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CONSEQUENCES OF BOUNDS ON LONGITUDINAL EMITTANCE GROWTH FOR THE DESIGN OF RECIRCULATING LINEAR ACCELERATORS

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Abstract

Recirculating linear accelerators (RLAs) are a cost-effective method for the acceleration of muons for a muon collider in energy ranges from a couple GeV to a few 10s of GeV. Muon beams generally have longitudinal emittances that are large for the RF frequency that is used, and it is important to limit the growth of that longitudinal emittance. This has particular consequences for the arc design of the RLAs. I estimate the longitudinal emittance growth in an RLA arising from the RF nonlinearity. Given an emittance growth limitation and other design parameters, one can then compute the maximum momentum compaction in the arcs. I describe how to obtain an approximate arc design satisfying these requirements based on the deisgn in [1]. Longitudinal dynamics also determine the energy spread in the beam, and this has consequences on the transverse phase advance in the linac. This in turn has consequences for the arc design due to the need to match beta functions. I combine these considerations to discuss design parameters for the acceleration of muons for a collider in an RLA from 5 to 63 GeV.

INTRODUCTION

For the muon accelerator scenario described in [2], they envision an RLA accelerating from 5 to 63 GeV, which would accelerate beams for two stages of a Higgs factory, and would be reused for a higher energy collider with a substantially larger emittance. Important machine parameters are given in Tables 1 and 2. Details of the rationale for many of these parameters can be found in [3].

An RLA allows rapid acceleration while making multiple passes through a linac by having a beam splitter at the ends of the linac and arcs for each energy that return the beam to the linac. Having large numbers of arcs at the splitter can become a problem, particularly for large beam emittances and small energy spacings. A "dogbone" RLA geometry helps address this [4]. Bogacz [1] has described a technique to design the "droplet" arcs for this lattice; I will adopt that basic technique here.

I first discuss a generic technique to obtain a droplet arc design according to Bogacz' concept, and obtain from that an approximation to the time of flight dependence on energy in the arc. I then compute an approximation to the longitudinal emittance growth which depends on that time of flight parameter. I finally put these together to determine parameters for the RLA that will meet the requirements.

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Table 1: Beam and Machine Parameters [2]. The left two columns are for a Higgs factory, the rightmost column for a higher energy collider.

Particles per bunch ($\times 10^{12}$)	2	4	2
Longitudinal emittance (mm)	1.0	1.5	70
Transverse emittance (μm)	400	200	25
Initial total energy (GeV)	5	5	5
Final total energy (GeV)	63	63	63
Maximum emittance growth (%)		10	
Maximum decay loss (%)		11.0	
Physical aperture (σ)		4.5	

Table 2: Assumed Cavity Parameters

Frequency (MHz)	325	650
Gradient (MV/m)	20	25
Maximum cells per cavity	4	5
Maximum cavity passes	9	3
Additional length at each end (cells)	1.5	

DROPLET ARC

The structure of a droplet arc is described in Table 3 and shown in Fig. 1. From the linac to the middle, the sections consist of

- Dispersion matching cells from the linac to the arc
- Outward bending cells

Table 3: Droplet Arc Structure

Cells	Bend Angle	Cell Length
2	$-\theta_m/2$	L_m
n_o	$-\theta$	L
2	0	L
n_i	θ	L
2	0	L
n_o	$-\theta$	L
2	$-\theta_m/2$	L_m

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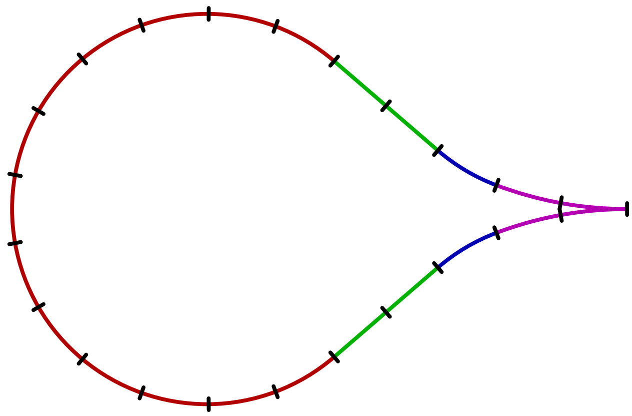

Figure 1: Droplet arc for $n_o = 1$ and $n_i = 13$. The linac is connected at the right.

- Dispersion flip cells, swapping the dispersion from the inside to the outside of the droplet
- Inward bending cells

All cells have a 90° betatron phase advance.

In a small angle approximation, dispersion matching requires that

$$L\theta = L_m\theta_m. \tag{1}$$

Geometric constraints lead to the equations

$$(n_i - 2n_o)\theta - 2\theta_m = \pi \tag{2}$$

$$\begin{aligned} 2L_m\theta(1-\cos\theta_m) + L\theta_m[\cos\theta_m - \cos(n_o\theta+\theta_m)] \\ + 2L\theta\theta_m\sin(n_o\theta+\theta_m) = L\theta_m\sin(n_i\theta/2) \end{aligned} \tag{3}$$

Starting with $\theta \approx \theta_m$, one finds that $n_i = 5n_o + 8$. Define $r = \theta_m/\theta = L/L_m$; then

$$\theta = \pi/(3n_o + 8 - 2r) \tag{4}$$

and $r \approx 1$ solves the equation

$$\begin{aligned} (2-r^2)\cos\frac{\pi r}{3n_o+8-2r} + 2r^2\cos\frac{\pi(n_o+r)}{3n_o+8-2r} \\ - \frac{2r^2\pi}{3n_o+8-2r}\sin\frac{\pi(n_o+r)}{3n_o+8-2r} - 2 = 0 \end{aligned} \tag{5}$$

The time of flight of a particle through the arc is $T_0 + T_1\Delta$, where Δ is the energy deviation from a design energy. Assuming thin lens quadrupoles, all spaces between occupied by dipoles, small bend angles, highly relativistic particles, $r = 1$, and ignoring dipole focusing, I find

$$T_1 \approx \frac{7\pi^2(7n_o+9)L}{144(n_o+2)^2pc^2} \approx \frac{7\pi^3(7n_o+9)}{432(n_o+2)^3qBc^2} \tag{6}$$

where B is the average dipole field and p is the arc design momentum.

LONGITUDINAL EMITTANCE GROWTH

I will now approximate the longitudinal emittance growth by assuming that the incoming longitudinal distribution is elliptical and that due to the nonlinearity in the RF, it filaments into a shape with a larger effective longitudinal emittance. The particles move in phase space along closed curves (assuming integrability) that are not ellipses due to nonlinearity. If the beam were matched to the linear motion, the area of the curves that the particles move on is in some cases larger than the curves they would move on if there were no nonlinearity. The result is an effective emittance growth.

Begin with the assumption that all arcs have the same T_1, and treat the RF contribution as a single nonlinear kick. Then in the notation of [5], the map can be written as

Table 4: Parameters to Limit Longitudinal Emittance Growth

ϵ (mm)	1.5	1.5	70	70
$\omega/2\pi$ (MHz)	325	650	325	650
T_1 (ps/GeV)	1567	299	435	83
ϕ (deg)	7	6	25	22
σ_E (MeV)	22	50	283	647

$$e^{-:H_{20}:}e^{-:H_{n0}:} \tag{7}$$

$$e^{-:H_{20}:} = e^{:T_1\Delta^2/4:}e^{:(V\omega\sin\phi)\tau^2/2:}e^{:T_1\Delta^2/4:} \tag{8}$$

$$e^{-:H_{n0}:} = e^{-:T_1\Delta^2/4:}e^{-:H_{RF,n}:}e^{:T_1\Delta^2/4:} \tag{9}$$

$$\begin{aligned} H_{RF,n} = \frac{V}{\omega}\sin(\omega\tau+\phi) \\ - \frac{V}{\omega}\sin\phi - V\tau\cos\phi + \frac{1}{2}(V\omega\sin\phi)\tau^2 \end{aligned} \tag{10}$$

where V is the energy gain per linac pass, ϕ is the RF phase, ω is the angular RF frequency, and τ is the time of flight deviation.

I first apply a linear transformation to τ and Δ to convert the map into the form

$$e^{-\mu:q^2+p^2:/2}e^{-:H_{n1}(q,p):} \tag{11}$$

where μ is the longitudinal phase advance per linac-arc pair. This map is then normalized with a third-order Lie operator $e^{:A_3:}$; in principle a fourth order operator is also required for the normalization, but it will not lead to any emittance growth. The emittance growth is found by computing the average value of the action in the nonlinearly normalized variables of values that lie uniformly distributed on the circle $q^2 + p^2 = 2J$ in the linearly normalized variables, and averaging over a distribution in J. For this case,

$$\Delta\epsilon = -\frac{5}{48}\frac{U^2\omega^4T_1^3}{\mu^2\sin^3\mu}\langle J^2\rangle \tag{12}$$

with $U = V\cos\phi$ being the energy gain per linac pass and

$$2\sin(\mu/2) = -\sqrt{T_1V\omega\sin\phi} \tag{13}$$

For a Gaussian distribution, $\langle J^2\rangle = 2\epsilon^2$ where ϵ is the emittance.

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Given μ, one can solve for T_1:

$$T_1 = \left[-\frac{48}{5}\frac{(\Delta\epsilon/\epsilon)\mu^2\sin^3\mu}{U^2\omega^4\epsilon(\langle J^2\rangle/\epsilon^2)}\right]^{1/3} \tag{14}$$

The lattice parameters will be more favorable with larger T_1, and this expression is maximized when $\mu \approx -1.91$, and I will henceforth use this value for μ. This synchrotron phase advance is large enough that there could be issues with the breakup of the RF bucket. One can then compute the RF phase and energy spread

$$\phi = \tan^{-1}\frac{4\sin^2(\mu/2)}{T_1 U\omega} \qquad \sigma_E = \sqrt{-\frac{\epsilon}{T_1}\tan\frac{\mu}{2}} \tag{15}$$

For a Gaussian distribution and the parameters in Tables 1 and 2, I find the resulting parameters in Table 4.

RLA DESIGN

To make the match from the linac to the arc perform optimally, the beta functions at the end of the linac should be similar to those at the end of the arc. For a thin lens FODO cell with a full cell length L, the geometric mean of the beta functions at the quadrupoles is

$$\sqrt{\beta_D\beta_F} = (L/2)\csc(\mu_\perp/2) \tag{16}$$

where $\mu_\perp$ is the betatron phase advance per cell. Assume that betatron oscillations become unstable for particles with energy $E_0 - k\sigma_E$, where E_0 is the linac injection energy and k is the "physical aperture" factor from Table 1. Then the phase advance $\mu_\perp$ at the end of the linac, if all linac cells and quadrupoles are identical, is given by

$$\sin\frac{\mu_\perp}{2} = \frac{E_0 - k\sigma_E}{E_0 + U}. \tag{17}$$

and the ratio of the first arc cell length to the linac cell length to match the linac to to the arc is

$$\frac{1}{\sqrt{2}}\frac{E_0 + U}{E_0 - k\sigma_E}. \tag{18}$$

Assuming linac quadrupoles with 4 T at 4.5σ and 25 cm of space on either side of the quadrupoles and one linac cavity between the quadrupoles, optimal design parameters for cost and performance are given in Table 5. Designs can be constructed that have reasonable performance for the 1.5 mm longitudinal emittance, but the results are unacceptable for the 70 mm longitudinal emittance. For 70 mm longitudinal emittance, I instead accept a (significant!) betatron mismatch between the linacs and the arcs, and instead fix the maximum fields in the arc magnets. The resulting designs are shown in Table 6.

The most significant concerns are for the large longitudinal emittance. Viable designs exist, but require a challenging match at the beginning of the arc. Energy spreads in the linac are a concern for 650 MHz, while switchyard crowding is challenging for the 325 MHz solution. Using two stages or a racetrack geometry will relieve some of the issues, though both will worsen the switchyard crowding.

Table 5: Droplet arc parameters when the linac and arc have the same geometric average of their beta functions at the points where they meet.

ϵ (mm)	1.5	1.5	70	70
$\omega/2\pi$ (MHz)	325	650	325	650
Linac passes	9	3	9	3
$\mu_\perp$ end (deg.)	51	23	38	10
Cells per cavity	2	5	2	2
Linac cell length (m)	6.46	5.53	6.46	4.15
n_o	0	6	7	46
Decay loss (%)	8.8	5.3	17.4	20.7
L, first arc (m)	11.2	20.1	15.5	34.4
Total arc length (km)	4.3	3.3	24.0	32.5

Table 6: Droplet arc parameters for a beam with a 70 mm longitudinal emittance with no beta function matching requirement between the arc and linac. Quadrupole fields at 4.5σ are 2/3 of the dipole fields.

$\omega/2\pi$ (MHz)	325	325	650	650
Linac passes	9	9	3	3
Arc dipole field (T)	1.5	6.0	1.5	6.0
n_o	11	5	28	15
L, first arc (m)	5.43	3.57	5.34	3.44
Total arc length (km)	12.0	4.3	3.2	1.2
Decay loss (%)	12.3	9.1	6.2	5.3

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