TIME DOMAIN SIMULATIONS OF DETUNED ACCELERATING CAVITIES FOR TWO BEAM APPLICATIONS

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Abstract

A multi-harmonic accelerating cavity that has its fundamental and harmonic mode frequency detuned away from the bunch repetition frequency could provide the basis for a beam driven wakefield accelerator with high transformer ratios. The excitation of multiple harmonic eigenmodes will allow high gradients to be achieved without encouraging the onset of rf breakdown or pulsed surface heating. This accelerating cavity will be introduced, and time domain simulations verifying the theory will be shown.

INTRODUCTION

Cavities that have their fundamental accelerating mode frequency detuned away from the bunch repetition frequency could form the basis of a high gradient collinear two beam accelerator structure with high transformer ratios [1]. The field that is excited by the high current drive beam undergoes a phase shift that arises from the detuning, and the cavity can be designed such that the drive bunches are being decelerated by the steady state field. High transformer ratios can be achieved by interleaving the drive beam with a low current test beam, such that the test bunches are being accelerated as they traverse the cavity chain. A cavity of this design would remove the need for the Power Extraction Transfer System (PETS) found in the CLIC design [2].

A multi-harmonic cavity that operates at high gradients could act as an alternative cavity design for CLIC. Cavities of this type have unconventional surface electric and magnetic field profiles that can potentially lower the surface field emission and/or pulsed surface heating without compromising the accelerating gradient [3]. Two particular phenomena found in multi-harmonic cavities provide the main motivation for their use: (a) the anode-cathode effect, which can be found in an asymmetric multi-harmonic cavity that relies on fields pointing into one wall (cathode-like) to be significantly smaller than fields pointing away (anode-like) from the same wall. This effect will raise the work function barrier to supress field and secondary emission, and (b) a reduction in the surface heating by lowering the average $H_\parallel^2$ along the surface.

Both of these concepts can be combined to give a cavity that is capable of operating at high gradients with reduced damage from pulsed surface heating, while maintaining high transformer ratios and collinear acceleration. This paper will verify some of the fundamental principles introduced here by time domain simulations. First, the transformer ratio and the surface fields will be verified for a single mode cavity. Then a third harmonic cavity structure will be introduced and

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similar comparisons presented that show the pulsed surface heating reduction.

SINGLE MODE SIMULATIONS

A three cell π-mode standing wave cavity was designed with a fundamental frequency of 11.7292 GHz and the conductivity of the cavity walls was adjusted to give a quality factor of 500. The field in the cavity is excited by a traversing high current drive bunch, but the field undergoes a phase shift given by [4]

$$\tan \phi = -2Q\delta \tag{1}$$

where Q is the cavity quality factor, $\delta = (f_c - f_d)/f_c$ is the magnitude of the detuning, f_c is the cavity frequency and f_d is the drive bunch repetition frequency. For test bunch offsets of $t = \pi/2$, it emerges that $T = -2Q\delta$. Therefore, when exciting this cavity with a train of bunches with repetition frequency $f_d = 11.9942$ GHz, there is an anticipated transformer ratio $T = 22.59$. It can be shown that when considering the test bunch excited field, the transformer ratio is given by [1]

$$T = \frac{\varsigma - 2Q\delta}{1 + 2Q\delta\varsigma}, \tag{2}$$

where ς is the modified current ratio given by $\varsigma = \Theta_T I_T / \Theta_D I_D$ where $\Theta_{T,D}$ is the transit time factor and $I_{T,D}$ are the currents of the drive and test bunches respectively [5]. Here, both bunches traversing the cavity have $\beta \approx 1$. The cavity was excited by a train of 1 pC bunches with $\sigma_z = 4$ mm using ACE3P [6]. A bunch repetition frequency of 5.9971 GHz was employed (CLIC rf frequency, every other bucket filled) such that a Gaussian distribution of width $5\sigma_z$ did not result in any overlapping in the tails. Field monitors were placed along the cavity axis, and the bunch train continually excited fields in the cavity until steady state was reached. The steady state surface fields were then extracted.

To calculate the transformer ratio of a specific mode, only the fields from that mode should be considered. Therefore, a frequency filter was applied to the E_z field from each field monitor, such that the higher orer modes were excluded. The transformer ratio was calculated by determining the time offset of the drive bunches through the steady state field. The definition $T = \frac{\Delta W_T}{\Delta W_D}$ was used, where $\Delta W_{T,D}$ refers to the energy gained by a test bunch and the energy lost by the drive bunch respectively. The cavity Q and f_c were determined from the E_z profile from the probe at the center of the middle cell. The arrival time of each drive bunch at this probe was calculated and the field that the drive bunch

experienced was recored. Then, a numerical fit was made to

$$E(t) = A\cos(2\pi(f_c - f_d) + \phi_C)\exp\left(-\frac{2\pi f_c t}{2Q}\right) + C, \quad (3)$$

where A, C and ϕ_C are constants. A typical result of which can be found in Fig. 1. As the simulation time increases, the cavity mode decays and the bunches begin to be steadily decelerated.
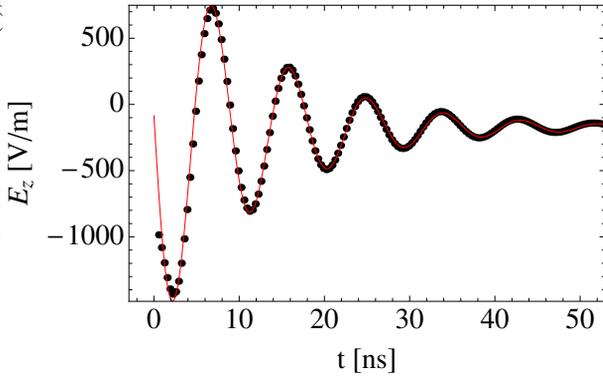
Figure 1: A fit of Eq. 3 to the drive bunch locations to determine cavity parameters. Red is the fit and the drive bunches are the black points.

Initially, the transformer ratio was calculated for the case where no test bunches were present (unloaded), then an additional test beam was interleaved, and the transformer ratio calculated and compared to the analytical. A snapshot of the steady state field profile can be found in Fig. 2, which also shows the drive bunch and test bunch locations.
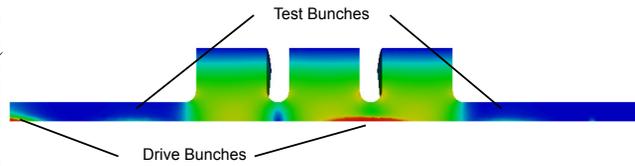
Figure 2: A snapshot of the steady state field profile. Test bunch and drive bunch locations are labelled.

Table 1: Summary of Time Domain Results for a Three Cell Pillbox Cavity with $T = 22.5$ and $Q = 500$

Parameter	Eigenmode	Time Domain
f_c [GHz]	11.7292	11.7197
Q_c	500.0	481.0
$T = -2Q_c\delta$	22.59	22.53
T	-	23.22
$T_{bl,single}$	-	-20.09
$T_{bl,multi}$	-	-17.65
$E_{s,max}/E_0$	4.63	4.38
$H_{s,max}/E_0$ [mA/V]	8.51	8.91

A summary of the simulation results can be found in table 1. The cavity Q and f_c were found to differ slightly in the

time domain from the eigenmode result. These numerical differences arise due to the intrisically different numerical schemes used in the eigensolver compared to that in the time domain method - despite the fact that identical meshing is used in both cases. The anticipated transformer ratio should relate the cavity parameters that have been simulated, not the values that are expected. Therefore, the expected transformer ratio is $T = 22.53$. The calculated transformer ratio was $T = 23.22$, which is a difference from the expected of 3%. The sensitivity of this calculation is fundamentally based upon the accuracy with which the drive bunch arrival time can be determined. The time step used in the time domain simulation was $dt = 1$ ps, a decrease in this value increases the accuracy of the calculation, but greatly increases simulation time. The sensitivity of the calculation to small errors increases for higher transformer ratios as it is dependant on $\tan \phi$. As $\phi \to \pi/2$, a small error can give rise to a much larger shift. The surface fields excited were within 5% of the eigenmode values. Differences arise due to the shifting cavity frequency, which reduces the field flatness and causes more intense fields in one cell, rather than equally strengthed fields in all cells.

MULTI-HARMONIC SIMULATIONS

A third harmonic detuned cavity was designed in order to show that pulsed surface heating reduction could be achieved while maintaining a high transformer ratio. For a beam driven cavity where the accelerating mode is a π-mode standing wave, the third harmonic is the most suitable harmonic mode. This is because the transformer ratio for a mode whose longitudinal distribution is odd is -1. Additionally, the mode seperation between the TM_{011} and the TM_{020} modes for a detuned cavity become very small (≈ 1 MHz), making accurate excitation of one mode over the other very difficult. When performing eigenmode simulations, each mode is normalised to 100 MV/m so that when a fraction of the third harmonic is included in the simulation, the total accelerating gradient remains unchanged.

The transformer ratio for a multi-harmonic cavity is given by

$$T = -\frac{\sin 2\phi_1 - \chi \sin 2\phi_3}{2(\cos^2 \phi_1 + \chi \cos^2 \phi_3)} \quad (4)$$

where the contribution from the third harmonic is given by

$$\chi = \frac{I_3 R_3}{I_1 R_1}, \quad (5)$$

$R_{1,3}$ are the shunt impedances of the first and third harmonic, and $I_{1,3}$ are the current components that are determined from the gaussian width and $\phi_{1,3}$ are the detuning angles of each mode. In order to ensure that both modes are strongly excited by the drive beam

$$\sigma_z \ll \frac{\lambda_1}{\pi\sqrt{2h^2 - 2}}. \quad (6)$$

If $\phi_1 = -\phi_3$, then $T = -2Q\delta$. The cavity geometry can be found in Fig. 3.
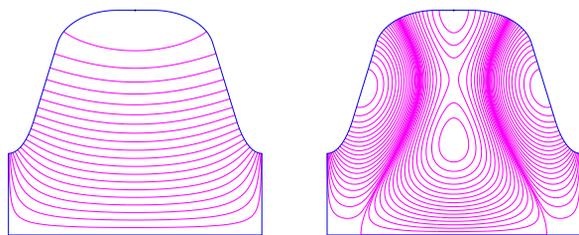
Figure 3: Field contours for the TM_{010} mode (left) and TM_{012} mode (right). Geometry is based on the TESLA cavity, but with a re-optimisation in order to tune it for X-band and to tune the third harmonic.

Following a similar procedure to that found in the first section, the cavity was excited by a drive bunch train with reptition frequency $f_d = 11.9942/2$ GHz (CLIC rf frequency, every other bucket filled). Two simulations were performed, the first was with a bunch train with $\sigma_z = 4$ mm. This is so only the fundamental mode gets strongly excited. Then, using the same geometry and mesh, the cavity field was excited with a bunch train that has $\sigma_z = 1$ mm. Here, the anticipated transformer ratio is on the order of 10 to reduce the sensitivity of the simulations to small errors.

Table 2: Summary of Time Domain Results for a Third Harmonic Cavity

Parameter	TM_{010}	TM_{012}	$TM_{010}+TM_{012}$
σ_z [mm]	4	1	1
f_c [GHz]	11.888	36.2693	-
Q_c	500.0	672.0	-
$T_{analytical}$	8.94	-10.63	9.01
$T_{simulation}$	8.85	-10.27	9.04

A summary of the transformer ratio calculations can be found in Table 2. It can be seen that there is very good agreement between the analytic transformer ratio and the values calculated from the simulation (within 0.5%).

Figure 4 shows the average $H_\parallel^2$ along the surface of the third harmonic cavity. The upper plot shows the result from a single cell simulation in Superfish for just the single TM_{010} mode and for $TM_{010}+TM_{012}$, while the lower plot shows the same parameter but calculated in the time domain using the data from the middle cell from the two different simulations. The maximum anticipated surface field reduction for the eigenmode case was 19.3%. It was found in the time domain that there was a maximum reduction of 13.3%, corresponding to a difference between eigenmode and time domain of 6%. This difference arises from several places. Firstly, a frequency filter can not be applied to the surface data (only extracting surface data from steady state). This means that the surface field does not exclusively contain only the modes of interest, but may also contain a small contribution from other higher order modes. The second is that, as with the single mode case, the frequencies of each of the modes are not the same as with the eigenmode. The third harmonic
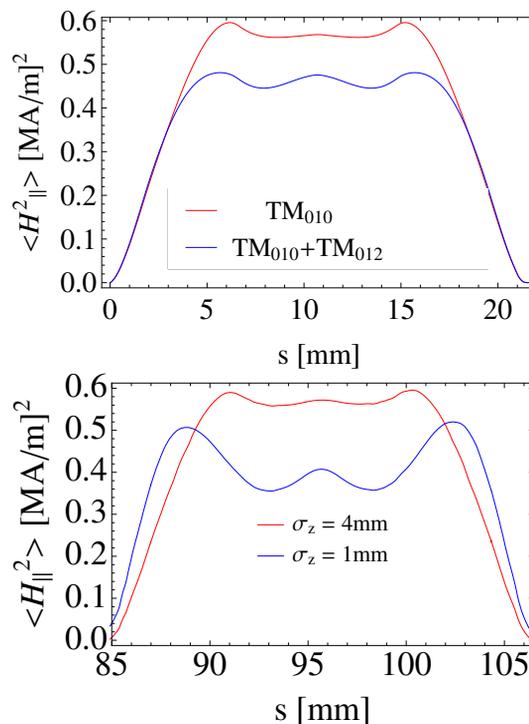
Figure 4: Average of the magnetic field squared as a function of path length along the surface. The top plot shows the eigenmode result, whereas the bottom plot shows the time domain result for just the middle cell of multi-harmonic cavity chain.

mode is much more sensitive to these shifts due to the higher frequency, therefore what was initially a relatively flat field can be shifted extensively, changing the field strength in each cell.

These results demonstrate there is a clear potential to reduce the pulsed temperature rise in multi-harmonic cavities.

CONCLUSION

The transformer ratio for a detuned cavity has been investigated in both the single and multi-harmonic cases. Large transformer ratios are anticipated for both cavity structures we evaluated. T values of more than 8 for the single mode cavity and more than 10 for the multi-harmonic case were obtained. A surface field reduction in the latter case is predicted of more than 13%.

ACKNOWLEDGMENT

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3: Alternative Particle Sources and Acceleration Techniques

A15 - New Acceleration Techniques

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