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MIXING AND SPACE-CHARGE EFFECTS IN FREE-ELECTRON LASERS*

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Abstract

This work aims to understand the single pass FEL dynamics with an initially cold beam, through a semi-analytical model, based in a group's previous works in beams [1, 2]. The central point of the model is the compressibility factor, which allows establishing the transition from Compton to Raman regimes. The model is useful also to perform analytical estimates of the elapsed time until the onset of mixing and the saturated amplitude of the radiation field. Semi-analytical and full simulations results are compared, showing a good agreement.

INTRODUCTION

Free-electron lasers are devices that efficiently convert the kinetic energy of a relativistic electron beam into the energy of electromagnetic radiation. An electromagnetic wave (called laser or radiation) copropagates with the electron beam, which passes through a static and periodic magnetic field generated by a wiggler. Due the presence of the laser and wiggler fields, the electrons lose velocity, giving their energy to the laser. This single pass FEL, with initially cold beam, in general, is well explored in literature, both in Compton and Raman regimes [3-5].

In a FEL, there is interaction between the electrons and the ponderomotive well (formed through the superposition of the wiggler and laser electromagnetic fields) and among themselves. The last interaction is called space-charge effect.

When the electric charge is small in the system, the ponderomotive well mainly drive the particle dynamics, and the particles are attracted to the bottom of the well. In this case, electric repulsion is weak, and the particles revolve as a whole around themselves in the particle phase-space. This regime is called Compton.

But, when the charge increases, the mixing process in the phase-space become different (Raman regime). Electric repulsion offers resistance against the ponderomotive well, and the process is similar to the case of magnetically focused charged beams [1,6].

The main target of this work is review the paper [7] using more adequate parameters to FEL operation, establishing a threshold between Compton and Raman regimes. We made it through a semi-analytical approach based on the compressibility factor, whose zeroes indicate the onset of mixing in phase-space. This semi-analytical model can provide an estimative of the time and the position in the ponderomotive well for the onset of mixing and the saturated amplitude of the radiation field.

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PHYSICAL MODEL

A complete description of FEL dynamics must include laser, electron phase and energy evolutions and space-charge effects, which occurs due a longitudinal electric field. We start with the laser and wiggler (w) fields. They are described by the respective vector potentials (with $\hat{e} = (\hat{x} + i\,\hat{y})/\sqrt{2}$)

$$\frac{e}{mc^2}\vec{\mathcal{A}}_w(z) = a_w(e^{-i\,k_w z} + c.c.)\,\hat{e}\ , \qquad (1)$$

$$\frac{e}{mc^2}\vec{\mathcal{A}}(z) = -i\left[a(z)e^{i(k\,z-\omega\,t)} - c.c.\right]\hat{e}. \qquad (2)$$

The dimensionless laser amplitude $a(z)$ is a slowly varying function of z. As for the space-charge contribution, to satisfy the periodic boundary conditions, we consider a thin electron beam moving at the center of the pipe. An equivalent physical picture is of a beam propagating along the z axis with two grounded plates located at $y = \pm L/2$. Based on a sheet beam model [8], Poisson equation is solved, demanding 2π periodicity for the variable $\theta = k_p z - \omega t$, where $k_p = k + k_w$ is the ponderomotive wave number and θ is the particle phase in the ponderomotive potential. In the limit of large values of L, the electric field generated at θ by one particle of unitary charge located at θ' can be expressed as the following periodic saw-tooth function, which is the dimensionless Green's function for the electric field:

$$E_z{}^G(\theta,\theta') = sign(\theta-\theta')[\pi - Abs(\theta-\theta')]. \qquad (3)$$

Therefore, the total electric field at particle phase θ (where $\eta^2 = \omega_p{}^2/\omega^2$, and ω_p is the plasma frequency)

$$E_z(\theta) = \eta^2\langle E_z{}^G(\theta,\theta')\rangle. \qquad (4)$$

From Lorentz equation, we write (where $\gamma = [(1+|a_{TOT}|^2)/(1-v_z{}^2)]^{1/2}$ is the relativistic Lorentz factor and v_z is the longitudinal velocity):

$$\frac{d\gamma_j}{dz} = -\frac{a_w}{2\gamma_j}\left(ae^{i\theta_j} + c.c.\right) + v_{zj}v_p E_z(\theta_j). \qquad (5)$$

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It is well known maximum growth rate does occur, in general, for non-resonant beam velocity. Then, we can define a parameter that measures the difference between beam (v_p) and ponderomotive field ($v_p{}'$) velocities. This parameter is called detuning ($\nu = \left(v_p - v_p{}'\right)/v_p$), and it appears in the electron phase, which is written as

$$\frac{d\theta}{dz} = \frac{v_z}{v_p{}'} - 1. \qquad (6)$$

The last equation is obtained solving the wave equation, considering a slowly varying envelope approximation, for the stimulated radiation amplitude, a

$$\frac{d}{dz}a = \eta^2 \langle v_{zi} \rangle \, a_w \left\langle \frac{e^{-i\theta}}{2\gamma} \right\rangle - i\,\eta^2 \langle v_{zi} \rangle \left\langle \frac{1}{2\gamma} \right\rangle a. \qquad (7)$$

These normalized equations (5), (6) and (7) (with $t \to \omega t$, $v \to v/c$ and $v_p \approx k/k_p$) constitute a closed set that completely describes FEL dynamics.

SEMI-ANALYTICAL MODEL

Linear analysis is a proper way to understand parameter regions that led to instability. More than that, linear analysis provides stimulated radiation amplitude growth rate (which is almost linear until the onset of the mixing process). The semi-analytical model considers a linear wave dynamics and introduces the nonlinear particle dynamics by exploring the connection between particle phase and energy.

Starting with linear wave dynamics, it is assumed that initial radiation amplitude is too small, and particle phase and energy, θ and γ, can be written as $\tau = \tau_0 + \delta\tau$. Introducing this linearization in the previous equations, making use of the collective complex variable description developed [9] (using that $X = \langle \delta\theta e^{-i\tilde{\theta}_0} \rangle$, $Y = \langle \delta\gamma e^{-i\tilde{\theta}_0} \rangle$, $\mathcal{D}_a = (\partial v_z / \partial |a_{TOT}|^2)_{z=0}$ and $\mathcal{D}_\gamma = (\partial v_z / \partial \gamma^2)_{z=0}$), and through a change in phase, necessary to satisfy initial equilibrium condition, we build a linear set of equations that describes the laser evolution.

Considering the distribution remains acceptably uniform and the number of particles left of a given particle is almost constant until the onset of the mixing process, some approximations are done and Eqs. (5) and (6) are connected, resulting in a second order ODE for θ, which depends upon the initial particle phase θ_0. Deriving this equation with respect to θ_0 and defining $\partial\theta/\partial\theta_0 \equiv C$ as the compressibility [2], we obtain

$$\frac{d^2}{dz^2}C = -\,i\,C\,a_w \frac{\mathcal{D}_\gamma}{v_p}\left(\tilde{a}\,e^{i\theta} + c.c.\right) + 2 v_p \eta^2 \gamma_r \mathcal{D}_\gamma (1 - C). \qquad (8)$$

Compressibility depends on the z and θ_0. When $C \to 0$, it means that particles located in the vicinity of θ_0 in $z = 0$, overtake each other at time z in the coordinate $\theta(z, \theta_0)$. So, the time and position related to the onset of mixing process are obtained from the initial phase θ_0 which minimizes the time until $C \to 0$, and this is the physical meaning of the compressibility. The present model is not valid after the onset of mixing.

RESULTS

In this chapter, we briefly discuss about some results provided by the semi-analytical model. In Fig. 1, by setting $a_w = 0.5$, $a(z=0) = -i\,a_w\,10^{-5}$ and $v_p = 0.99$ (these parameters are used in whole work, unless a_w for Fig. 2), we look for parameters η and ν that lead to instability and we plot curves that correspond to the maximum growth rate *via* semi-analytical model (yellow line) and the limit of instability (black lines). The colours indicate the time until the onset of mixing *via* full set of equations integration. For a fixed charge value, the detuning value that maximizes the growth rate is the same that minimizes the time elapsed until the onset of mixing.

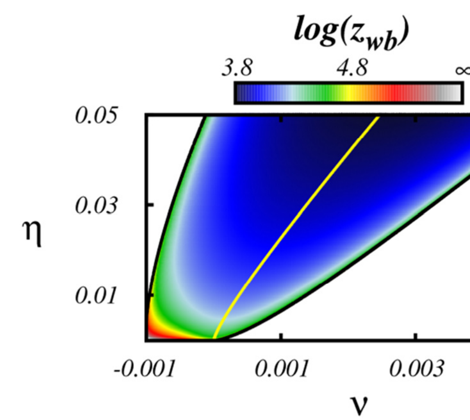

Figure 1: Map of parameter space. White regions represent stability (no laser field growth). Colours indicate the time until the onset of mixing *via* full particle simulation. Yellow line means the detuning which maximizes the growth rate for a specific η *via* linear analysis. Black lines delimit the instability region, through linear analysis.

The curve that maximizes the growth rate is of great importance in the laboratory, because it leads to a reduction of the wiggler length. Thus, our goal is to establish the threshold value of the charge (η) which is responsible for the transition between the regimes over the yellow line. We set as a condition for this transition that if, increasing the charge value, space-charge effects add a full extra cycle oscillation (around the initial value, which is equal to 1) in compressibility evolution, then the regime changes to Raman. Full simulations are compared with model results in Fig. 2, for different a_w values, showing a good agreement.

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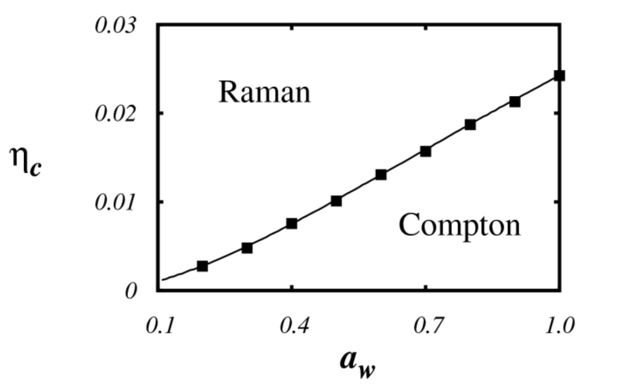

Figure 2: Critical charge η_c for different a_w values. Filled squares represent full set of equations while solid line represents semi-analytical model results. The curve is an interface between Raman (above) and Compton (below) regions.

A way to understand the difference between Compton and Raman mixing processes is to analyse the phase-space configuration *via* full simulations just after the onset of the mixing. In Fig. 3a, in Compton regime ($\eta < \eta_c$), the relaxation proceeds as the particles distribution mostly revolves as whole around itself in phase-space. While in Fig. 3b, where $\eta > \eta_c$ (Raman regime), relaxation is better described in terms of jets of particles emanating from the core of the distribution.

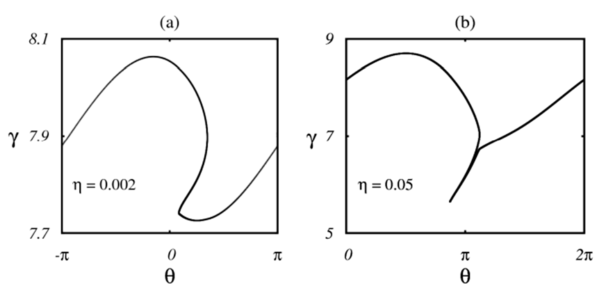

Figure 3: Phase-space snapshots immediately after the onset of mixing. Panel (a) is for Compton regime ($\eta = 0.002$), while panel (b) is for Raman regime ($\eta = 0.05$).

Even though the ponderomotive well is not stationary, the mixing process in FEL is similar to wave-braking in magnetostatically confined beams. Making a comparison, Compton (Raman) regime bear some resemblance with fast (slow) wave-breaking [2].

CONCLUSION

In the present work, a linear analysis was developed, providing the parameters that lead to instability and the laser growth rate. In addition, we applied the compressibility factor in free-electron lasers to build a semi-analytical model capable to delimit Compton and Raman regimes, introducing nonlinearities caused by the electrons distribution. Making use of this delimitation, we compared the phase-space of the system immediately after the onset of mixing for Compton and Raman regimes. There is a significant difference in the form which the mixing occurs and, certainly, the difference is caused due space-charge effects.

Finally, the results given by the model were compared with wave-particle simulations, showing a good agreement. This way, we may consider compressibility a helpful tool to model FEL dynamics.

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