ISBN: 978-3-95450-168-7 doi:10.18429/JACoW-IPAC2015-THPF137

BEAM DYNAMICS EFFECTS OF HIGH ORDER MULTIPOLES IN NON-AXISYMMETRIC SUPERCONDUCTING RF CAVITIES

Z. He*, Y. Zhang, J. Wei, FRIB, Michigan State University, USA

Abstract

Non-axisymmetric superconducting RF cavities have been widely used in accelerator facilities. Because of the geometry, electric and magnetic multipole components, including steering terms, quadrupole terms, and higher order terms, would arise and have potential effects on beam dynamics. In this paper, we start with a simple linac periodic structure to study the effects of higher order terms. The action is defined as a figure of merit to quantify the effects. After that, we move to a more realistic situation of FRIB linac segment 1 (LS1). Multipole terms of quarter wave resonators (QWRs) are firstly calculated using multipole expansion scheme. Then, the scheme is tested using the FRIB linac lattice with QWRs, and the effects of higher order terms on FRIB LS1 are estimated.

INTRODUCTION

The non-axisymmetric geometry of the superconducting RF cavities would produce all kinds of multipole components. In 1992, Marco Cavenago firstly described dipole and quadrupole terms in a non-axisymmetric QWR [1]. Then, Alberto Facco came up with an analytical approach to calculate steering terms in 2001 [2]. Later works [3–5], based on the idea of field multipole expansion, made several attempts to study the cavity multipole components under different scenarios. Even though efforts have been made to extract the higher order terms by multipole expansion, the beam dynamics effects coming from it have never received systematic study. In this paper, we are going to develop a systematic scheme to estimate beam dynamics effects coming from multipole components, and the scheme is then applied to FRIB LS1.

FIGURE OF MERIT

It is well known that the particle action, described in Eq.1, is a constant for a linear lattice with no error.

$$J = \frac{1}{2\beta}[y^2 + (\beta y' + \alpha y)^2]. \tag{1}$$

The transformation between action-angle coordinate system and x-x' phase space coordinate system is decribed in Eq.2. It can be easily found out that, the beam envelope of a beam bunch can be determined by the beta-function and the maximum particle action.

$$\begin{cases} y = \sqrt{2\beta J}\cos\psi \\ y' = -\sqrt{\frac{2J}{\beta}}[\sin\psi + \alpha\cos\psi] \end{cases} \tag{2}$$

Therefore, particle action can be chosen as the figure of merit [6–8].

ACTION KICK FOR SINGLE HIGH ORDER TERMS

Particle action is a constant for perfect linear lattice, however, when a non-linear high order kick occurs, particle action changes. The action kick coming from high order terms can be estimated by Eq.3 [7], where J represents particle action, $B_0\rho$ is the magnetic rigidity. B_{ref} and R_0 are reference magnetic field and radius. n is the multipole order. a_n is multipole coefficient, which can be both normal term or skewed term.

$$\frac{\Delta J_{x,y}}{J_{x,y}} \sim \frac{(2J)^{\frac{n-1}{2}}}{4\pi B_0\rho}\frac{B_{ref}}{R_0^n}\int a_n\beta_{x,y}^{\frac{n+1}{2}} \tag{3}$$

Subsequently, we take the specific example of sextupole to derivate the maximum action kick. Assume the sextupole kick is small, and take horizontal direction as an example, we can get Eq.4:

$$\begin{aligned} \Delta J_x &\sim \sqrt{2\beta_x J_x}\sin\psi_x \cdot g_2(2\beta_x\cos^2\psi_x - 2\beta_y J_y\cos^2\psi_y) \\ &\leq 2\sqrt{2}\beta_x^{\frac{3}{2}}J_x^{\frac{3}{2}}g_2\sin\psi_x\cos^2\psi_x \end{aligned} \tag{4}$$

Furthermore, we test Eq.4 using a simple lattice, which can be seen in Fig. 1(a). A sextupole kick is induced at 4.28 m with strength $g_2 = 0.05$. The red-plus line indicates the predicted maximum action after sextupole kick, which agrees pretty well with particle tracking result. Fig. 1(b) shows the predicted maximum action kick vs sextupole strength. Blue line is the result coming from 10000 particle tracking. The green star is the result coming from the model. We can see that, two results agree with each other pretty well.

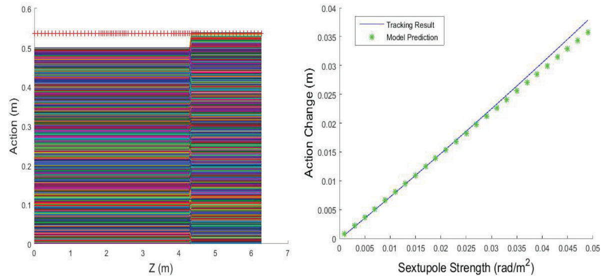
ACTION KICK FOR MULTIPLE HIGH ORDER TERMS

Reference [7] has pointed out that, if phase advance can be neglected, multiple high order terms, after scaled by β-function, can be added up directly. However, in a superconducting linac, higher order multipoles caused by RF cavities would usually separate with each other by a certain phase advance. As a result, high order terms would not cancel out even with same absolute value and opposite sign.

We derive the formulism for multiple high order kicks separated by a non-negligible phase advance. We start with magnetic field and electric field can be treated similarly. We'll still based on the idea of small kick approximation and

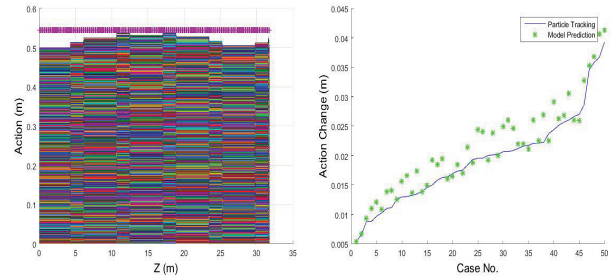
* hez@frib.msu.edu

ISBN: 978-3-95450-168-7 doi:10.18429/JACoW-IPAC2015-THPF137



(a) (b)

Figure 1: (a) Action plot of the beam with single sextupole kick. (b) Maximum action kick vs sextupole strength, comparison between model and particle tracking.



(a) (b)

Figure 2: (a) Action plot of beam with ten sextupole kicks. (b) Maximum action kick calculated by model and particle tracking for different cases.

expand action change to first order of momentum change. The momentum change can be expressed as Eq.5:

$$\begin{cases} \Delta x' = -\frac{1}{B\rho}\int B_y ds \\ \Delta y' = \frac{1}{B\rho}\int B_x ds \end{cases} \tag{5}$$

Where B_x and B_y can be expressed in multipole form as Eq.6:

$$\begin{cases} B_y = \Big[b_n \sum_{k=0,even}^{n} (-1)^{\frac{k}{2}} C_k^n x^{n-k} y^k \\ -a_n \sum_{k=1,odd}^{n} (-1)^{\frac{k-1}{2}} C_k^n x^{n-k} y^k \Big] B_0 \\ B_x = \Big[b_n \sum_{k=0,odd}^{n} (-1)^{\frac{k-1}{2}} C_k^n x^{n-k} y^k \\ +a_n \sum_{k=0,even}^{n} (-1)^{\frac{k}{2}} C_k^n x^{n-k} y^k \Big] B_0 \end{cases} \tag{6}$$

Using action-angle coordinate system and assuming similar β-function and action for horizontal and vertical direction, and assuming slow change of β-function, we then obtain Eq.7 to calculate action change from one of the multiple high order kicks:

$$\frac{\Delta J_{x,y}}{J_{x,y}} = 2^{\frac{n+1}{2}} g_n \beta_{x,y}^{\frac{n+1}{2}} J_{x,y}^{\frac{n-1}{2}} \sin\psi_{x,y} \Phi(\psi_x, \psi_y) \tag{7}$$

Where g_n is the multipole strength defined as $g_n = \mp\frac{B_0 L_{eff}}{B\rho} a_n$ or $g_n = \mp\frac{B_0 L_{eff}}{B\rho} b_n$, minus sign is for horizontal magnetic field case and plus sign is for vertical magnetic field case. $\Phi(\psi_x, \psi_y)$ is a phase factor. We can see that the single multipole kick scaling law is compatible with Eq.3.

The total action kick caused by number of m multipole kicks can be calculated by adding up action kick calculated using Eq.7 at each multipole position. If the lattice we are interested in satisfies certain conditions, we can further simplify the result into Eq.8. The conditions are, beta value at each multipole position is similar; horizontal and vertical β-function and beam emittance are similar.

$$\frac{\Delta J_{tot}}{J_{x,y}} = 2^{\frac{n+1}{2}} \beta_{x,y}^{\frac{n+1}{2}} J_{x,y}^{\frac{n-1}{2}} \sum_{i=1}^{m} g_n^{(i)} \sin\psi_{x,y}^{(i)} \Phi(\psi_x^{(i)}, \psi_y^{(i)}) \tag{8}$$

The total action kick can be maximized by maximizing the sum factor in Eq.8 by choosing proper initial phases $\psi_x^{(1)}$ and $\psi_y^{(1)}$.

The scheme is then tested on a simple lattice. Result of one case of 10 distributed sextupole kicks can be seen in Fig. 2(a). The maximum possible action kick is predicted by model and plotted as the magenta-plus line. As shown in Fig. 2(a), the action tracking result coming from 10000 particles is well-bounded by the model prediction value.

Subsequently, we studied additional cases. The maximum action kick predicted by the model is benchmarked with particle tracking as is shown in Fig. 2(b). X axis is case number, 50 random sextupole kicks sets are generated and maximum action kick is predicted both by model (green star) and and 20000 particle tracking (blue line). We confirm that Eq.8 is a good metric to predict maximum action kick for multiple high order terms.

THE RESONANCE CONDITION

Considering the worst situation when all high order kicks described in Eq.7 have the same sign and the same value, the total action kick is still finite. We can then define the resonance-safe threshold as Eq.9.

$$g_n < \frac{P_{max}}{mC} 2^{-\frac{n+1}{2}} \beta_{x,y}^{-\frac{n+1}{2}} J_{x,y}^{-\frac{n-1}{2}} \tag{9}$$

Where g_n is the threshold multipole strength, P_{max} is a design value determining how much percentage of action growth we are allowing. m is the multipole kick number distributed along the whole linac, C is the maximum value of phase factor $\sin\psi_{x,y}\Phi(\psi_x, \psi_y)$. β and J represent β-function and action respectively.

Eq.9 can be an easy and useful metric to determine if a multipole component can be a threat to beam dynamics or not.

APPLICATION TO FRIB LINAC SEGMENT

FRIB, a new national user facility, funded by the U.S. Department of Energy, is now under construction at MSU [9].

ISBN: 978-3-95450-168-7 doi:10.18429/JACoW-IPAC2015-THPF137

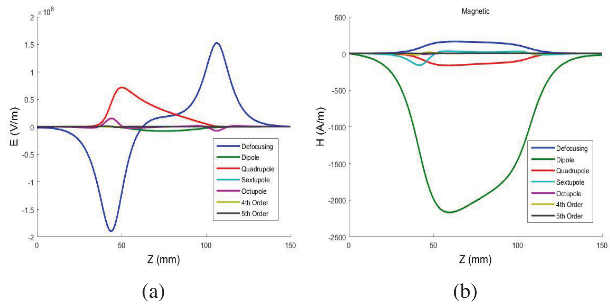


Figure 3: (a) Electric multipole streghth curve. (b) Magnetic multipole streghth curve.

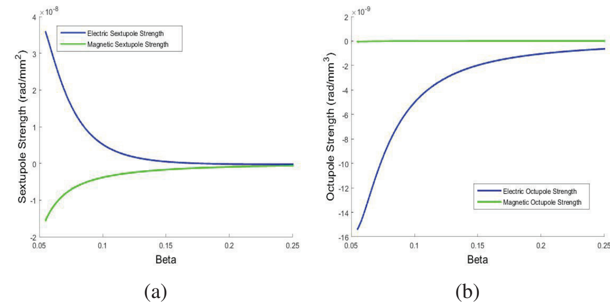


Figure 4: (a) Sextupole strength vs β. (b) Octupole strength vs β.

We choose FRIB linac segment 1 (LS1) as an example and use the above scheme to estimate the high order effects on beam dynamics.

Cavity Field Multipole Expansion

We use the scheme developed in reference [3] to handle this problem. It is usually enough to use traditional multipole components derived from a 2D Laplace equation to do higher order term estimation. We can do the field multipole expansion for each discrete transverse plane of electromagnetic field along longitudinal direction and plot out the multipole strength curve as can be seen in Fig. 3.

Multipole Kick Effects Estimation

To estimate g_n, contribution from both magnetic multipoles and electric multipoles should be added.Because the electromagnetic field is time varying, the time-of-flight effect should be taken into account as is described in reference [3]. For a single cavity, g_n can be estimated using Eq.10:

$$g_n^{tot} = \left[\frac{q}{\gamma\beta^2 E_s} E_0 L_{eff} a_n (T\cos\phi_{0,E} - S\sin\phi_{0,E})\right]_E \mp \left[\frac{1}{B\rho} B_0 L_{eff} b_n ((T\cos\phi_{0,B} - S\sin\phi_{0,B})\right]_B \quad (10)$$

Subscript E means electric component and subscript B means magnetic component. $E_0 L_{eff} a_n$ ($B_0 L_{eff} b_n$) is the integrated multipole electic (magnetic) voltage. T and S are transit time factors. Similar formula can also be derived with skewed electric terms b_n and magnetic terms a_n.

Using the multipole components drawn in Fig.3, we can calculate the sextupole strength and octupole strength of FRIB 0.085 QWR, which can be seen in Fig. 4. After that, we can use Eq. 9 to estimate the level of effects of high order multipoles on the beam. We take the real parameter coming from FRIB LS1. We pick $P_{max} = 0.01$, $m = 88$, $C = 0.3$. The average $\beta_{x,y}$ of LS1 is 1.7 m, the average $J_{x,y}$ of LS1 is 7e-6 m, for sextupole where $n = 2$, the resulting estimation $g_{2,e} = 2e-8(rad/mm^2)$, for octupole where $n = 3$, the resulting estimation $g_{3,e} = 5e-9(rad/mm^3)$. We can see that both $g_{2,esti}$ and $g_{3,esti}$ are at the similar level of calculated g_2 and g_3 plotted in Fig. 4. That means, even if at resonance, when all sextupole and octupole effects are added up, the total action (or emittance) growth would still be around 1% level.

CONCLUSION

A scheme based on action kick to estimate the effects of superconducting RF cavity multipole components on beam dynamics is established. The scheme starts with single multipole kick, and then is extended to multiple high order kicks.Then, an easy scheme of estimating multipole effects using resonance-safe threshold is setup. After that, the scheme developed above is applied to estimate the effects of high order terms on FRIB. Multipole components of 0.085 QWR are extracted. The resonance-safe threshold for sextupole and octupole of FRIB LS1 are estimated and it is confirmed that even at worst case when all sextupole and octupole effects are added up, the total action (or emittance) growth would around 1% level.

ACKNOWLEDGMENT

The work is supported by the U.S. National Science Foundation under Grant No. PHY-11-02511, and the U.S. Department of Energy Office of Science under Cooperative Agreement DE-SC0000661.

REFERENCES

[1] Marco Cavenago. *NIMA*, 311(1):19–29, 1992.

[2] Alberto Facco et al. *PRSTAB*, 14(7):070101, 2011.

[3] Z He et al. In *Proc. of LINAC*, volume 12, 2012.

[4] RG Olave et al. *Proc. LINAC12, Israel, MOPB072*, 2012.

[5] SU De Silva et al. *PRSTAB*, 16(1):012004, 2013.

[6] J Wei. Error compensation in insertion-region magnets. *Part. Accel.*, 55:193–202, 1996.

[7] J Wei et al. Interaction region analysis for a high-field hadron collider. *New Directions for High-Energy Physics*, page 351, 1996.

[8] J Wei et al. Us-lhc ir magnet error analysis and compensation. In *Proc. of EPAC*, pages 380–382, 1998.

[9] Jie Wei et al. *FRYBA3, NA-PAC*, 13, 2013.