 3 shows the variation in bunch centroid energy, showing the greater variation for case 1, with one source of RF phase jitter.

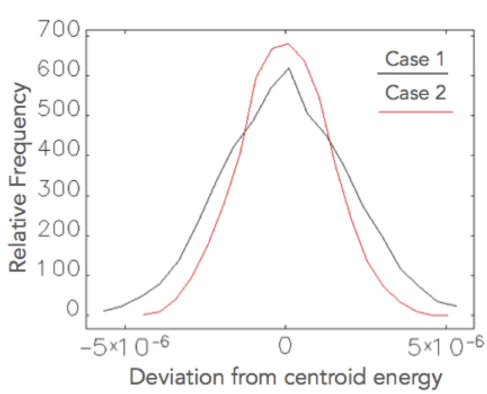


Figure 3: Elegant simulation comparing the two cases of one or two independent sources of RF phase jitter.

HARMONIC LINEARIZATION

Turning our focus to harmonic linearization we can determine if introducing an independent source of jitter can improve the sensitivity of the compression ratio due to RF phase jitter. Two new cases are defined (see Fig. 4).

The gradient function quantifies the sensitivity of the compression ratio due to jitter for case 1 and case 2 respectively,

$$|\nabla C_1| = \sqrt{\left(\frac{\partial C_1}{\partial \phi_{eq}}\right)^2 + \left(\frac{\partial C_1}{\partial \phi_h}\right)^2}, \tag{11}$$

$$|\nabla C_2| = \sqrt{\left(\frac{\partial C_2}{\partial \phi_1}\right)^2 + \left(\frac{\partial C_2}{\partial \phi_2}\right)^2 + \left(\frac{\partial C_2}{\partial \phi_h}\right)^2}. \tag{12}$$

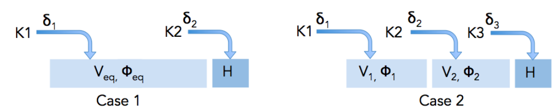


Figure 4: Schematic of two cases of harmonic linearization being compared for RF phase jitter sensitivity, where 'H' labels the harmonic structure, and K_1, and K_2, are the klystrons providing independent sources of RF phase jitter.

Using LCLS parameters of $V_{eq} = 153\ V$, $\phi_{eq} = -39.0°$, $V_h = 18\ V$ and $\phi_h = 0°$ [2], we can find a set of ϕ_1 and ϕ_2 that will produce the exact same compression (in terms of phase space output for the same R_{56}), with a greater robustness to phase jitter. This is shown graphically in Fig. 5, where regions of this 3D plot where the ratio of $|\nabla C_2|$ to $|\nabla C_1|$ is smallest, are the regions where the system is most robust to RF phase jitter. In Fig. 6 we have taken a slice of Fig. 5, plotting the ratio $|\nabla C_2|/|\nabla C_1|$ for $\phi_1 = \phi_{eq} = 0.890$. Also shown in Fig. 6 is the percentage difference in the compression ratio experienced when moving away from $\phi_2 = 0.890$. Increasing ϕ_2 by a small margin will decrease the compression ratio but will improve the jitter sensitivity of the compression.

For example, if ϕ_2 is increased to 0.9123, then the compression ratio is decreased by 5.0%, and the sensitivity due to RF phase jitter (i.e. the ratio of $|\nabla C_2|$ to $|\nabla C_1|$) is increased to 0.631. Increasing ϕ_2 by a greater amount to 0.9370, the compression ratio is decreased by 10.0% and the sensitivity due to RF phase jitter is increased to 0.559.

OVER VERSUS UNDER-COMPRESSION

Over-compressing the bunch can reduce the sensitivity due to phase jitter. This can be shown by starting with Eq. (9), and finding the two solutions of R_{56} that will give the same magnitude for the compression ratio. These expressions are then substituted into the expression for ∇C. Figure 7 shows the change in CR expected for a phase jitter of $\Delta\phi = 1°$ for various nominal values of CR.

ISBN 978-3-95450-147-2

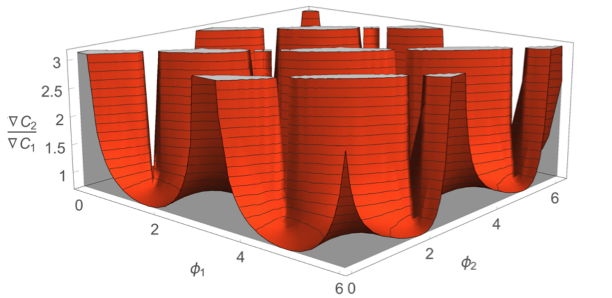


Figure 5: Ratio of the sensitivities of the compression ratio due to RF phase jitter for case 1 and 2.

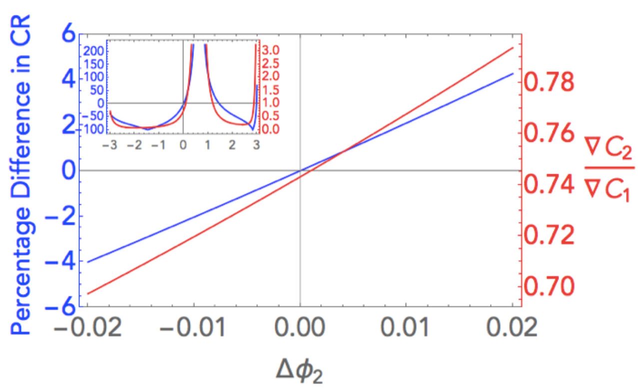


Figure 6: Ratio of the sensitivities of the compression ratio due to RF phase jitter for case 1 and 2, where $\phi_1 = \phi_{eq} = 0.890$.

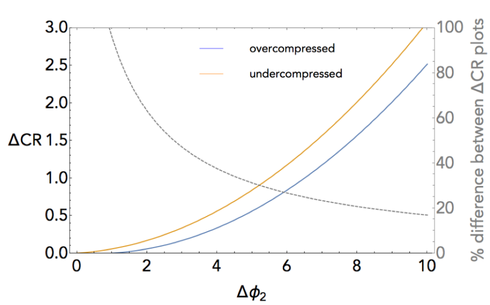
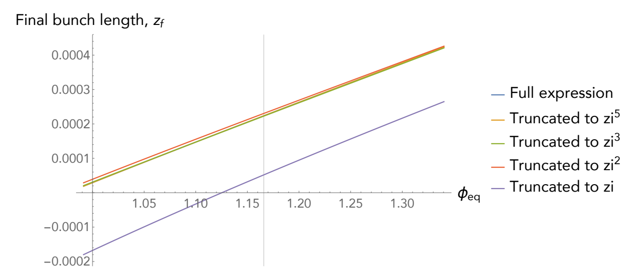


Figure 7: The change in CR expected for a phase jitter of $\Delta\phi = 1°$ for various nominal values of |CR|.

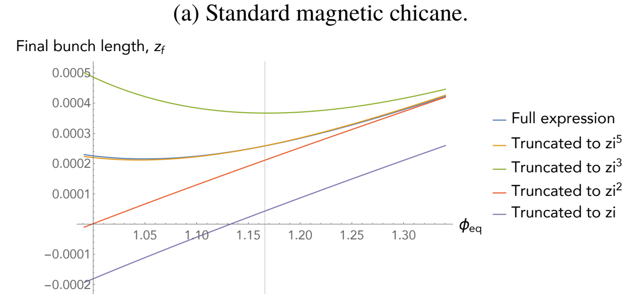
PHASE JITTER FOR DIFFERENT COMPRESSOR DESIGNS

An alternate compressor design, designed to avoid the current spikes typical when bunches are compressed to a large degree is briefly outlined in [3]. This compressor has values of longitudinal dispersion that vary from the relationships that govern a standard magnetic chicane of $T_{566} = -3/2R_{56}$, and $U_{5666} = -2R_{56}$. Instead the new design has R_{56}=−12.22 mm, T_{566} = 9.9 mm, and U_{5666} = 2.45 m. Figure 8a shows the variation in final bunch length with phase, ϕ_{eq} for a standard chicane. Figure 8b shows the variation in final bunch length for the alternative compressor design outlined in [3]. These plots shows firstly, that the expression Eq. (8) truncated to first order in z_i could be an adequate approximation for the final bunch length, to within 3.9%. However the gradient of the Fig. 8b plots (i.e. the sensitivity due to ϕ_{eq}) varies with each truncation.

Comparing the $\frac{dz_f}{d\phi_{eq}}$ calculation on the complete expression (no truncation in z_i) of Eq. (8) for the standard chicane and the alternate design reveal a 3.4 % difference between the two, with the smaller sensitivity to phase jitter favoring the alternate design. If the compression ratio is relaxed down to 2, then the difference between sensitivity to phase jitter jumps to 42.2 % in favor of the new design.



(a) Standard magnetic chicane.



(b) Non-standard chicane.

Figure 8: Final bunch length variation with ϕ_{eq}, showing ±10° either side of the nominal 66.8°.

CONCLUSION

This paper found the following conclusions:

1. Introducing additional sources of jitter can reduce the sensitivity of the compression ratio to RF phase jitter by allowing for the possibility of destructive interference of the RF voltage change through jitter. n additional sources of jitter will reduce the sensitivity of the compression ratio to RF phase jitter by a factor of $\frac{1}{\sqrt{n}}$.
2. Over-compressing in the bunch compressor can achieve a reduced sensitivity to phase jitter when compared to an under-compressed bunch with the same magnitude compression ratio.
3. A chicane with longitudinal dispersion values that vary from the standard arrangement (i.e. $T_{566} = -3/2R_{56}$) can improve the sensitivity of the final bunch length to phase jitter. The compressor design studied saw an improvement to the jitter sensitivity of 3.4 % and up to 42.2% for a smaller compression ratio of 2.

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ISBN 978-3-95450-147-2