# CHARACTERIZATION OF VISIBLE SYNCHROTRON RADIATION POLARIZATION AT SPEAR3* 

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## Abstract

Schwinger's equations predict the angular- and spectral distribution of synchrotron radiation across a wide band of the electromagnetic spectrum. Using a visible-light diagnostic beam line, it is possible to characterize the electric field polarization state as a function of vertical observation angle and compare with theory. Complications include accounting for $\sigma$ - and $\pi$-mode transmission factors at mirror surfaces and precise alignment of the polarizing optics with the principle beam axes. The Stokes parameters are measured and beam polarization ellipse reported.

## INTRODUCTION

Synchrotron radiation has both partially-coherent and fully polarized electromagnetic field properties, each used for a wide range of scientific research. With the advent of high-power, short-pulse free-electron lasers, applications depending on the spatial beam coherence are increasing at a rapid rate. Similarly elliptically polarized x-rays have become an increasingly powerful tool to study magnetic dichroism and chirality at both storage rings and FEL facilities. At SPEAR3, a visible-light diagnostic beam line originally constructed to study transverse and longitudinal electron beam dynamics has recently been used to study both the transverse spatial coherence [1] and polarization state of the beam [2].

The polarization state can be characterized by measuring beam power transmitted through linear polarizing elements oriented at systematic angles with respect to the beam axis. In this way is it possible to obtain 'slice' measurements of the beam polarization ellipse which can be combined to concisely express the beam polarization state in terms of the Stokes parameters $[3,4]$. Using the visible SR component, polarization measurements can be made with conventional optical elements in a relatively straightforward manner.

To cross-check the measurements it is also possible to calculate the Stokes parameters and corresponding beam polarization ellipse using Schwinger's equations for the SR field [5]. The 'classical' Schwinger equations express $\delta$ the electric field in terms of both radiation frequency and, Conveniently, vertical observation angle. Hence it is possible to both measure and calculate the SR polarization state in the visible light regime as a function of vertical observation angle.
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## SPEAR3 DIAGNOSTIC BEAM LINE

The visible-light diagnostic beam line at SPEAR3 contains a horizontal 'cold finger' to block the on-axis hard x-ray beam followed by a rhodium-coated mirror to horizontally reflect the SR light down the beam line [6]. The cold finger intercepts $\pm 0.6 \mathrm{mrad}$ of the beam at the midplane and the remaining beam line apertures have


Figure 1: Visible light extraction mirror. Note $E$-field attenuation and relative phase shift upon reflection.
an acceptance of $3.5 \mathrm{mrad} \times 6 \mathrm{mrad}$. For measurements reported here only $\pm 2.2 \mathrm{mrad}$ was available in the vertical plane. As indicated in Fig. 1, the dipole-radiation SR beam hits the Rh mirror at a shallow angle of $9^{\circ}$ causing attenuation and phase shift of the $\sigma$-mode (horizontal) and $\pi$-mode (vertical) visible light components.
For beam polarization measurements, we constructed a remote-controlled system with rotatable polarizer, 532 nm bandpass filter ( BP ) and power meter all on a continuousscan vertical stage [7]. An insertable quarter waveplate (QWP) shown in Fig. 2 is optically matched to the bandpass filter and used to determine beam helicity for Stokes parameter measurements $[3,4]$.


Figure 2: Stokes parameter measurement apparatus.

## SR FIELD REPRESENTATION

The well-known Schwinger equations for synchrotron radiation express the SR radiation field in terms angular frequency $\omega$ and vertical emission angle $\psi^{\dagger}$ [5]
$E_{x 0} \triangleq \tilde{E}_{\sigma}(\omega, \psi)=C_{0} K_{2 / 3}(\omega, \psi)$
$E_{y 0} \triangleq \tilde{E}_{\pi}(\omega, \psi)=C_{0} \frac{-i \gamma \psi}{\sqrt{1+\gamma^{2} \psi^{2}}} K_{1 / 3}(\omega, \psi)$.
where $\mathrm{C}_{0}$ is a function of $\omega$ and $\psi, \gamma$ is the electron beam Lorentz factor and $K$ are modified Bessel functions.

[^0]The vertical field amplitude at frequency $\omega, E_{y 0}$, is an odd function of $\psi, 90^{\circ}$ out of phase with respect to $E_{x 0}$ and the light is fully (albeit elliptically) polarized. For visible light, the characteristic opening angle $\psi_{r m s}$ is a few mrad. Field amplitude nomenclature $E_{x 0}$ and $E_{y 0}$ is used below.


Figure 3 : Elliptically polarized light.

## The Polarization Ellipse

To simplify the analysis we adopt a plane wave model for the SR beam. The time-dependent electric field components resolved into the $x-y$ plane are then

$$
\begin{align*}
& E_{x}=E_{x 0} e^{-i(\omega t-k z+\delta)}  \tag{2a}\\
& E_{y}=E_{y 0} e^{-i(\omega t-k z)} \tag{2b}
\end{align*}
$$

where $E_{x 0}$ and $E_{y 0}$ are the time-averaged field amplitudes evaluated at frequency $\omega$. Eliminating the propagator term $\omega t-k z$ yields an equation for the beam polarization ellipse

$$
\begin{equation*}
\frac{E_{x}^{2}}{E_{x 0}^{2}}-2 \frac{E_{x} E_{y}}{E_{x 0} E_{y 0}} \cos (\delta)+\frac{E_{y}^{2}}{E_{y 0}^{2}}=\sin ^{2}(\delta) \tag{3}
\end{equation*}
$$

where $\delta$ is the relative phase between $E_{x}$ and $E_{y}$. For a general plane wave, the field amplitudes $E_{x 0}$ and $E_{y 0}$ are not equal, the phase term $\delta$ can take on any value and the radiation is elliptically polarized (Fig. 3). For synchrotron radiation, $E_{x 0}$ and $E_{y 0}$ vary with vertical observation angle as per Eq. 1 and the relative phase $\delta$ is $90^{\circ}$ so that $\cos (\delta)=0$ and the polarization ellipse is upright.

## Stokes Parameters

Similar to applications in x-ray SR science, physical measurements of the radiation field result in loss of phase information. In order to reconstruct the beam polarization ellipse and hence characterize the beam polarization state, both $E_{x 0}$ and $E_{y 0}$, and the relative phase shift $\delta$ between $E_{x}$ and $E_{y}$ are required. The information can be extracted by measuring the Stokes parameters using the apparatus shown in Fig. 2. The Stokes parameters can be defined as
$S_{0}=E_{x 0}^{2}+E_{y 0}^{2}=I_{0^{o}}+I_{90^{\circ}}$
$S_{1}=E_{x 0}^{2}-E_{y 0}^{2}=I_{0} o-I_{90^{\circ}}$
$S_{2}=2 E_{x 0} E_{y 0} \cos (\delta)=I_{45^{\circ}}-I_{135^{\circ}}$
$S_{3}=2 E_{x 0} E_{y 0} \sin (\delta)=I_{45^{\circ}}^{Q W P}-I_{135^{\circ}}^{Q W P}$
where $I_{0^{o}}, I_{90^{o}}, I_{45^{\circ}}$ and $I_{135^{\circ}}$ are beam intensity measurements with the polarizer oriented along the
indicated axis and the superscript ' $Q W P$ ' indicates a quarter waveplate inserted. A schematic of the intensity measurement process is shown in Fig. 4. Measurement of $I_{0^{\circ}}$ and $I_{90^{\circ}}$ yield the $E_{x 0}$ and $E_{y 0}$ field amplitudes while measurement of $S_{2}$ yields the relative phase $\delta$.


Figure 4: Beam polarization ellipse indicating polarizer orientations for Stokes' parameter measurements. $\theta$ is the ellipse rotation angle.

The QWP with fast axis vertical introduces a $\pi / 2$ phase shift between the $x$ and $y$ field components to provide information on the polarization helicity $\left(S_{3}\right)$. Referring to Eq. 3, the QWP reverses the sign of the cross-correlation term thereby flipping the polarization ellipse about the x axis.

The process of determining the Stokes parameters provides a complete description of the beam polarization state and can be used for example to calculate the ellipse rotation angle,

$$
\begin{equation*}
\tan (2 \theta)=\frac{2 E_{x 0} E_{y 0} \cos (\delta)}{E_{x 0}^{2}-E_{y 0}^{2}}=\frac{s_{2}}{S_{1}} \tag{5}
\end{equation*}
$$

## EXPERIMENTAL RESULTS

Figure 5 shows the measured beam intensities $E_{x 0}^{2}$ and $E_{y 0}^{2}$ as a function of vertical elevation angle with the beam polarizer oriented in the $x$ - and $y$-directions, respectively. The shadow of the cold finger and associated edge diffraction effects are clear.


Figure 5: Horizontal (SR $\sigma$-mode) and vertical (SR $\pi$-mode) field intensities as a function of vertical elevation angle.
The theoretical intensity values calculated from squares of Eq. 1 are superimposed. Of significance, a scaling factor of 0.3 has been applied to the calculated value for $E_{x 0}^{2}$ to match the measured data. We attribute this factor to differences in reflectivity for the horizontal and vertical electric field components at the rhodium and subsequent metal mirror surfaces. Using the $E_{y 0}^{2}$ curve as reference, the un-scaled theoretical curve for $E_{x 0}^{2}$ is shown as a dashed line.

Figure 6: Field intensities measured with polarizer oriented $45^{\circ}$ and $135^{\circ}$ w.r.t. horizontal axis.
Similarly Fig. 6 shows the measured beam intensity data with the polarizer oriented $45^{\circ}$ and $135^{\circ}$ with respect to the horizontal axis. Using calculated $E_{x 0}$ and $E_{y 0}$ field amplitudes from Fig. $5(\mid$ field $\mid=\sqrt{\text { intensity }})$ and the same mirror reflectivity factor ( $\sqrt{0.3}$ for field), Eqs. 1 can be projected into the $45^{\circ}$ and $135^{\circ}$ axes and again squared to compare theoretical intensity values with measurement. The resulting intensity curves are superimposed on the measured data in Fig. 6. The slight imbalance above and below the midplane may be due to an electron beam pointing error.
To match the measurements in Fig. 6, the relative phase factor $\delta$ was adjusted from $90^{\circ}$ to $58^{\circ}$. The phase difference is agaion attributed to different phase shifts for the horizontal and vertical field components upon reflection at the mirror surfaces. The theoretical curve (no factor of 0.3 and $90^{\circ}$ phase) is plotted in Fig. 6.
Following similar steps we found the calculated Stokes parameters agree well with the measured Stokes parameters over the observable range of vertical elevation angles, including data taken with the phase-shifting quarter waveplate $\left(S_{3}\right)$. Using the model expressions for field amplitude


Figure 7: Measured SR beam polarization ellipse as a function of vertical elevation angle (solid lines). Polarization ellipse with quadrature phase and relative intensity factor 0.3 removed (dashed). and relative phase $\delta$, we can therefore plot the SR beam polarization ellipse as a function of elevation angle. The result is shown in Fig. 7 with solid curves representing measured data. The effect of sign reversal in the vertical $\mathrm{SR} \pi$-mode $E$-field component at the horizontal midplane is clear. For the 'ideal' SR beam, the relative phase $\delta=90^{\circ}$ so the ellipses are upright.
Of interest, the polarization ellipse rotation angle $\theta$ can be calculated from knowledge of the field amplitudes and
phase or from the Stokes parameters using


Figure 8: Polarization ellipse rotation angle as a function of vertical elevation angle.
Eq. 5. As seen in Fig. 8 the measurement agrees well with theory. For unperturbed synchrotron radiation (no mirrors) the rotation angle is zero.

## CONCLUSIONS

Taking into account both amplitude and phase shift of the visible SR beam during transport through the SPEAR3 diagnostic beam line, the measured SR beam polarization state is found to be in good agreement with theory. In particular the characteristic shape of both the $\sigma$ and $\pi$-mode beam intensity profiles measured along the vertical axis agree with calculated values at 532 nm . The agreement indicates that Schwinger's classical-field SR equations are valid in the visible portion of the spectrum and that the measurement apparatus is reliable. Based on this information it is possible to measure and model the Stokes parameters and beam polarization ellipse.

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[^0]:    $\dagger$ standard frequency and angle normalizations implied

