

# COMPENSATING TUNE SPREAD INDUCED BY SPACE CHARGE IN BUNCHED BEAMS

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## Abstract

The effects of space charge play a significant role in modern-day accelerators, frequently constraining the beam parameters attainable in an accelerator or in an accelerator chain. They also can limit the luminosity of hadron colliders operating either at low energies or with sub-TeV high-brightness hadron beams. The latter is applied for strongly cooled proton and ion beams in eRHIC – the proposed future electron-ion collider at Brookhaven National Laboratory. Using an appropriate electron beam would compensate both the tune shift and the tune spread in the hadron beam in a coasting beam. But these methods cannot compensate space charge tune spread in a bunched hadron beam. In this paper we propose and evaluate a novel idea of using a co-propagating electron bunch with miss-matched longitudinal velocity to compensate the space charge induced tune-shift and tune spread.

## INTRODUCTION

The paper is motivated by developing a high-energy high-luminosity electron-ion collider at Brookhaven National Laboratory called eRHIC [1] with a very short (5 cm RMS) and strongly cooled proton and ion beams with normalized transverse emittances of 0.2 mm·mrad.

Space-charge effects have been known in accelerator physics for a half of the century. There is an extensive literature [2-23] describing the effects of space charge. A nonlinear space-charge force induces an irreducible transverse tune-spread, i.e. the tune dependence on both the hadron's longitudinal position inside the bunch,  $z$ , and the amplitude of the transverse oscillations.

It is well known that space-charge effects fall as a high power of the beam's relativistic factor:

$$\Delta Q_{sc} \approx -\frac{Z^2 r_p}{A} \frac{N_o}{4\pi\beta_h^2 \gamma^3 \varepsilon} \frac{C}{\sqrt{2\pi\sigma_z}} \quad (1)$$

where  $C$  is the ring's circumference,  $Z$  is the charge, and  $A$  is the atomic number of the hadron (e.g., an ion, for proton  $Z = A = 1$ ),  $r_p = e^2 / m_p c^2$  is the classical radius of the proton,  $\gamma^2 = 1 / (1 - \beta^2)$  is the relativistic factor of hadron beam,  $N_o$  is number of hadrons in the bunch with an RMS bunch length of  $\sigma_z$ , and  $\varepsilon$  is the beam's transverse emittance.

Naturally, the maximum tune shift is experienced by the particles in the center of the beam, while the particles with large amplitude of oscillations experience a smaller

value of the tune shift. The overall tune-spread is determined by its value for the center particles.

We are presenting here a strongly compressed version of our studies. Detailed description is published in full-size (24-page long) article [24].

For a round beam with equal emittances, the transverse tune shifts depend on particle's location inside the bunch as follows:

$$\delta Q_{x,y} = \delta Q_{sc}(z) \cdot f_{x,y};$$

$$\delta Q_{sc}(z) = -\frac{C}{4\pi\varepsilon} \frac{1}{\beta^2 \gamma^3} \frac{Z^2 r_p}{A} \cdot \frac{N_o}{\sqrt{2\pi\sigma_z}} \cdot e^{-\frac{z^2}{2\sigma_z^2}}; \quad (2)$$

$$f_x = \left\langle \frac{2}{1 + \sqrt{\beta_y / \beta_x}} \right\rangle; f_y = \left\langle \frac{2}{1 + \sqrt{\beta_x / \beta_y}} \right\rangle.$$

Since the longitudinal motion of hadrons usually is very slow (e.g.  $Q_s \ll Q_{x,y}$ ), the tune of the particle depends not only on the amplitudes (actions) of the transverse oscillations, but also on their longitudinal the location within the bunch.

One practically important feature of the space-charge effects is a very strong dependence on the relativistic factor,  $\gamma$ :  $\delta Q_{sc} \propto \gamma^{-3} / (1 - \gamma^{-2})$ . While the power one of  $\gamma$  naturally comes from the increasing rigidity of the beam, the  $\gamma^{-2}$  comes from the effective cancelling of the forces from the electric and magnetic fields induced by the beam

$$\vec{F}_\perp = eZ \left( \vec{E}_\perp + \beta_o \left[ \hat{z} \times \vec{B}_\perp \right] \right) \equiv \frac{eZ \cdot \vec{E}_\perp}{\gamma^2}. \quad (3)$$

Several practical schemes were suggested for space-charge tune shift and tune spread compensation by colliding an electron beam with hadron beam (e.g., an electron lens), or employing an electron column induced in a residual gas [25-29]. The tune shift given by the colliding beam does not suffer from  $\gamma_h^{-2}$  cancelation: for round electron beam having RMS size of  $\sigma_e$  and the interaction length of  $L$ , the tune shift is given by the following:

$$\delta Q_{x,y,el} = \frac{Z}{A\beta_h\gamma_h} \frac{r_p}{4\pi\sigma_e^2} \cdot \frac{I_e}{ec\beta_e} (1 + \beta_e^2) \cdot L \langle \beta_{x,y} \rangle; \quad (4)$$

where  $\beta_e = v_e / c$  is the normalized velocity and of the electron beam.

Comparing eqs. (2) and (4) one can conclude that electron beam's current

$$I_e \approx \frac{C}{2\gamma_h^2 L} \cdot \frac{\beta_e}{\beta_h} \cdot I_p; \quad I_p = \frac{ecZN_o\beta_h}{\sqrt{2\pi\sigma_z}}; \quad (5)$$

can be used to compensate for the space-charge induced tune shift. Invariably, the interaction length is much smaller than the ring's circumference e.g.,  $\eta = L/C \ll 1$ . We can compensate for this shortcoming by having large relativistic factor,  $2\gamma_h^2\beta_h/\beta_e \gg 1$ . This means that the electron current in such electron lens can be modest, and frequently, it can be comparable to the hadron beam's current.

As explained in [25,29], by selecting a proper transverse distribution of electron beam, we can match the dependence of the space-charge tune shift on the transverse amplitudes. However, in a bunched beam, the space-charge tune shift depends on the hadron's position within the hadron bunch,  $z$ . Thus, for a bunched beam, at best these schemes could reduce the space-charge tune spread by a half.

Using a co-propagating electron beam with the same relativistic velocity  $\beta_e = \beta_h$  (as in electron cooling schemes) and the same longitudinal distribution offers an opportunity of compensating for both the transverse and longitudinal dependences of the space-charge field. Unfortunately, the compensating beam suffers from  $\gamma_h^{-2}$  cancelation, and such a scheme would require a very large electron beam current:

$$I_e \approx \frac{C}{L} \cdot I_p \gg I_p. \quad (6)$$

This unfavourable scaling makes such a scheme impractical, especially for hadron beams in large colliders. For example, eRHIC would be operating hadron beams with peak current  $\sim 10$  A (and duration of 0.4 nsec). Using a 30 m of the 3.8 km RHIC circumference for such a space-charge compensator would require having an electron bunch with a peak current  $\sim 1.2$  kA, and the bunch charge  $\sim 4,000$  nC. Such an e-beam simply does not exist.

We propose to use the co-propagating scheme, but with mismatched relativistic factors (e.g. velocities) of the two beams. Such mode offers the possibility of diminishing the reduction factor while keeping under control the slippage between the beams.

## THE IDEA UNDERLYING THE METHOD

The idea for the proposed method is based on a simple observation that the relativistic canceling is proportional to  $\gamma^2$ , while the velocity of the particles weakly depends on  $\gamma$  for  $\gamma > 2$ . To be exact, we consider a co-propagating relativistic e-beam having a nearly identical bunch profile as the hadrons, but having a different relativistic factor (see Fig. 1). Hence, the slippage of the e-beam with the respect to the hadron beam is small compared with the length of the interaction section,  $L$ :

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#### A16 - Advanced Concepts

$$\Delta z = \frac{L}{\beta_h} (\beta_e - \beta_h) \approx \frac{L}{2} \left( \frac{1}{\gamma_h^2} - \frac{1}{\gamma_e^2} \right) \quad (7)$$

Since using low-energy electron beams is economically favorable, let us assume that relativistic factor of hadron is significantly larger than that of the e-beam:  $\gamma_h^2 \gg \gamma_e^2$ .

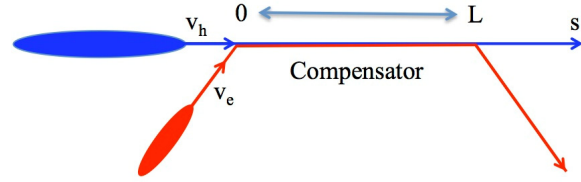


Figure 1: A layout of the interaction region of the space-charge compensator. Electron beam co-propagates through straight section with length  $L$ , and is extracted. Their velocities  $v_e \equiv c\beta_e$  and  $v_h \equiv c\beta_h$  can differ. It is easy to show:

$$F_{hr} \approx -Ze \frac{2I_h(z)}{\gamma_h^2 cr} \int_0^r f_h(x,s) x dx; \quad (8)$$

$$F_{er} \approx -Ze \frac{I_e(z)}{\gamma_e^2 cr} \int_0^r f_e(x,s) x dx; \quad \Delta z \approx -\frac{L}{2\gamma_e^2}.$$

Then the electron beam current required for compensating the space charge tune shift in proton beam (using  $N_c \geq 1$  compensators) can be estimated as:

$$I_e \approx -2I_h \cdot \frac{\gamma_e^2}{\gamma_h^2} \cdot \frac{C}{N_c L}; \quad (9)$$

e.g. it scales as  $\gamma_e^2$ . As we can see in next section, that slippage  $\Delta z$  should be comparable to the hadron bunch length which limits interaction length to about:

$$L \sim 2\gamma_e^2 \sigma_z, \quad (10)$$

and turns (9) into

$$\frac{I_e}{I_h} \approx -\frac{C}{N_c \sigma_z \gamma_h^2}. \quad (11)$$

For example,  $\gamma_h = 100$  and the RHIC circumference  $C=3.8$  km the required ratio is the following:

$$\frac{I_e}{I_h} \sim \frac{1}{N_c} \cdot \frac{1}{\sigma_z (nsec)} \quad (12)$$

This means that electron beam's peak current can be of the same order as that of the hadrons.

## THE METHOD

We now consider the method for finite velocities without any limitations, and an electron beam having longitudinal profile determined by its current

$$I_e \left( t - \frac{s}{v_e} \right). \quad (13)$$

The integrated effect is denoted by the following expression:

$$L \cdot \bar{I}_e(t) = c \frac{\beta_e \beta_h}{\beta_h - \beta_e} \int_0^{\Delta t} I_e(t + \zeta - \Delta t) d\zeta \quad (14)$$

with the slippage given by

$$c\Delta t = L(\beta_h - \beta_e) / \beta_e \beta_h. \quad (15)$$

To assess the value of allowable slippage by the deconvolving equation (19), assuming that the shape of  $\bar{I}_e(t)$  repeats that of the hadron beam  $I_h(t)$ , e.g.,

$$\frac{\bar{I}_e(t)}{\bar{I}_e(0)} = \frac{I_h(t)}{I_h(0)} = q(t) \quad (16)$$

or in other words:

$$\int_0^{\Delta t} g(t + \zeta) d\zeta = q(t); \quad I_e(t - \Delta t) = I_o g(t); \quad (17)$$

with value of  $I_o$  be chosen to compensate the tune shift for the hadron in the center of the bunch.

Our paper [24] discusses deconvolution of eq. (17) in full details. Here we briefly summarize the results: there are simple solutions:

$$g_+(t) = -\sum_{m=0}^{\infty} q'(t + m\Delta t); \quad g_-(t) = +\sum_{m=1}^{\infty} q'(z - m\Delta t). \quad (18)$$

It is evident that a linear combination of the solutions (18)

$$g_\alpha(t) = \alpha g_+(t) + (1 - \alpha) g_-(t)$$

is a solution and it is likely that  $g_{1/2}$  can be of practical interest. For a rather general physics assumptions, these functions converge to zero at one of the infinities:  $g_+(z)_{z \rightarrow +\infty} \rightarrow 0$ ;  $g_-(z)_{z \rightarrow -\infty} \rightarrow 0$ . This is not necessarily true for the other sign. While mathematical properties of the solutions mostly are of academic interest, there is an additional, very practical issue. By definition  $q(t)$  is a non-negative function. Similarly, the sign of the e-beam current is always negative, and  $I_e(t) \leq 0$ . Thus, any practical deconvolution cannot change the sign, and choosing  $I_o < 0$  requires  $g(t)$  being a positive function. The natural parameter determining the behavior and “positivity” of  $g_\pm(t)$  is determined by the ratio between the slippage,  $\Delta t$ , and the RMS length of the hadron bunch,  $\sigma_t$ :  $\tau = \Delta t / \sigma_t$ .

The following shortly summarizes our findings. First, for  $\tau \leq 1$ , both  $g_\pm(t)$  solutions converge very well within the typical physical aperture of  $\pm 5\sigma_z$ . For  $\tau \leq 1$ , the deconvolutions  $g_\pm(t)$  are nearly identical (see Fig. 3) and are positively defined within the interval  $t/\sigma_t \in \{-5, 5\}$ . For  $\tau = 1$ , the difference between  $g_\pm(t)$  is less than  $\pm 10^{-7} g_\pm(0)$ . This simply means that, for practical purposes with  $\tau \leq 1$ , the compensating error will be not be defined by the deconvolution function, but by other practical means. Second, for values

of  $\tau$  exceeding unity, the situation changes rather rapidly (see Fig. 2). The practical conclusion from these studies is that  $\tau = 1.5$  is a natural boundary, where a  $g_{1/2}(t)$  deconvolution works very well.

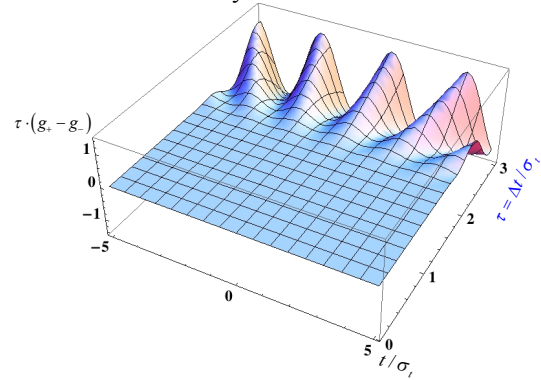


Figure 2: 3D-plot of  $\tau \cdot (g_+(t) - g_-(t))$  (vertical axis) for Gaussian convolution function with  $t = z/\sigma_z \in (-5, 5)$  being a horizontal axis, and the third axis is  $\tau = \Delta z / \sigma_z \in \{0, 3\}$ .

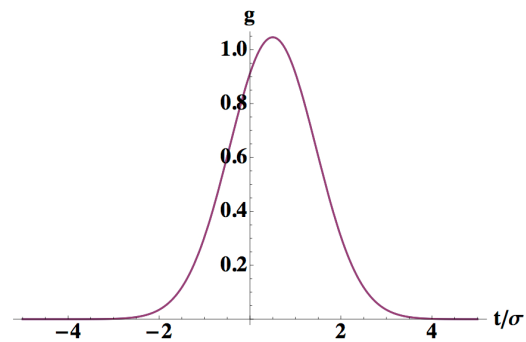


Figure 3: Graphs of  $g_\pm(t)$  for deconvolution for Gaussian distribution (27) with  $\Delta z = \sigma_z$ . We note that function are practically indistinguishable

## CONCLUSION

We proposed a novel method of compensating space-charge-induced tune spread in bunched hadron beams. We showed that it is possible to compensate for both the tune shift and the tune spread with significant accuracy.

We found that space-charge tune spread for eRHIC’s 250 GeV proton beam can be removed by three 3m long compensators with 1.35 MeV e-beam and peak current of 10.6 A [24].

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